APPENDIX. Proof of Exercise 60.

In[ ]:= `proofEx60["ProofNotebook"]`

Axiom 1

We are given that:

`∀x (Dnew[x]⇒A[x])`

Axiom 2

We are given that:

`∀x (!R[x]⇒Dnew[x])`

Axiom 3

We are given that:

`∀x (!Enew[x]⇒!R[x])`

Axiom 4

We are given that:

`∀x (!Cnew[x]⇒!Enew[x])`

Axiom 5

We are given that:

`∀x (!K[x]⇒!Cnew[x])`

Axiom 6

We are given that:

`∀x (!B[x]⇒!K[x])`

Axiom 7

We are given that:

`∀x (!M[x]⇒!B[x])`

Axiom 8

We are given that:

`∀x (!L[x]⇒!M[x])`

Axiom 9

We are given that:

`∀x (!Nnew[x]⇒!L[x])`

Axiom 10

We are given that:

`∀x (H[x]⇒!Nnew[x])`

Hypothesis 1

We would like to show that:

```
∀x (H[x]⇒A[x])
```

Equationalized Axiom 1

We generate the "equationalized" axiom:

```
x1==(x1||(x2&&!x2))
```

Equationalized Axiom 2

We generate the "equationalized" axiom:

```
x1==(x1&&(x2||!x2))
```

Equationalized Axiom 3

We generate the "equationalized" axiom:

```
(x1||x2)==(x2||x1)
```

Equationalized Axiom 4

We generate the "equationalized" axiom:

```
(x1||(x2&&x3))==((x1||x2)&&(x1||x3))
```

Equationalized Axiom 5

We generate the "equationalized" axiom:

```
(R[x1]||Dnew[x1])==(a0||!a0)
```

Equationalized Axiom 6

We generate the "equationalized" axiom:

```
(Nnew[x1]||!L[x1])==(a0||!a0)
```

Equationalized Axiom 7

We generate the "equationalized" axiom:

```
(M[x1]||!B[x1])==(a0||!a0)
```

Equationalized Axiom 8

We generate the "equationalized" axiom:

```
(L[x1]||!M[x1])==(a0||!a0)
```

Equationalized Axiom 9

We generate the "equationalized" axiom:

```
(K[x1]||!Cnew[x1])==(a0||!a0)
```

Equationalized Axiom 10

We generate the "equationalized" axiom:

```
(Enew[x1]||!R[x1])==(a0||!a0)
```

Equationalized Axiom 11

We generate the "equationalized" axiom:

```
(Cnew[x1]||!Enew[x1])==(a0||!a0)
```

Equationalized Axiom 12

We generate the "equationalized" axiom:

```
(B[x1]||!K[x1])==(a0||!a0)
```

Equationalized Axiom 13

We generate the "equationalized" axiom:

```
(!Dnew[x1]||A[x1])==(a0||!a0)
```

Equationalized Axiom 14

We generate the "equationalized" axiom:

```
(!H[x1]||!Nnew[x1])==(a0||!a0)
```

Equationalized Axiom 15

We generate the "equationalized" axiom:

```
((x1&&x2)||(x1&&x3))==(x1&&(x2||x3))
```

Equationalized Axiom 16

We generate the "equationalized" axiom:

```
(x1&&x2)==(x2&&x1)
```

Equationalized Hypothesis 1

We generate the "equationalized" hypothesis:

```
(!H[x0]||A[x0])==(a0||!a0)
```

Critical Pair Lemma 1

The following expressions are equivalent:

```
((x1&&!x1)||x2)==x2
```

Proof

Note that the input for the rule:

```
x1_||x2_↔x2_||x1_
```

contains a subpattern of the form:

```
x1_||x2_
```

which can be unified with the input for the rule:

```
x1_||(x2_&&!x2_)→x1
```

where these rules follow from Equationalized Axiom 3 and Equationalized Axiom 1 respectively.

Critical Pair Lemma 2

The following expressions are equivalent:

```
(x1||(x2&&!x1))==(x1||x2)
```

Proof

Note that the input for the rule:

`(x1_||x2_)&&(x1_||x3_)→x1||(x2&&x3)`

contains a subpattern of the form:

`(x1_||x2_)&&(x1_||x3_)`

which can be unified with the input for the rule:

`x1_&&(x2_||!x2_)→x1`

where these rules follow from Equationalized Axiom 4 and Equationalized Axiom 2 respectively.

Critical Pair Lemma 3

The following expressions are equivalent:

`(x1||(x2&&x3&&!x3))==((x1||x2)&&x1)`

Proof

Note that the input for the rule:

`(x1_||x2_)&&(x1_||x3_)→x1||(x2&&x3)`

contains a subpattern of the form:

`x1_||x3_`

which can be unified with the input for the rule:

`x1_||(x2_&&!x2_)→x1`

where these rules follow from Equationalized Axiom 4 and Equationalized Axiom 1 respectively.

Substitution Lemma 1

It can be shown that:

`(Dnew[x1]||R[x1])==(a0||!a0)`

Proof

We start by taking Equationalized Axiom 5, and apply the substitution:

`x1_||x2_→x2||x1`

which follows from Equationalized Axiom 3.

Substitution Lemma 2

It can be shown that:

`(!L[x1]||Nnew[x1])==(a0||!a0)`

Proof

We start by taking Equationalized Axiom 6, and apply the substitution:

`x1_||x2_→x2||x1`

which follows from Equationalized Axiom 3.

Substitution Lemma 3

It can be shown that:

`(!L[x1]||Nnew[x1])==(Dnew[x0]||R[x0])`

Proof

We start by taking Substitution Lemma 2, and apply the substitution:

```
a0||!a0→Dnew[x0]||R[x0]
```

which follows from Substitution Lemma 1.

Substitution Lemma 4

It can be shown that:

```
(!B[x1]||M[x1]) == (a0||!a0)
```

Proof

We start by taking Equationalized Axiom 7, and apply the substitution:

```
x1_||x2_→x2||x1
```

which follows from Equationalized Axiom 3.

Substitution Lemma 5

It can be shown that:

```
(!B[x1]||M[x1]) == (Dnew[x0]||R[x0])
```

Proof

We start by taking Substitution Lemma 4, and apply the substitution:

```
a0||!a0→Dnew[x0]||R[x0]
```

which follows from Substitution Lemma 1.

Substitution Lemma 6

It can be shown that:

```
(!M[x1]||L[x1]) == (a0||!a0)
```

Proof

We start by taking Equationalized Axiom 8, and apply the substitution:

```
x1_||x2_→x2||x1
```

which follows from Equationalized Axiom 3.

Substitution Lemma 7

It can be shown that:

```
(!M[x1]||L[x1]) == (Dnew[x0]||R[x0])
```

Proof

We start by taking Substitution Lemma 6, and apply the substitution:

```
a0||!a0→Dnew[x0]||R[x0]
```

which follows from Substitution Lemma 1.

Substitution Lemma 8

It can be shown that:

```
(!Cnew[x1]||K[x1]) == (a0||!a0)
```

Proof

We start by taking Equationalized Axiom 9, and apply the substitution:

x1_||x2_→x2||x1

which follows from Equationalized Axiom 3.

Substitution Lemma 9

It can be shown that:

(!Cnew[x1]||K[x1]) == (Dnew[x_0]||R[x_0])

Proof

We start by taking Substitution Lemma 8, and apply the substitution:

a_0||!a_0→Dnew[x_0]||R[x_0]

which follows from Substitution Lemma 1.

Substitution Lemma 10

It can be shown that:

(!R[x1]||Enew[x1]) == (a_0||!a_0)

Proof

We start by taking Equationalized Axiom 10, and apply the substitution:

x1_||x2_→x2||x1

which follows from Equationalized Axiom 3.

Substitution Lemma 11

It can be shown that:

(!R[x1]||Enew[x1]) == (Dnew[x_0]||R[x_0])

Proof

We start by taking Substitution Lemma 10, and apply the substitution:

a_0||!a_0→Dnew[x_0]||R[x_0]

which follows from Substitution Lemma 1.

Substitution Lemma 12

It can be shown that:

(!Enew[x1]||Cnew[x1]) == (a_0||!a_0)

Proof

We start by taking Equationalized Axiom 11, and apply the substitution:

x1_||x2_→x2||x1

which follows from Equationalized Axiom 3.

Substitution Lemma 13

It can be shown that:

(!Enew[x1]||Cnew[x1]) == (Dnew[x_0]||R[x_0])

Proof

We start by taking Substitution Lemma 12, and apply the substitution:

$a_0 || !a_0 \to Dnew[x_0] || R[x_0]$

which follows from Substitution Lemma 1.

Substitution Lemma 14

It can be shown that:

$(!K[x1] || B[x1]) == (a_0 || !a_0)$

Proof

We start by taking Equationalized Axiom 12, and apply the substitution:

`x1_||x2_→x2||x1`

which follows from Equationalized Axiom 3.

Substitution Lemma 15

It can be shown that:

$(!K[x1] || B[x1]) == (Dnew[x_0] || R[x_0])$

Proof

We start by taking Substitution Lemma 14, and apply the substitution:

$a_0 || !a_0 \to Dnew[x_0] || R[x_0]$

which follows from Substitution Lemma 1.

Substitution Lemma 16

It can be shown that:

$(A[x1] || !Dnew[x1]) == (a_0 || !a_0)$

Proof

We start by taking Equationalized Axiom 13, and apply the substitution:

`x1_||x2_→x2||x1`

which follows from Equationalized Axiom 3.

Substitution Lemma 17

It can be shown that:

$(A[x1] || !Dnew[x1]) == (Dnew[x_0] || R[x_0])$

Proof

We start by taking Substitution Lemma 16, and apply the substitution:

$a_0 || !a_0 \to Dnew[x_0] || R[x_0]$

which follows from Substitution Lemma 1.

Substitution Lemma 18

It can be shown that:

$!L[x1\_] || Nnew[x1\_] \to A[x_0] || !Dnew[x_0]$

Proof

We start by taking Substitution Lemma 3, and apply the substitution:

```
Dnew[x0]||R[x0]→A[x0]||!Dnew[x0]
```

which follows from Substitution Lemma 17.

Substitution Lemma 19

It can be shown that:

```
!K[x1_]||B[x1_]→A[x0]||!Dnew[x0]
```

Proof

We start by taking Substitution Lemma 15, and apply the substitution:

```
Dnew[x0]||R[x0]→A[x0]||!Dnew[x0]
```

which follows from Substitution Lemma 17.

Substitution Lemma 20

It can be shown that:

```
!Enew[x1_]||Cnew[x1_]→A[x0]||!Dnew[x0]
```

Proof

We start by taking Substitution Lemma 13, and apply the substitution:

```
Dnew[x0]||R[x0]→A[x0]||!Dnew[x0]
```

which follows from Substitution Lemma 17.

Substitution Lemma 21

It can be shown that:

```
!R[x1_]||Enew[x1_]→A[x0]||!Dnew[x0]
```

Proof

We start by taking Substitution Lemma 11, and apply the substitution:

```
Dnew[x0]||R[x0]→A[x0]||!Dnew[x0]
```

which follows from Substitution Lemma 17.

Substitution Lemma 22

It can be shown that:

```
!Cnew[x1_]||K[x1_]→A[x0]||!Dnew[x0]
```

Proof

We start by taking Substitution Lemma 9, and apply the substitution:

```
Dnew[x0]||R[x0]→A[x0]||!Dnew[x0]
```

which follows from Substitution Lemma 17.

Substitution Lemma 23

It can be shown that:

```
!M[x1_]||L[x1_]→A[x0]||!Dnew[x0]
```

Proof

We start by taking Substitution Lemma 7, and apply the substitution:

We start by taking Substitution Lemma 7, and apply the substitution:

```
Dnew[x0]||R[x0]→A[x0]||!Dnew[x0]
```

which follows from Substitution Lemma 17.

Substitution Lemma 24

It can be shown that:

```
!B[x1_]||M[x1_]→A[x0]||!Dnew[x0]
```

Proof

We start by taking Substitution Lemma 5, and apply the substitution:

```
Dnew[x0]||R[x0]→A[x0]||!Dnew[x0]
```

which follows from Substitution Lemma 17.

Substitution Lemma 25

It can be shown that:

```
(!H[x1]||!Nnew[x1])==(Dnew[x0]||R[x0])
```

Proof

We start by taking Equationalized Axiom 14, and apply the substitution:

```
a0||!a0→Dnew[x0]||R[x0]
```

which follows from Substitution Lemma 1.

Substitution Lemma 26

It can be shown that:

```
(!H[x1]||!Nnew[x1])==(A[x0]||!Dnew[x0])
```

Proof

We start by taking Substitution Lemma 25, and apply the substitution:

```
Dnew[x0]||R[x0]→A[x0]||!Dnew[x0]
```

which follows from Substitution Lemma 17.

Critical Pair Lemma 4

The following expressions are equivalent:

```
(!H[x1]||(!Nnew[x1]&&x2))==((A[x0]||!Dnew[x0])&&(!H[x1]||x2))
```

Proof

Note that the input for the rule:

```
(x1_||x2_)&&(x1_||x3_)→x1||(x2&&x3)
```

contains a subpattern of the form:

```
x1_||x2_
```

which can be unified with the input for the rule:

```
!H[x1_]||!Nnew[x1_]→A[x0]||!Dnew[x0]
```

where these rules follow from Equationalized Axiom 4 and Substitution Lemma 26 respectively.

Critical Pair Lemma 5

The following expressions are equivalent:

`(x1&&(x2||!x1))==(x1&&x2)`

PROOF

Note that the input for the rule:

`(x1_&&x2_)||(x1_&&x3_)→x1&&(x2||x3)`

contains a subpattern of the form:

`(x1_&&x2_)||(x1_&&x3_)`

which can be unified with the input for the rule:

`x1_||(x2_&&!x2_)→x1`

where these rules follow from Equationalized Axiom 15 and Equationalized Axiom 1 respectively.

Critical Pair Lemma 6

The following expressions are equivalent:

`((x1||!x1)&&x2)==x2`

PROOF

Note that the input for the rule:

`x1_&&x2_↔x2_&&x1_`

contains a subpattern of the form:

`x1_&&x2_`

which can be unified with the input for the rule:

`x1_&&(x2_||!x2_)→x1`

where these rules follow from Equationalized Axiom 16 and Equationalized Axiom 2 respectively.

Critical Pair Lemma 7

The following expressions are equivalent:

`(x1&&x2)==(x1&&(!x1||x2))`

PROOF

Note that the input for the rule:

`(x1_&&!x1_)||x2_→x2`

contains a subpattern of the form:

`(x1_&&!x1_)||x2_`

which can be unified with the input for the rule:

`(x1_&&x2_)||(x1_&&x3_)→x1&&(x2||x3)`

where these rules follow from Critical Pair Lemma 1 and Equationalized Axiom 15 respectively.

Critical Pair Lemma 8

The following expressions are equivalent:

`(x1||x2)==(x1||(!x1&&x2))`

PROOF

Note that the input for the rule:

```
(x1_||!x1_)&&x2_→x2
```
contains a subpattern of the form:
```
(x1_||!x1_)&&x2_
```
which can be unified with the input for the rule:
```
(x1_||x2_)&&(x1_||x3_)→x1||(x2&&x3)
```
where these rules follow from Critical Pair Lemma 6 and Equationalized Axiom 4 respectively.

Critical Pair Lemma 9

The following expressions are equivalent:
```
(x1||!x1)==(x2||!x2)
```
Proof

Note that the input for the rule:
```
(x1_||!x1_)&&x2_→x2
```
contains a subpattern of the form:
```
(x1_||!x1_)&&x2_
```
which can be unified with the input for the rule:
```
x1_&&(x2_||!x2_)→x1
```
where these rules follow from Critical Pair Lemma 6 and Equationalized Axiom 2 respectively.

Critical Pair Lemma 10

The following expressions are equivalent:
```
(x1||x1)==x1
```
Proof

Note that the input for the rule:
```
x1_||(x2_&&!x1_)→x1||x2
```
contains a subpattern of the form:
```
x1_||(x2_&&!x1_)
```
which can be unified with the input for the rule:
```
x1_||(x2_&&!x2_)→x1
```
where these rules follow from Critical Pair Lemma 2 and Equationalized Axiom 1 respectively.

Critical Pair Lemma 11

The following expressions are equivalent:
```
(x1||(x1&&x2))==(x1&&(x1||x2))
```
Proof

Note that the input for the rule:
```
(x1_||x2_)&&(x1_||x3_)→x1||(x2&&x3)
```
contains a subpattern of the form:
```
x1_||x2_
```
which can be unified with the input for the rule:
```
x1_||x1_→x1
```

where these rules follow from Equationalized Axiom 4 and Critical Pair Lemma 10 respectively.

Critical Pair Lemma 12

The following expressions are equivalent:

`(x1||(x2&&x1))==((x1||x2)&&x1)`

Proof

Note that the input for the rule:

`(x1_||x2_)&&(x1_||x3_)→x1||(x2&&x3)`

contains a subpattern of the form:

`x1_||x3_`

which can be unified with the input for the rule:

`x1_||x1_→x1`

where these rules follow from Equationalized Axiom 4 and Critical Pair Lemma 10 respectively.

Substitution Lemma 27

It can be shown that:

`(x1||(x2&&x1))==(x1&&(x1||x2))`

Proof

We start by taking Critical Pair Lemma 12, and apply the substitution:

`x1_&&x2_→x2&&x1`

which follows from Equationalized Axiom 16.

Critical Pair Lemma 13

The following expressions are equivalent:

`(!R[x1]&&Dnew[x1])==(!R[x1]&&(a0||!a0))`

Proof

Note that the input for the rule:

`x1_&&(x2_||!x1_)→x1&&x2`

contains a subpattern of the form:

`x2_||!x1_`

which can be unified with the input for the rule:

`"0"`

where these rules follow from Critical Pair Lemma 5 and Substitution Lemma 1 respectively.

Substitution Lemma 28

It can be shown that:

`(!R[x1]&&Dnew[x1])==!R[x1]`

Proof

We start by taking Critical Pair Lemma 13, and apply the substitution:

`x1_&&(x2_||!x2_)→x1`

which follows from Equationalized Axiom 2.

Critical Pair Lemma 14

The following expressions are equivalent:

```
(x1&&x1)==x1
```

Proof

Note that the input for the rule:

```
x1_&&(!x1_||x2_)→x1&&x2
```

contains a subpattern of the form:

```
x1_&&(!x1_||x2_)
```

which can be unified with the input for the rule:

```
x1_&&(x2_||!x2_)→x1
```

where these rules follow from Critical Pair Lemma 7 and Equationalized Axiom 2 respectively.

Critical Pair Lemma 15

The following expressions are equivalent:

```
True
```

Proof

Note that the input for the rule:

```
x1_||(!x1_&&x2_)→x1||x2
```

contains a subpattern of the form:

```
!x1_&&x2_
```

which can be unified with the input for the rule:

```
x1_&&x1_→x1
```

where these rules follow from Critical Pair Lemma 8 and Critical Pair Lemma 14 respectively.

Substitution Lemma 29

It can be shown that:

```
(Dnew[x1]&&!R[x1])==!R[x1]
```

Proof

We start by taking Substitution Lemma 28, and apply the substitution:

```
x1_&&x2_→x2&&x1
```

which follows from Equationalized Axiom 16.

Critical Pair Lemma 16

The following expressions are equivalent:

```
(R[x1]||Dnew[x1])==(R[x1]||!R[x1])
```

Proof

Note that the input for the rule:

```
x1_||(x2_&&!x1_)→x1||x2
```

contains a subpattern of the form:

contains a subpattern of the form:

`x2_&&!x1_`

which can be unified with the input for the rule:

`Dnew[x1_]&&!R[x1_]→!R[x1]`

where these rules follow from Critical Pair Lemma 2 and Substitution Lemma 29 respectively.

Critical Pair Lemma 17

The following expressions are equivalent:

`(x1&&x1)==(x1&&(x1||!x1))`

Proof

Note that the input for the rule:

`x1_&&(x2_||!x1_)→x1&&x2`

contains a subpattern of the form:

`x2_||!x1_`

which can be unified with the input for the rule:

`x1_||!x1_→x1||!x1`

where these rules follow from Critical Pair Lemma 5 and Critical Pair Lemma 15 respectively.

Substitution Lemma 30

It can be shown that:

`(x1&&x1)==x1`

Proof

We start by taking Critical Pair Lemma 17, and apply the substitution:

`x1_&&(x2_||!x2_)→x1`

which follows from Equationalized Axiom 2.

Substitution Lemma 31

It can be shown that:

`(x1&&x1)==x1`

Proof

We start by taking Substitution Lemma 30, and apply the substitution:

`x1_&&x2_→x2&&x1`

which follows from Equationalized Axiom 16.

Substitution Lemma 32

It can be shown that:

`True`

Proof

We start by taking Substitution Lemma 31, and apply the substitution:

`x1_&&x1_→x1`

which follows from Critical Pair Lemma 14.

Critical Pair Lemma 18

The following expressions are equivalent:

`(!x1||x2)==(!x1||(x1&&x2))`

Proof

Note that the input for the rule:

`x1_||(!x1_&&x2_)→x1||x2`

contains a subpattern of the form:

`!x1_`

which can be unified with the input for the rule:

`x1_→x1`

where these rules follow from Critical Pair Lemma 8 and Substitution Lemma 32 respectively.

Critical Pair Lemma 19

The following expressions are equivalent:

`(!x1&&x2)==(!x1&&(x2||x1))`

Proof

Note that the input for the rule:

`x1_&&(x2_||!x1_)→x1&&x2`

contains a subpattern of the form:

`!x1_`

which can be unified with the input for the rule:

`x1_→x1`

where these rules follow from Critical Pair Lemma 5 and Substitution Lemma 32 respectively.

Substitution Lemma 33

It can be shown that:

(A[x1]||!Dnew[x1])==(Dnew[x_0]||R[x_0])

Proof

We start by taking Substitution Lemma 17, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 32.

Substitution Lemma 34

It can be shown that:

(!L[x1]||Nnew[x1])==(A[x_0]||!Dnew[x_0])

Proof

We start by taking Substitution Lemma 18, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 32.

Substitution Lemma 35

Substitution Lemma 35

It can be shown that:

`(!K[x1]||B[x1])==(A[x_0]||!Dnew[x_0])`

PROOF

We start by taking Substitution Lemma 19, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 32.

Substitution Lemma 36

It can be shown that:

`(!Enew[x1]||Cnew[x1])==(A[x_0]||!Dnew[x_0])`

PROOF

We start by taking Substitution Lemma 20, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 32.

Substitution Lemma 37

It can be shown that:

`(!R[x1]||Enew[x1])==(A[x_0]||!Dnew[x_0])`

PROOF

We start by taking Substitution Lemma 21, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 32.

Substitution Lemma 38

It can be shown that:

`(!Cnew[x1]||K[x1])==(A[x_0]||!Dnew[x_0])`

PROOF

We start by taking Substitution Lemma 22, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 32.

Substitution Lemma 39

It can be shown that:

`(!M[x1]||L[x1])==(A[x_0]||!Dnew[x_0])`

PROOF

We start by taking Substitution Lemma 23, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 32.

Substitution Lemma 40

It can be shown that:

`(!B[x1]||M[x1])==(A[x0]||!Dnew[x0])`

PROOF

We start by taking Substitution Lemma 24, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 32.

Critical Pair Lemma 20

The following expressions are equivalent:

`(x1&&x1&&x2)==(x1&&(!x1||x2))`

PROOF

Note that the input for the rule:

`x1_&&(!x1_||x2_)→x1&&x2`

contains a subpattern of the form:

`!x1_||x2_`

which can be unified with the input for the rule:

`!x1_||(x1_&&x2_)→!x1||x2`

where these rules follow from Critical Pair Lemma 7 and Critical Pair Lemma 18 respectively.

Substitution Lemma 41

It can be shown that:

`(x1&&x1&&x2)==(x1&&x2)`

PROOF

We start by taking Critical Pair Lemma 20, and apply the substitution:

`x1_&&(!x1_||x2_)→x1&&x2`

which follows from Critical Pair Lemma 7.

Substitution Lemma 42

It can be shown that:

`(R[x1]||Dnew[x1])==(x0||!x0)`

PROOF

We start by taking Critical Pair Lemma 16, and apply the substitution:

`x1_||!x1_→x0||!x0`

which follows from Critical Pair Lemma 9.

Critical Pair Lemma 21

The following expressions are equivalent:

`(R[x1]||(x2&&Dnew[x1]))==((R[x1]||x2)&&(x0||!x0))`

PROOF

Note that the input for the rule:

`(x1_||x2_)&&(x1_||x3_)→x1||(x2&&x3)`

contains a subpattern of the form:

`x1_||x3_`

which can be unified with the input for the rule:

`R[x1_]||Dnew[x1_]→x0||!x0`

where these rules follow from Equationalized Axiom 4 and Substitution Lemma 42 respectively.

Substitution Lemma 43

It can be shown that:

`(R[x1]||(x2&&Dnew[x1]))==(R[x1]||x2)`

Proof

We start by taking Critical Pair Lemma 21, and apply the substitution:

`x1_&&(x2_||!x2_)→x1`

which follows from Equationalized Axiom 2.

Substitution Lemma 44

It can be shown that:

`(A[x1]||!Dnew[x1])==(R[x0]||Dnew[x0])`

Proof

We start by taking Substitution Lemma 33, and apply the substitution:

`x1_||x2_→x2||x1`

which follows from Equationalized Axiom 3.

Substitution Lemma 45

It can be shown that:

`(A[x1]||!Dnew[x1])==(x0||!x0)`

Proof

We start by taking Substitution Lemma 44, and apply the substitution:

`R[x1_]||Dnew[x1_]→x0||!x0`

which follows from Substitution Lemma 42.

Critical Pair Lemma 22

The following expressions are equivalent:

`(Dnew[x1]&&A[x1])==(Dnew[x1]&&(x0||!x0))`

Proof

Note that the input for the rule:

`!x1_&&(x2_||x1_)→!x1&&x2`

contains a subpattern of the form:

`x2_||x1_`

which can be unified with the input for the rule:

`A[x1_]||!Dnew[x1_]→x0||!x0`

where these rules follow from Critical Pair Lemma 19 and Substitution Lemma 45 respectively.

Substitution Lemma 46

It can be shown that:

`(Dnew[x1]&&A[x1]) == (Dnew[x1]&&(x0||!x0))`

Proof

We start by taking Critical Pair Lemma 22, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 32.

Substitution Lemma 47

It can be shown that:

`(Dnew[x1]&&A[x1]) == Dnew[x1]`

Proof

We start by taking Substitution Lemma 46, and apply the substitution:

`x1_&&(x2_||!x2_)→x1`

which follows from Equationalized Axiom 2.

Substitution Lemma 48

It can be shown that:

`(Dnew[x1]&&A[x1]) == Dnew[x1]`

Proof

We start by taking Substitution Lemma 47, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 32.

Substitution Lemma 49

It can be shown that:

`(A[x1]&&Dnew[x1]) == Dnew[x1]`

Proof

We start by taking Substitution Lemma 48, and apply the substitution:

`x1_&&x2_→x2&&x1`

which follows from Equationalized Axiom 16.

Substitution Lemma 50

It can be shown that:

`(Nnew[x1]||!L[x1]) == (A[x0]||!Dnew[x0])`

Proof

We start by taking Substitution Lemma 34, and apply the substitution:

`x1_||x2_→x2||x1`

which follows from Equationalized Axiom 3.

which follows from Equationalized Axiom 3.

Substitution Lemma 51

It can be shown that:

`(Nnew[x1]||!L[x1])==(x_0||!x_0)`

Proof

We start by taking Substitution Lemma 50, and apply the substitution:

`A[x1_]||!Dnew[x1_]→x_0||!x_0`

which follows from Substitution Lemma 45.

Critical Pair Lemma 23

The following expressions are equivalent:

`(L[x1]&&Nnew[x1])==(L[x1]&&(x_0||!x_0))`

Proof

Note that the input for the rule:

`!x1_&&(x2_||x1_)→!x1&&x2`

contains a subpattern of the form:

`x2_||x1_`

which can be unified with the input for the rule:

`Nnew[x1_]||!L[x1_]→x_0||!x_0`

where these rules follow from Critical Pair Lemma 19 and Substitution Lemma 51 respectively.

Substitution Lemma 52

It can be shown that:

`(L[x1]&&Nnew[x1])==(L[x1]&&(x_0||!x_0))`

Proof

We start by taking Critical Pair Lemma 23, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 32.

Substitution Lemma 53

It can be shown that:

`(L[x1]&&Nnew[x1])==L[x1]`

Proof

We start by taking Substitution Lemma 52, and apply the substitution:

`x1_&&(x2_||!x2_)→x1`

which follows from Equationalized Axiom 2.

Substitution Lemma 54

It can be shown that:

`(L[x1]&&Nnew[x1])==L[x1]`

Proof

PROOF

We start by taking Substitution Lemma 53, and apply the substitution:

```
x1_→x1
```

which follows from Substitution Lemma 32.

Substitution Lemma 55

It can be shown that:

```
(B[x1]||!K[x1])==(A[x0]||!Dnew[x0])
```

PROOF

We start by taking Substitution Lemma 35, and apply the substitution:

```
x1_||x2_→x2||x1
```

which follows from Equationalized Axiom 3.

Substitution Lemma 56

It can be shown that:

```
(B[x1]||!K[x1])==(x0||!x0)
```

PROOF

We start by taking Substitution Lemma 55, and apply the substitution:

```
A[x1_]||!Dnew[x1_]→x0||!x0
```

which follows from Substitution Lemma 45.

Critical Pair Lemma 24

The following expressions are equivalent:

```
(K[x1]&&B[x1])==(K[x1]&&(x0||!x0))
```

PROOF

Note that the input for the rule:

```
!x1_&&(x2_||x1_)→!x1&&x2
```

contains a subpattern of the form:

```
x2_||x1_
```

which can be unified with the input for the rule:

```
B[x1_]||!K[x1_]→x0||!x0
```

where these rules follow from Critical Pair Lemma 19 and Substitution Lemma 56 respectively.

Substitution Lemma 57

It can be shown that:

```
(K[x1]&&B[x1])==(K[x1]&&(x0||!x0))
```

PROOF

We start by taking Critical Pair Lemma 24, and apply the substitution:

```
x1_→x1
```

which follows from Substitution Lemma 32.

Substitution Lemma 58

Substitution Lemma 58

Out[•]=

It can be shown that:

`(K[x1]&&B[x1])==K[x1]`

PROOF

We start by taking Substitution Lemma 57, and apply the substitution:

`x1_&&(x2_||!x2_)→x1`

which follows from Equationalized Axiom 2.

Substitution Lemma 59

It can be shown that:

`(K[x1]&&B[x1])==K[x1]`

PROOF

We start by taking Substitution Lemma 58, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 32.

Substitution Lemma 60

It can be shown that:

`(B[x1]&&K[x1])==K[x1]`

PROOF

We start by taking Substitution Lemma 59, and apply the substitution:

`x1_&&x2_→x2&&x1`

which follows from Equationalized Axiom 16.

Substitution Lemma 61

It can be shown that:

$(Cnew[x1] || !Enew[x1]) == (A[x_0] || !Dnew[x_0])$

PROOF

We start by taking Substitution Lemma 36, and apply the substitution:

`x1_||x2_→x2||x1`

which follows from Equationalized Axiom 3.

Substitution Lemma 62

It can be shown that:

$(Cnew[x1] || !Enew[x1]) == (x_0 || !x_0)$

PROOF

We start by taking Substitution Lemma 61, and apply the substitution:

$A[x1\_] || !Dnew[x1\_] \to x_0 || !x_0$

which follows from Substitution Lemma 45.

Critical Pair Lemma 25

The following expressions are equivalent:

(Enew[x1]&&Cnew[x1])==(Enew[x1]&&(x_0||!x_0))

PROOF

Note that the input for the rule:

!x1_&&(x2_||x1_)→!x1&&x2

contains a subpattern of the form:

x2_||x1_

which can be unified with the input for the rule:

Cnew[x1_]||!Enew[x1_]→x_0||!x_0

where these rules follow from Critical Pair Lemma 19 and Substitution Lemma 62 respectively.

Substitution Lemma 63

It can be shown that:

(Enew[x1]&&Cnew[x1])==(Enew[x1]&&(x_0||!x_0))

PROOF

We start by taking Critical Pair Lemma 25, and apply the substitution:

x1_→x1

which follows from Substitution Lemma 32.

Substitution Lemma 64

It can be shown that:

(Enew[x1]&&Cnew[x1])==Enew[x1]

PROOF

We start by taking Substitution Lemma 63, and apply the substitution:

x1_&&(x2_||!x2_)→x1

which follows from Equationalized Axiom 2.

Substitution Lemma 65

It can be shown that:

(Enew[x1]&&Cnew[x1])==Enew[x1]

PROOF

We start by taking Substitution Lemma 64, and apply the substitution:

x1_→x1

which follows from Substitution Lemma 32.

Substitution Lemma 66

It can be shown that:

(Cnew[x1]&&Enew[x1])==Enew[x1]

PROOF

We start by taking Substitution Lemma 65, and apply the substitution:

x1_&&x2_→x2&&x1

which follows from Equationalized Axiom 16.

Substitution Lemma 67

It can be shown that:

(Enew[x1]||!R[x1])==(A[x_0]||!Dnew[x_0])

Proof

We start by taking Substitution Lemma 37, and apply the substitution:

x1_||x2_→x2||x1

which follows from Equationalized Axiom 3.

Substitution Lemma 68

It can be shown that:

(Enew[x1]||!R[x1])==(x_0||!x_0)

Proof

We start by taking Substitution Lemma 67, and apply the substitution:

A[x1_]||!Dnew[x1_]→x_0||!x_0

which follows from Substitution Lemma 45.

Critical Pair Lemma 26

The following expressions are equivalent:

(R[x1]&&Enew[x1])==(R[x1]&&(x_0||!x_0))

Proof

Note that the input for the rule:

!x1_&&(x2_||x1_)→!x1&&x2

contains a subpattern of the form:

x2_||x1_

which can be unified with the input for the rule:

Enew[x1_]||!R[x1_]→x_0||!x_0

where these rules follow from Critical Pair Lemma 19 and Substitution Lemma 68 respectively.

Substitution Lemma 69

It can be shown that:

(R[x1]&&Enew[x1])==(R[x1]&&(x_0||!x_0))

Proof

We start by taking Critical Pair Lemma 26, and apply the substitution:

x1_→x1

which follows from Substitution Lemma 32.

Substitution Lemma 70

It can be shown that:

(R[x1]&&Enew[x1])==R[x1]

`(R[x1]&&Enew[x1])==R[x1]`

PROOF

We start by taking Substitution Lemma 69, and apply the substitution:

`x1_&&(x2_||!x2_)→x1`

which follows from Equationalized Axiom 2.

Substitution Lemma 71

It can be shown that:

`(R[x1]&&Enew[x1])==R[x1]`

PROOF

We start by taking Substitution Lemma 70, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 32.

Substitution Lemma 72

It can be shown that:

`(Enew[x1]&&R[x1])==R[x1]`

PROOF

We start by taking Substitution Lemma 71, and apply the substitution:

`x1_&&x2_→x2&&x1`

which follows from Equationalized Axiom 16.

Substitution Lemma 73

It can be shown that:

$(K[x1]||!Cnew[x1])==(A[x_0]||!Dnew[x_0])$

PROOF

We start by taking Substitution Lemma 38, and apply the substitution:

`x1_||x2_→x2||x1`

which follows from Equationalized Axiom 3.

Substitution Lemma 74

It can be shown that:

$(K[x1]||!Cnew[x1])==(x_0||!x_0)$

PROOF

We start by taking Substitution Lemma 73, and apply the substitution:

$A[x1\_]||!Dnew[x1\_]\to x_0||!x_0$

which follows from Substitution Lemma 45.

Critical Pair Lemma 27

The following expressions are equivalent:

$(Cnew[x1]\&\&K[x1])==(Cnew[x1]\&\&(x_0||!x_0))$

PROOF

Note that the input for the rule:

`!x1_&&(x2_||x1_)→!x1&&x2`

contains a subpattern of the form:

`x2_||x1_`

which can be unified with the input for the rule:

`K[x1_]||!Cnew[x1_]→x0||!x0`

where these rules follow from Critical Pair Lemma 19 and Substitution Lemma 74 respectively.

Substitution Lemma 75

It can be shown that:

`(Cnew[x1]&&K[x1])==(Cnew[x1]&&(x0||!x0))`

PROOF

We start by taking Critical Pair Lemma 27, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 32.

Substitution Lemma 76

It can be shown that:

`(Cnew[x1]&&K[x1])==Cnew[x1]`

PROOF

We start by taking Substitution Lemma 75, and apply the substitution:

`x1_&&(x2_||!x2_)→x1`

which follows from Equationalized Axiom 2.

Substitution Lemma 77

It can be shown that:

`(Cnew[x1]&&K[x1])==Cnew[x1]`

PROOF

We start by taking Substitution Lemma 76, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 32.

Substitution Lemma 78

It can be shown that:

`(L[x1]||!M[x1])==(A[x0]||!Dnew[x0])`

PROOF

We start by taking Substitution Lemma 39, and apply the substitution:

`x1_||x2_→x2||x1`

which follows from Equationalized Axiom 3.

Substitution Lemma 79

It can be shown that:

`(L[x1]||!M[x1])==(x0||!x0)`

Proof

We start by taking Substitution Lemma 78, and apply the substitution:

`A[x1_]||!Dnew[x1_]→x0||!x0`

which follows from Substitution Lemma 45.

Critical Pair Lemma 28

The following expressions are equivalent:

`(M[x1]&&L[x1])==(M[x1]&&(x0||!x0))`

Proof

Note that the input for the rule:

`!x1_&&(x2_||x1_)→!x1&&x2`

contains a subpattern of the form:

`x2_||x1_`

which can be unified with the input for the rule:

`L[x1_]||!M[x1_]→x0||!x0`

where these rules follow from Critical Pair Lemma 19 and Substitution Lemma 79 respectively.

Substitution Lemma 80

It can be shown that:

`(M[x1]&&L[x1])==(M[x1]&&(x0||!x0))`

Proof

We start by taking Critical Pair Lemma 28, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 32.

Substitution Lemma 81

It can be shown that:

`(M[x1]&&L[x1])==M[x1]`

Proof

We start by taking Substitution Lemma 80, and apply the substitution:

`x1_&&(x2_||!x2_)→x1`

which follows from Equationalized Axiom 2.

Substitution Lemma 82

It can be shown that:

`(M[x1]&&L[x1])==M[x1]`

Proof

We start by taking Substitution Lemma 81, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 32.

Substitution Lemma 83

It can be shown that:

`(L[x1]&&M[x1])==M[x1]`

Proof

We start by taking Substitution Lemma 82, and apply the substitution:

`x1_&&x2_→x2&&x1`

which follows from Equationalized Axiom 16.

Substitution Lemma 84

It can be shown that:

`(M[x1]||!B[x1])==(A[x0]||!Dnew[x0])`

Proof

We start by taking Substitution Lemma 40, and apply the substitution:

`x1_||x2_→x2||x1`

which follows from Equationalized Axiom 3.

Substitution Lemma 85

It can be shown that:

`(M[x1]||!B[x1])==(x0||!x0)`

Proof

We start by taking Substitution Lemma 84, and apply the substitution:

`A[x1_]||!Dnew[x1_]→x0||!x0`

which follows from Substitution Lemma 45.

Critical Pair Lemma 29

The following expressions are equivalent:

`(B[x1]&&M[x1])==(B[x1]&&(x0||!x0))`

Proof

Note that the input for the rule:

`!x1_&&(x2_||x1_)→!x1&&x2`

contains a subpattern of the form:

`x2_||x1_`

which can be unified with the input for the rule:

`M[x1_]||!B[x1_]→x0||!x0`

where these rules follow from Critical Pair Lemma 19 and Substitution Lemma 85 respectively.

Substitution Lemma 86

It can be shown that:

`(B[x1]&&M[x1])==(B[x1]&&(x0||!x0))`

Proof

We start by taking Critical Pair Lemma 29, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 32.

Substitution Lemma 87

It can be shown that:

`(B[x1]&&M[x1])==B[x1]`

Proof

We start by taking Substitution Lemma 86, and apply the substitution:

`x1_&&(x2_||!x2_)→x1`

which follows from Equationalized Axiom 2.

Substitution Lemma 88

It can be shown that:

`(B[x1]&&M[x1])==B[x1]`

Proof

We start by taking Substitution Lemma 87, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 32.

Critical Pair Lemma 30

The following expressions are equivalent:

`(R[x1]||A[x1])==(R[x1]||Dnew[x1])`

Proof

Note that the input for the rule:

`R[x1_]||(x2_&&Dnew[x1_])→R[x1]||x2`

contains a subpattern of the form:

`x2_&&Dnew[x1_]`

which can be unified with the input for the rule:

`A[x1_]&&Dnew[x1_]→Dnew[x1]`

where these rules follow from Substitution Lemma 43 and Substitution Lemma 49 respectively.

Substitution Lemma 89

It can be shown that:

`(R[x1]||A[x1])==(x0||!x0)`

Proof

We start by taking Critical Pair Lemma 30, and apply the substitution:

`R[x1_]||Dnew[x1_]→x_||!x_`

`R[x1_]||Dnew[x1_]→x0||!x0`

which follows from Substitution Lemma 42.

Substitution Lemma 90

It can be shown that:

`(A[x1]||R[x1])==(x0||!x0)`

Proof

We start by taking Substitution Lemma 89, and apply the substitution:

`x1_||x2_→x2||x1`

which follows from Equationalized Axiom 3.

Critical Pair Lemma 31

The following expressions are equivalent:

`(A[x1]||(x2&&R[x1]))==((A[x1]||x2)&&(x0||!x0))`

Proof

Note that the input for the rule:

`(x1_||x2_)&&(x1_||x3_)→x1||(x2&&x3)`

contains a subpattern of the form:

`x1_||x3_`

which can be unified with the input for the rule:

`A[x1_]||R[x1_]→x0||!x0`

where these rules follow from Equationalized Axiom 4 and Substitution Lemma 90 respectively.

Substitution Lemma 91

It can be shown that:

`(A[x1]||(x2&&R[x1]))==(A[x1]||x2)`

Proof

We start by taking Critical Pair Lemma 31, and apply the substitution:

`x1_&&(x2_||!x2_)→x1`

which follows from Equationalized Axiom 2.

Critical Pair Lemma 32

The following expressions are equivalent:

`(A[x1]||Enew[x1])==(A[x1]||R[x1])`

Proof

Note that the input for the rule:

`A[x1_]||(x2_&&R[x1_])→A[x1]||x2`

contains a subpattern of the form:

`x2_&&R[x1_]`

which can be unified with the input for the rule:

`Enew[x1_]&&R[x1_]→R[x1]`

where these rules follow from Substitution Lemma 91 and Substitution Lemma 72 respectively.

Substitution Lemma 92

It can be shown that:

`(A[x1]||Enew[x1])==(x0||!x0)`

Proof

We start by taking Critical Pair Lemma 32, and apply the substitution:

`A[x1_]||R[x1_]→x0||!x0`

which follows from Substitution Lemma 90.

Critical Pair Lemma 33

The following expressions are equivalent:

`(A[x1]||(x2&&Enew[x1]))==((A[x1]||x2)&&(x0||!x0))`

Proof

Note that the input for the rule:

`(x1_||x2_)&&(x1_||x3_)→x1||(x2&&x3)`

contains a subpattern of the form:

`x1_||x3_`

which can be unified with the input for the rule:

`A[x1_]||Enew[x1_]→x0||!x0`

where these rules follow from Equationalized Axiom 4 and Substitution Lemma 92 respectively.

Substitution Lemma 93

It can be shown that:

`(A[x1]||(x2&&Enew[x1]))==(A[x1]||x2)`

Proof

We start by taking Critical Pair Lemma 33, and apply the substitution:

`x1_&&(x2_||!x2_)→x1`

which follows from Equationalized Axiom 2.

Critical Pair Lemma 34

The following expressions are equivalent:

`(A[x1]||Cnew[x1])==(A[x1]||Enew[x1])`

Proof

Note that the input for the rule:

`A[x1_]||(x2_&&Enew[x1_])→A[x1]||x2`

contains a subpattern of the form:

`x2_&&Enew[x1_]`

which can be unified with the input for the rule:

`Cnew[x1_]&&Enew[x1_]→Enew[x1]`

where these rules follow from Substitution Lemma 93 and Substitution Lemma 66 respectively.

where these rules follow from Substitution Lemma 93 and Substitution Lemma 80 respectively.

Substitution Lemma 94

It can be shown that:

```
(A[x1]||Cnew[x1])==(x0||!x0)
```

Proof

We start by taking Critical Pair Lemma 34, and apply the substitution:

```
A[x1_]||Enew[x1_]→x0||!x0
```

which follows from Substitution Lemma 92.

Critical Pair Lemma 35

The following expressions are equivalent:

```
(A[x1]||(Cnew[x1]&&x2))==((x0||!x0)&&(A[x1]||x2))
```

Proof

Note that the input for the rule:

```
(x1_||x2_)&&(x1_||x3_)→x1||(x2&&x3)
```

contains a subpattern of the form:

```
x1_||x2_
```

which can be unified with the input for the rule:

```
A[x1_]||Cnew[x1_]→x0||!x0
```

where these rules follow from Equationalized Axiom 4 and Substitution Lemma 94 respectively.

Substitution Lemma 95

It can be shown that:

```
(A[x1]||(Cnew[x1]&&x2))==(A[x1]||x2)
```

Proof

We start by taking Critical Pair Lemma 35, and apply the substitution:

```
(x1_||!x1_)&&x2_→x2
```

which follows from Critical Pair Lemma 6.

Critical Pair Lemma 36

The following expressions are equivalent:

```
(A[x1]||K[x1])==(A[x1]||Cnew[x1])
```

Proof

Note that the input for the rule:

```
A[x1_]||(Cnew[x1_]&&x2_)→A[x1]||x2
```

contains a subpattern of the form:

```
Cnew[x1_]&&x2_
```

which can be unified with the input for the rule:

```
Cnew[x1_]&&K[x1_]→Cnew[x1]
```

where these rules follow from Substitution Lemma 95 and Substitution Lemma 77 respectively.

Substitution Lemma 96

It can be shown that:

`(A[x1]||K[x1])==(x0||!x0)`

Proof

We start by taking Critical Pair Lemma 36, and apply the substitution:

`A[x1_]||Cnew[x1_]→x0||!x0`

which follows from Substitution Lemma 94.

Critical Pair Lemma 37

The following expressions are equivalent:

`(A[x1]||(K[x1]&&x2))==((x0||!x0)&&(A[x1]||x2))`

Proof

Note that the input for the rule:

`(x1_||x2_)&&(x1_||x3_)→x1||(x2&&x3)`

contains a subpattern of the form:

`x1_||x2_`

which can be unified with the input for the rule:

`A[x1_]||K[x1_]→x0||!x0`

where these rules follow from Equationalized Axiom 4 and Substitution Lemma 96 respectively.

Substitution Lemma 97

It can be shown that:

`(A[x1]||(K[x1]&&x2))==(A[x1]||x2)`

Proof

We start by taking Critical Pair Lemma 37, and apply the substitution:

`(x1_||!x1_)&&x2_→x2`

which follows from Critical Pair Lemma 6.

Substitution Lemma 98

It can be shown that:

`(x1||(x2&&x3&&!x3))==(x1&&(x1||x2))`

Proof

We start by taking Critical Pair Lemma 3, and apply the substitution:

`x1_&&x2_→x2&&x1`

which follows from Equationalized Axiom 16.

Critical Pair Lemma 38

The following expressions are equivalent:

`(x1&&(x1||x2))==(x1||(x2&&!x2))`

Proof

Note that the input for the rule:

`x1_||(x2_&&x3_&&!x3_)↔x1_&&(x1_||x2_)`

contains a subpattern of the form:

`x2_&&x3_&&!x3_`

which can be unified with the input for the rule:

`x1_&&x1_&&x2_→x1&&x2`

where these rules follow from Substitution Lemma 98 and Substitution Lemma 41 respectively.

Substitution Lemma 99

It can be shown that:

`(x1&&(x1||x2))==x1`

Proof

We start by taking Critical Pair Lemma 38, and apply the substitution:

`x1_||(x2_&&!x2_)→x1`

which follows from Equationalized Axiom 1.

Substitution Lemma 100

It can be shown that:

`x1_||(x2_&&x1_)→x1`

Proof

We start by taking Substitution Lemma 27, and apply the substitution:

`x1_&&(x1_||x2_)→x1`

which follows from Substitution Lemma 99.

Substitution Lemma 101

It can be shown that:

`x1_||(x1_&&x2_)→x1`

Proof

We start by taking Critical Pair Lemma 11, and apply the substitution:

`x1_&&(x1_||x2_)→x1`

which follows from Substitution Lemma 99.

Critical Pair Lemma 39

The following expressions are equivalent:

`x1==(x1&&(x2||x1))`

Proof

Note that the input for the rule:

`x1_&&(x1_||x2_)→x1`

contains a subpattern of the form:

`x1_||x2_`

which can be unified with the input for the rule:

which can be unified with the input for the rule:

`x1_||x2_↔x2_||x1_`

where these rules follow from Substitution Lemma 99 and Equationalized Axiom 3 respectively.

Critical Pair Lemma 40

The following expressions are equivalent:

`(x1&&(x2||x1||x3))==(x1||(x1&&x3))`

Proof

Note that the input for the rule:

`(x1_&&x2_)||(x1_&&x3_)→x1&&(x2||x3)`

contains a subpattern of the form:

`x1_&&x2_`

which can be unified with the input for the rule:

`x1_&&(x2_||x1_)→x1`

where these rules follow from Equationalized Axiom 15 and Critical Pair Lemma 39 respectively.

Substitution Lemma 102

It can be shown that:

`(x1&&(x2||x1||x3))==x1`

Proof

We start by taking Critical Pair Lemma 40, and apply the substitution:

`x1_||(x1_&&x2_)→x1`

which follows from Substitution Lemma 101.

Critical Pair Lemma 41

The following expressions are equivalent:

`(x1&&x2)==(x1&&x2&&(x2||x3))`

Proof

Note that the input for the rule:

`x1_&&(x2_||x1_||x3_)→x1`

contains a subpattern of the form:

`x2_||x1_`

which can be unified with the input for the rule:

`Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[2442,x1_&&(x1_|`

where these rules follow from Substitution Lemma 102 and Substitution Lemma 100 respectively.

Critical Pair Lemma 42

The following expressions are equivalent:

`(x1&&x2)==(x1&&x2&&(x1||x3))`

Proof

Note that the input for the rule:

`x1_&&(x2_||x1_||x3_)→x1`

contains a subpattern of the form:

`x2_||x1_`

which can be unified with the input for the rule:

`` Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[2443,x1_&&(x1_| ``

where these rules follow from Substitution Lemma 102 and Substitution Lemma 101 respectively.

Critical Pair Lemma 43

The following expressions are equivalent:

`(B[x1]&&M[x1])⩵(B[x1]&&(M[x1]||x2))`

Proof

Note that the input for the rule:

`x1_&&x2_&&(x2_||x3_)→x1&&x2`

contains a subpattern of the form:

`x1_&&x2_`

which can be unified with the input for the rule:

`B[x1_]&&M[x1_]→B[x1]`

where these rules follow from Critical Pair Lemma 41 and Substitution Lemma 88 respectively.

Substitution Lemma 103

It can be shown that:

`B[x1]⩵(B[x1]&&(M[x1]||x2))`

Proof

We start by taking Critical Pair Lemma 43, and apply the substitution:

`B[x1_]&&M[x1_]→B[x1]`

which follows from Substitution Lemma 88.

Critical Pair Lemma 44

The following expressions are equivalent:

`(L[x1]&&Nnew[x1])⩵(L[x1]&&(Nnew[x1]||x2))`

Proof

Note that the input for the rule:

`x1_&&x2_&&(x2_||x3_)→x1&&x2`

contains a subpattern of the form:

`x1_&&x2_`

which can be unified with the input for the rule:

`L[x1_]&&Nnew[x1_]→L[x1]`

where these rules follow from Critical Pair Lemma 41 and Substitution Lemma 54 respectively.

Substitution Lemma 104

It can be shown that:

`L[x1]==(L[x1]&&(Nnew[x1]||x2))`

Proof

We start by taking Critical Pair Lemma 44, and apply the substitution:

`L[x1_]&&Nnew[x1_]→L[x1]`

which follows from Substitution Lemma 54.

Critical Pair Lemma 45

The following expressions are equivalent:

`(B[x1]&&K[x1])==(K[x1]&&(B[x1]||x2))`

Proof

Note that the input for the rule:

`x1_&&x2_&&(x1_||x3_)→x1&&x2`

contains a subpattern of the form:

`x1_&&x2_`

which can be unified with the input for the rule:

`B[x1_]&&K[x1_]→K[x1]`

where these rules follow from Critical Pair Lemma 42 and Substitution Lemma 60 respectively.

Substitution Lemma 105

It can be shown that:

`K[x1]==(K[x1]&&(B[x1]||x2))`

Proof

We start by taking Critical Pair Lemma 45, and apply the substitution:

`B[x1_]&&K[x1_]→K[x1]`

which follows from Substitution Lemma 60.

Critical Pair Lemma 46

The following expressions are equivalent:

`(L[x1]&&M[x1])==(M[x1]&&(L[x1]||x2))`

Proof

Note that the input for the rule:

`x1_&&x2_&&(x1_||x3_)→x1&&x2`

contains a subpattern of the form:

`x1_&&x2_`

which can be unified with the input for the rule:

`L[x1_]&&M[x1_]→M[x1]`

where these rules follow from Critical Pair Lemma 42 and Substitution Lemma 83 respectively.

Substitution Lemma 106

It can be shown that:

`M[x1]==(M[x1]&&(L[x1]||x2))`

`M[x1]==(M[x1]&&(L[x1]||x2))`

PROOF

We start by taking Critical Pair Lemma 46, and apply the substitution:

`L[x1_]&&M[x1_]→M[x1]`

which follows from Substitution Lemma 83.

Substitution Lemma 107

It can be shown that:

`(!H[x1]||(!Nnew[x1]&&x2))==((x0||!x0)&&(!H[x1]||x2))`

PROOF

We start by taking Critical Pair Lemma 4, and apply the substitution:

`A[x1_]||!Dnew[x1_]→x0||!x0`

which follows from Substitution Lemma 45.

Substitution Lemma 108

It can be shown that:

`(!H[x1]||(!Nnew[x1]&&x2))==(!H[x1]||x2)`

PROOF

We start by taking Substitution Lemma 107, and apply the substitution:

`(x1_||!x1_)&&x2_→x2`

which follows from Critical Pair Lemma 6.

Critical Pair Lemma 47

The following expressions are equivalent:

`(!H[x1]||Nnew[x1])==!H[x1]`

PROOF

Note that the input for the rule:

`!H[x1_]||(!Nnew[x1_]&&x2_)→!H[x1]||x2`

contains a subpattern of the form:

`!H[x1_]||(!Nnew[x1_]&&x2_)`

which can be unified with the input for the rule:

`x1_||(x2_&&!x2_)→x1`

where these rules follow from Substitution Lemma 108 and Equationalized Axiom 1 respectively.

Substitution Lemma 109

It can be shown that:

`(!H[x1]||Nnew[x1])==!H[x1]`

PROOF

We start by taking Critical Pair Lemma 47, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 32.

Substitution Lemma 110

It can be shown that:

```
(Nnew[x1]||!H[x1])==!H[x1]
```

Proof

We start by taking Substitution Lemma 109, and apply the substitution:

```
x1_||x2_→x2||x1
```

which follows from Equationalized Axiom 3.

Critical Pair Lemma 48

The following expressions are equivalent:

```
L[x1]==(L[x1]&&!H[x1])
```

Proof

Note that the input for the rule:

```
L[x1_]&&(Nnew[x1_]||x2_)→L[x1]
```

contains a subpattern of the form:

```
Nnew[x1_]||x2_
```

which can be unified with the input for the rule:

```
Nnew[x1_]||!H[x1_]→!H[x1]
```

where these rules follow from Substitution Lemma 104 and Substitution Lemma 110 respectively.

Critical Pair Lemma 49

The following expressions are equivalent:

```
(!H[x1])==(!H[x1]||L[x1])
```

Proof

Note that the input for the rule:

```
Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[2442,x1_&&(x1_|
```

contains a subpattern of the form:

```
x2_&&x1_
```

which can be unified with the input for the rule:

```
L[x1_]&&!H[x1_]→L[x1]
```

where these rules follow from Substitution Lemma 100 and Critical Pair Lemma 48 respectively.

Substitution Lemma 111

It can be shown that:

```
(!H[x1])==(L[x1]||!H[x1])
```

Proof

We start by taking Critical Pair Lemma 49, and apply the substitution:

```
x1_||x2_→x2||x1
```

which follows from Equationalized Axiom 3.

Critical Pair Lemma 50

The following expressions are equivalent:

```
M[x1]==(M[x1]&&!H[x1])
```

Proof

Note that the input for the rule:

```
M[x1_]&&(L[x1_]||x2_)→M[x1]
```

contains a subpattern of the form:

```
L[x1_]||x2_
```

which can be unified with the input for the rule:

```
L[x1_]||!H[x1_]→!H[x1]
```

where these rules follow from Substitution Lemma 106 and Substitution Lemma 111 respectively.

Critical Pair Lemma 51

The following expressions are equivalent:

```
(!H[x1])==(!H[x1]||M[x1])
```

Proof

Note that the input for the rule:

```
Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[2442,x1_&&(x1_|
```

contains a subpattern of the form:

```
x2_&&x1_
```

which can be unified with the input for the rule:

```
M[x1_]&&!H[x1_]→M[x1]
```

where these rules follow from Substitution Lemma 100 and Critical Pair Lemma 50 respectively.

Substitution Lemma 112

It can be shown that:

```
(!H[x1])==(M[x1]||!H[x1])
```

Proof

We start by taking Critical Pair Lemma 51, and apply the substitution:

```
x1_||x2_→x2||x1
```

which follows from Equationalized Axiom 3.

Critical Pair Lemma 52

The following expressions are equivalent:

```
B[x1]==(B[x1]&&!H[x1])
```

Proof

Note that the input for the rule:

```
B[x1_]&&(M[x1_]||x2_)→B[x1]
```

contains a subpattern of the form:

```
M[x1_]||x2_
```

which can be unified with the input for the rule:

`M[x1_]||!H[x1_]→!H[x1]`

where these rules follow from Substitution Lemma 103 and Substitution Lemma 112 respectively.

Critical Pair Lemma 53

The following expressions are equivalent:

`(!H[x1])==(!H[x1]||B[x1])`

Proof

Note that the input for the rule:

`` Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[2442,x1_&&(x1_| ``

contains a subpattern of the form:

`x2_&&x1_`

which can be unified with the input for the rule:

`B[x1_]&&!H[x1_]→B[x1]`

where these rules follow from Substitution Lemma 100 and Critical Pair Lemma 52 respectively.

Substitution Lemma 113

It can be shown that:

`(!H[x1])==(B[x1]||!H[x1])`

Proof

We start by taking Critical Pair Lemma 53, and apply the substitution:

`x1_||x2_→x2||x1`

which follows from Equationalized Axiom 3.

Critical Pair Lemma 54

The following expressions are equivalent:

`K[x1]==(K[x1]&&!H[x1])`

Proof

Note that the input for the rule:

`K[x1_]&&(B[x1_]||x2_)→K[x1]`

contains a subpattern of the form:

`B[x1_]||x2_`

which can be unified with the input for the rule:

`B[x1_]||!H[x1_]→!H[x1]`

where these rules follow from Substitution Lemma 105 and Substitution Lemma 113 respectively.

Critical Pair Lemma 55

The following expressions are equivalent:

`(A[x1]||!H[x1])==(A[x1]||K[x1])`

Proof

Note that the input for the rule:

Note that the input for the rule:

A[x1_]||(K[x1_]&&x2_)→A[x1]||x2

contains a subpattern of the form:

K[x1_]&&x2_

which can be unified with the input for the rule:

K[x1_]&&!H[x1_]→K[x1]

where these rules follow from Substitution Lemma 97 and Critical Pair Lemma 54 respectively.

Substitution Lemma 114

It can be shown that:

(A[x1]||!H[x1])==(x_0||!x_0)

Proof

We start by taking Critical Pair Lemma 55, and apply the substitution:

A[x1_]||K[x1_]→x_0||!x_0

which follows from Substitution Lemma 96.

Substitution Lemma 115

It can be shown that:

(A[x_0]||!H[x_0])==(a_0||!a_0)

Proof

We start by taking Equationalized Hypothesis 1, and apply the substitution:

x1_||x2_→x2||x1

which follows from Equationalized Axiom 3.

Substitution Lemma 116

It can be shown that:

(A[x_0]||!H[x_0])==(x_0||!x_0)

Proof

We start by taking Substitution Lemma 115, and apply the substitution:

x1_||!x1_→x_0||!x_0

which follows from Critical Pair Lemma 9.

Conclusion 1

We obtain the conclusion:

True

Proof

Take Substitution Lemma 116, and apply the substitution:

A[x1_]||!H[x1_]→x_0||!x_0

which follows from Substitution Lemma 114.