AN EQUATIONAL LOGIC PROOF OF LEWIS CARROLL’S PUZZLE #1 (“BABIES CAN’T MANAGE CROCODILES”)

Author: Wolfram Research
Minor modifications by: J. K. Horner
Last modified: 19 March 2020 by JKH

Platform:

Mathematica v12.1.0 Home Edition
Windows 10
Dell Inspiron 545
-- Intel Core2 Quad CPU Q8200, clocked at 2.33 GHz, 4 Cores
-- 8 GB RAM
-- 1 TB disk

References:

Carroll L. (1896). *Symbolic Logic*. Reprinted by Clarkson N. Potter, 1977.
Lemmon EJ. (1965). *Beginning Logic*. Thomas Nelson and Sons.
McCune WW and Padmanabhan R. (1996). *Automated Deduction in Equational Logic and Cubic Curves.* Springer.
Wolfram Research (2020). *Mathematica* v12.1.0 Home Edition. https://www.wolfram.com/mathematica-home-edition/. Accessed 19 March 2020.

Introduction

Mathematica v12.1.0 (Wolfram Research 2020), released March 2020, contains an enhancement of *Mathematica*'s equational-logic (McCune and Padmanabhan 1996) proof search function **FindEquationalProof.** The enhancement automatically converts *Mathematica* first-order predicate logic (similar to Lemmon 1965, Chap. 3) expressions to equational logic (prior versions did not provide this conversion). The v12.1.0 distribution contains an example (Lewis Carroll's Puzzle Number 1 (1896) showcasing the conversion capability. The example proves that "Babies can't manage crocodiles", given:

(a) All babies are illogical.
(b) Nobody is despised who can manage a crocodile.
(c) Illogical persons are despised.

I made minor modifications to the example provided in the Wolfram distribution; the results are shown below.

On the platform described above, *Mathematica* generates the entire proof in about 7 seconds.

Executable code and results

```
In[1]:= proofBabyCantManageCrocs =
         FindEquationalProof[Not[Exists[x, And[baby[x], manageCrocodile[x]]]],
          {ForAll[x, Implies[baby[x], Not[logical[x]]]],
           ForAll[x, Implies[manageCrocodile[x], Not[despised[x]]]],
           ForAll[x, Implies[Not[logical[x]], despised[x]]]}]

Out[1]= ProofObject[ Logic: Predicate/EquationalLogic   Steps: 100
                     Theorem: ∀x ! (baby[x] && manageCrocodile[x]) ]

In[2]:= proofBabyCantManageCrocs["ProofNotebook"]
```

Axiom 1

We are given that:

```
∀x (baby[x] ⇒ !logical[x])
```

Axiom 2

We are given that:

```
∀x (manageCrocodile[x] ⇒ !despised[x])
```

Axiom 3

We are given that:

```
∀x (!logical[x] ⇒ despised[x])
```

Hypothesis 1

We would like to show that:

```
∀x ! (baby[x] && manageCrocodile[x])
```

Equationalized Axiom 1

We generate the "equationalized" axiom:

```
x1 == (x1 || (x2 && !x2))
```

Equationalized Axiom 2

We generate the "equationalized" axiom:

```
x1 == (x1 && (x2 || !x2))
```

Equationalized Axiom 3

Equationalized Axiom 3

We generate the "equationalized" axiom:

`(x1 || x2) == (x2 || x1)`

Equationalized Axiom 4

We generate the "equationalized" axiom:

`(x1 || (x2&&x3)) == ((x1 || x2)&&(x1 || x3))`

Equationalized Axiom 5

We generate the "equationalized" axiom:

`(logical[x1] || despised[x1]) == (ạ0 || !ạ0)`

Equationalized Axiom 6

We generate the "equationalized" axiom:

`(!baby[x1] || !logical[x1]) == (ạ0 || !ạ0)`

Equationalized Axiom 7

We generate the "equationalized" axiom:

`(!manageCrocodile[x1] || !despised[x1]) == (ạ0 || !ạ0)`

Equationalized Axiom 8

We generate the "equationalized" axiom:

`((x1&&x2) || (x1&&x3)) == (x1&&(x2 || x3))`

Equationalized Axiom 9

We generate the "equationalized" axiom:

`(x1&&x2) == (x2&&x1)`

Equationalized Hypothesis 1

We generate the "equationalized" hypothesis:

`(ạ0 || !ạ0) == !(baby[x0]&&manageCrocodile[x0])`

Critical Pair Lemma 1

The following expressions are equivalent:

`((x1&&!x1) || x2) == x2`

Proof

Note that the input for the rule:

`x1_ || x2_ ↔ x2_ || x1_`

contains a subpattern of the form:

`x1_ || x2_`

which can be unified with the input for the rule:

`x1_ || (x2_&&!x2_) → x1`

where these rules follow from Equationalized Axiom 3 and Equationalized Axiom 1 respectively.

Critical Pair Lemma 2

The following expressions are equivalent:

`(x1||(x2&&!x1))==(x1||x2)`

PROOF

Note that the input for the rule:

`(x1_||x2_)&&(x1_||x3_)→x1||(x2&&x3)`

contains a subpattern of the form:

`(x1_||x2_)&&(x1_||x3_)`

which can be unified with the input for the rule:

`x1_&&(x2_||!x2_)→x1`

where these rules follow from Equationalized Axiom 4 and Equationalized Axiom 2 respectively.

Substitution Lemma 1

It can be shown that:

$(\text{despised}[x1]\,||\,\text{logical}[x1]) == (a_0\,||\,!a_0)$

PROOF

We start by taking Equationalized Axiom 5, and apply the substitution:

`x1_||x2_→x2||x1`

which follows from Equationalized Axiom 3.

Substitution Lemma 2

It can be shown that:

$(!\text{baby}[x1]\,||\,!\text{logical}[x1]) == (\text{despised}[x_0]\,||\,\text{logical}[x_0])$

PROOF

We start by taking Equationalized Axiom 6, and apply the substitution:

$a_0\,||\,!a_0 \to \text{despised}[x_0]\,||\,\text{logical}[x_0]$

which follows from Substitution Lemma 1.

Critical Pair Lemma 3

The following expressions are equivalent:

$(!\text{baby}[x1]\,||\,(!\text{logical}[x1]\,\&\&\,x2)) == ((\text{despised}[x_0]\,||\,\text{logical}[x_0])\,\&\&\,(!\text{baby}[x1]\,||\,x2))$

PROOF

Note that the input for the rule:

`(x1_||x2_)&&(x1_||x3_)→x1||(x2&&x3)`

contains a subpattern of the form:

`x1_||x2_`

which can be unified with the input for the rule:

$!\text{baby}[x1\_]\,||\,!\text{logical}[x1\_] \to \text{despised}[x_0]\,||\,\text{logical}[x_0]$

where these rules follow from Equationalized Axiom 4 and Substitution Lemma 2 respectively.

Substitution Lemma 3

It can be shown that:

It can be shown that:

(!manageCrocodile[x1]||!despised[x1])==(despised[x_0]||logical[x_0])

Proof

We start by taking Equationalized Axiom 7, and apply the substitution:

a_0||!a_0→despised[x_0]||logical[x_0]

which follows from Substitution Lemma 1.

Critical Pair Lemma 4

The following expressions are equivalent:

(!manageCrocodile[x1]||(!despised[x1]&&x2))==((despised[x_0]||logical[x_0])&&(!manageCroco

Proof

Note that the input for the rule:

(x1_||x2_)&&(x1_||x3_)→x1||(x2&&x3)

contains a subpattern of the form:

x1_||x2_

which can be unified with the input for the rule:

!manageCrocodile[x1_]||!despised[x1_]→despised[x_0]||logical[x_0]

where these rules follow from Equationalized Axiom 4 and Substitution Lemma 3 respectively.

Critical Pair Lemma 5

The following expressions are equivalent:

(x1&&(x2||!x1))==(x1&&x2)

Proof

Note that the input for the rule:

(x1_&&x2_)||(x1_&&x3_)→x1&&(x2||x3)

contains a subpattern of the form:

(x1_&&x2_)||(x1_&&x3_)

which can be unified with the input for the rule:

x1_||(x2_&&!x2_)→x1

where these rules follow from Equationalized Axiom 8 and Equationalized Axiom 1 respectively.

Critical Pair Lemma 6

The following expressions are equivalent:

((x1||!x1)&&x2)==x2

Proof

Note that the input for the rule:

x1_&&x2_↔x2_&&x1_

contains a subpattern of the form:

x1_&&x2_

which can be unified with the input for the rule:

x1_&&(x2_||!x2_)→x1

where these rules follow from Equationalized Axiom 9 and Equationalized Axiom 2 respectively.

Critical Pair Lemma 7

The following expressions are equivalent:

`(x1&&(x2||x3))==((x2&&x1)||(x1&&x3))`

Proof

Note that the input for the rule:

`(x1_&&x2_)||(x1_&&x3_)→x1&&(x2||x3)`

contains a subpattern of the form:

`x1_&&x2_`

which can be unified with the input for the rule:

`x1_&&x2_↔x2_&&x1_`

where these rules follow from Equationalized Axiom 8 and Equationalized Axiom 9 respectively.

Critical Pair Lemma 8

The following expressions are equivalent:

`(x1&&x2)==(x1&&(!x1||x2))`

Proof

Note that the input for the rule:

`(x1_&&!x1_)||x2_→x2`

contains a subpattern of the form:

`(x1_&&!x1_)||x2_`

which can be unified with the input for the rule:

`(x1_&&x2_)||(x1_&&x3_)→x1&&(x2||x3)`

where these rules follow from Critical Pair Lemma 1 and Equationalized Axiom 8 respectively.

Critical Pair Lemma 9

The following expressions are equivalent:

`x1==(x1&&!(x2&&!x2))`

Proof

Note that the input for the rule:

`x1_&&(x2_||!x2_)→x1`

contains a subpattern of the form:

`x2_||!x2_`

which can be unified with the input for the rule:

`(x1_&&!x1_)||x2_→x2`

where these rules follow from Equationalized Axiom 2 and Critical Pair Lemma 1 respectively.

Critical Pair Lemma 10

The following expressions are equivalent:

`(x1||x2)==(x1||(!x1&&x2))`

`(x1||x2)==(x1||(!x1&&x2))`

PROOF

Note that the input for the rule:

`(x1_||!x1_)&&x2_→x2`

contains a subpattern of the form:

`(x1_||!x1_)&&x2_`

which can be unified with the input for the rule:

`(x1_||x2_)&&(x1_||x3_)→x1||(x2&&x3)`

where these rules follow from Critical Pair Lemma 6 and Equationalized Axiom 4 respectively.

Critical Pair Lemma 11

The following expressions are equivalent:

`(x1||!x1)==(x2||!x2)`

PROOF

Note that the input for the rule:

`(x1_||!x1_)&&x2_→x2`

contains a subpattern of the form:

`(x1_||!x1_)&&x2_`

which can be unified with the input for the rule:

`x1_&&(x2_||!x2_)→x1`

where these rules follow from Critical Pair Lemma 6 and Equationalized Axiom 2 respectively.

Critical Pair Lemma 12

The following expressions are equivalent:

`x1==(x1||!(x2||!x2))`

PROOF

Note that the input for the rule:

`x1_||(x2_&&!x2_)→x1`

contains a subpattern of the form:

`x2_&&!x2_`

which can be unified with the input for the rule:

`(x1_||!x1_)&&x2_→x2`

where these rules follow from Equationalized Axiom 1 and Critical Pair Lemma 6 respectively.

Critical Pair Lemma 13

The following expressions are equivalent:

`(x1||x1)==x1`

PROOF

Note that the input for the rule:

`x1_||(x2_&&!x1_)→x1||x2`

contains a subpattern of the form:

x1_||(x2_&&!x1_)

which can be unified with the input for the rule:

x1_||(x2_&&!x2_)→x1

where these rules follow from Critical Pair Lemma 2 and Equationalized Axiom 1 respectively.

Critical Pair Lemma 14

The following expressions are equivalent:

(x1||(x1&&x2))==(x1&&(x1||x2))

Proof

Note that the input for the rule:

(x1_||x2_)&&(x1_||x3_)→x1||(x2&&x3)

contains a subpattern of the form:

x1_||x2_

which can be unified with the input for the rule:

x1_||x1_→x1

where these rules follow from Equationalized Axiom 4 and Critical Pair Lemma 13 respectively.

Critical Pair Lemma 15

The following expressions are equivalent:

(!logical[x1]&&despised[x1])==(!logical[x1]&&(a_0||!a_0))

Proof

Note that the input for the rule:

x1_&&(x2_||!x1_)→x1&&x2

contains a subpattern of the form:

x2_||!x1_

which can be unified with the input for the rule:

"0"

where these rules follow from Critical Pair Lemma 5 and Substitution Lemma 1 respectively.

Substitution Lemma 4

It can be shown that:

(!logical[x1]&&despised[x1])==!logical[x1]

Proof

We start by taking Critical Pair Lemma 15, and apply the substitution:

x1_&&(x2_||!x2_)→x1

which follows from Equationalized Axiom 2.

Critical Pair Lemma 16

The following expressions are equivalent:

(x1&&x1)==x1

PROOF

Note that the input for the rule:

`x1_&&(!x1_||x2_)→x1&&x2`

contains a subpattern of the form:

`x1_&&(!x1_||x2_)`

which can be unified with the input for the rule:

`x1_&&(x2_||!x2_)→x1`

where these rules follow from Critical Pair Lemma 8 and Equationalized Axiom 2 respectively.

Critical Pair Lemma 17

The following expressions are equivalent:

`True`

PROOF

Note that the input for the rule:

`x1_||(!x1_&&x2_)→x1||x2`

contains a subpattern of the form:

`!x1_&&x2_`

which can be unified with the input for the rule:

`x1_&&x1_→x1`

where these rules follow from Critical Pair Lemma 10 and Critical Pair Lemma 16 respectively.

Substitution Lemma 5

It can be shown that:

`(despised[x1]&&!logical[x1])==!logical[x1]`

PROOF

We start by taking Substitution Lemma 4, and apply the substitution:

`x1_&&x2_→x2&&x1`

which follows from Equationalized Axiom 9.

Critical Pair Lemma 18

The following expressions are equivalent:

`(logical[x1]||despised[x1])==(logical[x1]||!logical[x1])`

PROOF

Note that the input for the rule:

`x1_||(x2_&&!x1_)→x1||x2`

contains a subpattern of the form:

`x2_&&!x1_`

which can be unified with the input for the rule:

`despised[x1_]&&!logical[x1_]→!logical[x1]`

where these rules follow from Critical Pair Lemma 2 and Substitution Lemma 5 respectively.

Critical Pair Lemma 19

The following expressions are equivalent:

```
(x1||!x1)==!(x2&&!x2)
```

Proof

Note that the input for the rule:

```
x1_&&!(x2_&&!x2_)→x1
```

contains a subpattern of the form:

```
x1_&&!(x2_&&!x2_)
```

which can be unified with the input for the rule:

```
(x1_||!x1_)&&x2_→x2
```

where these rules follow from Critical Pair Lemma 9 and Critical Pair Lemma 6 respectively.

Critical Pair Lemma 20

The following expressions are equivalent:

```
(x1&&!x1)==!(x2||!x2)
```

Proof

Note that the input for the rule:

```
x1_||!(x2_||!x2_)→x1
```

contains a subpattern of the form:

```
x1_||!(x2_||!x2_)
```

which can be unified with the input for the rule:

```
(x1_&&!x1_)||x2_→x2
```

where these rules follow from Critical Pair Lemma 12 and Critical Pair Lemma 1 respectively.

Critical Pair Lemma 21

The following expressions are equivalent:

```
(x1&&x1)==(x1&&(x1||!x1))
```

Proof

Note that the input for the rule:

```
x1_&&(x2_||!x1_)→x1&&x2
```

contains a subpattern of the form:

```
x2_||!x1_
```

which can be unified with the input for the rule:

```
x1_||!x1_→x1||!x1
```

where these rules follow from Critical Pair Lemma 5 and Critical Pair Lemma 17 respectively.

Substitution Lemma 6

It can be shown that:

```
(x1&&x1)==x1
```

Proof

PROOF

We start by taking Critical Pair Lemma 21, and apply the substitution:

`x1_&&(x2_||!x2_)→x1`

which follows from Equationalized Axiom 2.

Substitution Lemma 7

It can be shown that:

`(x1&&x1)==x1`

PROOF

We start by taking Substitution Lemma 6, and apply the substitution:

`x1_&&x2_→x2&&x1`

which follows from Equationalized Axiom 9.

Substitution Lemma 8

It can be shown that:

`True`

PROOF

We start by taking Substitution Lemma 7, and apply the substitution:

`x1_&&x1_→x1`

which follows from Critical Pair Lemma 16.

Critical Pair Lemma 22

The following expressions are equivalent:

`(!x1||x2)==(!x1||(x1&&x2))`

PROOF

Note that the input for the rule:

`x1_||(!x1_&&x2_)→x1||x2`

contains a subpattern of the form:

`!x1_`

which can be unified with the input for the rule:

`x1_→x1`

where these rules follow from Critical Pair Lemma 10 and Substitution Lemma 8 respectively.

Critical Pair Lemma 23

The following expressions are equivalent:

`(!x1&&x2)==(!x1&&(x2||x1))`

PROOF

Note that the input for the rule:

`x1_&&(x2_||!x1_)→x1&&x2`

contains a subpattern of the form:

`!x1_`

which can be unified with the input for the rule:

`x1_→x1`

where these rules follow from Critical Pair Lemma 5 and Substitution Lemma 8 respectively.

Critical Pair Lemma 24

The following expressions are equivalent:

`(x1&&x1&&x2) == (x1&&(!x1||x2))`

Proof

Note that the input for the rule:

`x1_&&(!x1_||x2_)→x1&&x2`

contains a subpattern of the form:

`!x1_||x2_`

which can be unified with the input for the rule:

`!x1_||(x1_&&x2_)→!x1||x2`

where these rules follow from Critical Pair Lemma 8 and Critical Pair Lemma 22 respectively.

Substitution Lemma 9

It can be shown that:

`(x1&&x1&&x2) == (x1&&x2)`

Proof

We start by taking Critical Pair Lemma 24, and apply the substitution:

`x1_&&(!x1_||x2_)→x1&&x2`

which follows from Critical Pair Lemma 8.

Critical Pair Lemma 25

The following expressions are equivalent:

`(x1&&(x2||(x1&&x3))) == ((x1&&x2)||(x1&&x3))`

Proof

Note that the input for the rule:

`(x1_&&x2_)||(x1_&&x3_)→x1&&(x2||x3)`

contains a subpattern of the form:

`x1_&&x3_`

which can be unified with the input for the rule:

`x1_&&x1_&&x2_→x1&&x2`

where these rules follow from Equationalized Axiom 8 and Substitution Lemma 9 respectively.

Substitution Lemma 10

It can be shown that:

`(x1&&(x2||(x1&&x3))) == (x1&&(x2||x3))`

Proof

We start by taking Critical Pair Lemma 25, and apply the substitution:

We start by taking Critical Pair Lemma 25, and apply the substitution:

(x1_&&x2_)||(x1_&&x3_)→x1&&(x2||x3)

which follows from Equationalized Axiom 8.

Substitution Lemma 11

It can be shown that:

(logical[x1]||despised[x1])==(x_0||!x_0)

Proof

We start by taking Critical Pair Lemma 18, and apply the substitution:

x1_||!x1_→x_0||!x_0

which follows from Critical Pair Lemma 11.

Critical Pair Lemma 26

The following expressions are equivalent:

(logical[x1]||(despised[x1]&&x2))==((x_0||!x_0)&&(logical[x1]||x2))

Proof

Note that the input for the rule:

(x1_||x2_)&&(x1_||x3_)→x1||(x2&&x3)

contains a subpattern of the form:

x1_||x2_

which can be unified with the input for the rule:

logical[x1_]||despised[x1_]→x_0||!x_0

where these rules follow from Equationalized Axiom 4 and Substitution Lemma 11 respectively.

Substitution Lemma 12

Out[2]= It can be shown that:

(logical[x1]||(despised[x1]&&x2))==(logical[x1]||x2)

Proof

We start by taking Critical Pair Lemma 26, and apply the substitution:

(x1_||!x1_)&&x2_→x2

which follows from Critical Pair Lemma 6.

Critical Pair Lemma 27

The following expressions are equivalent:

(logical[x1]||!despised[x1])==logical[x1]

Proof

Note that the input for the rule:

logical[x1_]||(despised[x1_]&&x2_)→logical[x1]||x2

contains a subpattern of the form:

logical[x1_]||(despised[x1_]&&x2_)

which can be unified with the input for the rule:

```
x1_||(x2_&&!x2_)→x1
```

where these rules follow from Substitution Lemma 12 and Equationalized Axiom 1 respectively.

Critical Pair Lemma 28

The following expressions are equivalent:

```
(x1&&(x2||!x2))==(x1&&(x2||x1))
```

Proof

Note that the input for the rule:

```
x1_&&(x2_||(x1_&&x3_))→x1&&(x2||x3)
```

contains a subpattern of the form:

```
x2_||(x1_&&x3_)
```

which can be unified with the input for the rule:

```
x1_||(x2_&&!x1_)→x1||x2
```

where these rules follow from Substitution Lemma 10 and Critical Pair Lemma 2 respectively.

Substitution Lemma 13

It can be shown that:

```
x1==(x1&&(x2||x1))
```

Proof

We start by taking Critical Pair Lemma 28, and apply the substitution:

```
x1_&&(x2_||!x2_)→x1
```

which follows from Equationalized Axiom 2.

Critical Pair Lemma 29

The following expressions are equivalent:

```
x1==(x1&&(x1||x2))
```

Proof

Note that the input for the rule:

```
x1_&&(x2_||x1_)→x1
```

contains a subpattern of the form:

```
x2_||x1_
```

which can be unified with the input for the rule:

```
x1_||x2_↔x2_||x1_
```

where these rules follow from Substitution Lemma 13 and Equationalized Axiom 3 respectively.

Critical Pair Lemma 30

The following expressions are equivalent:

```
(!x1||x2||x1)==(!x1||x1)
```

Proof

Note that the input for the rule:

```
!x1_||(x1_&&x2_)→!x1||x2
```

`!x1_||(x1_&&x2_)→!x1||x2`

contains a subpattern of the form:

`x1_&&x2_`

which can be unified with the input for the rule:

`x1_&&(x2_||x1_)→x1`

where these rules follow from Critical Pair Lemma 22 and Substitution Lemma 13 respectively.

Substitution Lemma 14

It can be shown that:

`x1_||(x1_&&x2_)→x1`

Proof

We start by taking Critical Pair Lemma 14, and apply the substitution:

`x1_&&(x1_||x2_)→x1`

which follows from Critical Pair Lemma 29.

Substitution Lemma 15

It can be shown that:

`(!x1||x2||x1)==(x1||!x1)`

Proof

We start by taking Critical Pair Lemma 30, and apply the substitution:

`x1_||x2_→x2||x1`

which follows from Equationalized Axiom 3.

Critical Pair Lemma 31

The following expressions are equivalent:

`(!(x1||x2)&&!x2)==(!(x1||x2)&&(x2||!x2))`

Proof

Note that the input for the rule:

`!x1_&&(x2_||x1_)→!x1&&x2`

contains a subpattern of the form:

`x2_||x1_`

which can be unified with the input for the rule:

`!x1_||x2_||x1_→x1||!x1`

where these rules follow from Critical Pair Lemma 23 and Substitution Lemma 15 respectively.

Substitution Lemma 16

It can be shown that:

`(!(x1||x2)&&!x2)==!(x1||x2)`

Proof

We start by taking Critical Pair Lemma 31, and apply the substitution:

`x1_&&(x2_||!x2_)→x1`

which follows from Equationalized Axiom 2.

Substitution Lemma 17

It can be shown that:

`(!x1&&!(x2||x1))==!(x2||x1)`

Proof

We start by taking Substitution Lemma 16, and apply the substitution:

`x1_&&x2_→x2&&x1`

which follows from Equationalized Axiom 9.

Critical Pair Lemma 32

The following expressions are equivalent:

`(!x1)==(!x1||!(x2||x1))`

Proof

Note that the input for the rule:

`` Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[1551,x1_&&(x1_| ``

contains a subpattern of the form:

`x1_&&x2_`

which can be unified with the input for the rule:

`!x1_&&!(x2_||x1_)→!(x2||x1)`

where these rules follow from Substitution Lemma 14 and Substitution Lemma 17 respectively.

Substitution Lemma 18

It can be shown that:

`(!baby[x1]||(!logical[x1]&&x2))==((despised[x0]||logical[x0])&&(!baby[x1]||x2))`

Proof

We start by taking Critical Pair Lemma 3, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 8.

Substitution Lemma 19

It can be shown that:

`(!baby[x1]||(!logical[x1]&&x2))==((logical[x0]||despised[x0])&&(!baby[x1]||x2))`

Proof

We start by taking Substitution Lemma 18, and apply the substitution:

`x1_||x2_→x2||x1`

which follows from Equationalized Axiom 3.

Substitution Lemma 20

It can be shown that:

`(!baby[x1]||(!logical[x1]&&x2))==((x0||!x0)&&(!baby[x1]||x2))`

Proof

We start by taking Substitution Lemma 19, and apply the substitution:

`logical[x1_]||despised[x1_]→x0||!x0`

which follows from Substitution Lemma 11.

Substitution Lemma 21

It can be shown that:

`(!baby[x1]||(!logical[x1]&&x2))==(!baby[x1]||x2)`

Proof

We start by taking Substitution Lemma 20, and apply the substitution:

`(x1_||!x1_)&&x2_→x2`

which follows from Critical Pair Lemma 6.

Critical Pair Lemma 33

The following expressions are equivalent:

`(!baby[x1]||logical[x1])==!baby[x1]`

Proof

Note that the input for the rule:

`!baby[x1_]||(!logical[x1_]&&x2_)→!baby[x1]||x2`

contains a subpattern of the form:

`!baby[x1_]||(!logical[x1_]&&x2_)`

which can be unified with the input for the rule:

`x1_||(x2_&&!x2_)→x1`

where these rules follow from Substitution Lemma 21 and Equationalized Axiom 1 respectively.

Substitution Lemma 22

It can be shown that:

`(!baby[x1]||logical[x1])==!baby[x1]`

Proof

We start by taking Critical Pair Lemma 33, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 8.

Substitution Lemma 23

It can be shown that:

`(logical[x1]||!baby[x1])==!baby[x1]`

Proof

We start by taking Substitution Lemma 22, and apply the substitution:

`x1_||x2_→x2||x1`

which follows from Equationalized Axiom 3.

Critical Pair Lemma 34

The following expressions are equivalent:

`(baby[x1]&&logical[x1]) == (baby[x1]&&!baby[x1])`

Proof

Note that the input for the rule:

`!x1_&&(x2_||x1_)→!x1&&x2`

contains a subpattern of the form:

`x2_||x1_`

which can be unified with the input for the rule:

`logical[x1_]||!baby[x1_]→!baby[x1]`

where these rules follow from Critical Pair Lemma 23 and Substitution Lemma 23 respectively.

Substitution Lemma 24

It can be shown that:

`(baby[x1]&&logical[x1]) == (baby[x1]&&!baby[x1])`

Proof

We start by taking Critical Pair Lemma 34, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 8.

Substitution Lemma 25

It can be shown that:

`(baby[x1]&&logical[x1]) == (baby[x1]&&!baby[x1])`

Proof

We start by taking Substitution Lemma 24, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 8.

Critical Pair Lemma 35

The following expressions are equivalent:

`x1 == (x1||!(x2||!x1))`

Proof

Note that the input for the rule:

`!x1_||!(x2_||x1_)→!x1`

contains a subpattern of the form:

`!x1_`

which can be unified with the input for the rule:

`x1_→x1`

where these rules follow from Critical Pair Lemma 32 and Substitution Lemma 8 respectively.

Substitution Lemma 26

It can be shown that:

`x1==(x1||!(x2||!x1))`

Proof

We start by taking Critical Pair Lemma 35, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 8.

Critical Pair Lemma 36

The following expressions are equivalent:

`x1==(x1||!(!x1||x2))`

Proof

Note that the input for the rule:

`x1_||!(x2_||!x1_)→x1`

contains a subpattern of the form:

`x2_||!x1_`

which can be unified with the input for the rule:

`x1_||x2_↔x2_||x1_`

where these rules follow from Substitution Lemma 26 and Equationalized Axiom 3 respectively.

Critical Pair Lemma 37

The following expressions are equivalent:

`(x1&&!(x1||x2))==(x1&&!x1)`

Proof

Note that the input for the rule:

`x1_&&(!x1_||x2_)→x1&&x2`

contains a subpattern of the form:

`!x1_||x2_`

which can be unified with the input for the rule:

`x1_||!(!x1_||x2_)→x1`

where these rules follow from Critical Pair Lemma 8 and Critical Pair Lemma 36 respectively.

Substitution Lemma 27

It can be shown that:

`(x1&&!(x1||x2))==(x1&&!x1)`

Proof

We start by taking Critical Pair Lemma 37, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 8.

Critical Pair Lemma 38

The following expressions are equivalent:

(logical[x1]||!(despised[x1]||x2))==(logical[x1]||(despised[x1]&&!despised[x1]))

PROOF

Note that the input for the rule:

logical[x1_]||(despised[x1_]&&x2_)→logical[x1]||x2

contains a subpattern of the form:

despised[x1_]&&x2_

which can be unified with the input for the rule:

x1_&&!(x1_||x2_)→x1&&!x1

where these rules follow from Substitution Lemma 12 and Substitution Lemma 27 respectively.

Substitution Lemma 28

It can be shown that:

(logical[x1]||!(despised[x1]||x2))==(logical[x1]||!despised[x1])

PROOF

We start by taking Critical Pair Lemma 38, and apply the substitution:

logical[x1_]||(despised[x1_]&&x2_)→logical[x1]||x2

which follows from Substitution Lemma 12.

Substitution Lemma 29

It can be shown that:

(logical[x1]||!(despised[x1]||x2))==logical[x1]

PROOF

We start by taking Substitution Lemma 28, and apply the substitution:

logical[x1_]||!despised[x1_]→logical[x1]

which follows from Critical Pair Lemma 27.

Substitution Lemma 30

It can be shown that:

(baby[x1]&&logical[x1])==!(x_0||!x_0)

PROOF

We start by taking Substitution Lemma 25, and apply the substitution:

x1_&&!x1_→!(x_0||!x_0)

which follows from Critical Pair Lemma 20.

Substitution Lemma 31

It can be shown that:

(baby[x1]&&logical[x1])==(x_0&&!x_0)

PROOF

We start by taking Substitution Lemma 30, and apply the substitution:

!(x1_||!x1_)→x_0&&!x_0

```
!(x1_||!x1_)→x0&&!x0
```

which follows from Critical Pair Lemma 20.

Critical Pair Lemma 39

The following expressions are equivalent:

```
(baby[x1]&&(x2||logical[x1]))==((x2&&baby[x1])||(x0&&!x0))
```

Proof

Note that the input for the rule:

```
(x1_&&x2_)||(x2_&&x3_)→x2&&(x1||x3)
```

contains a subpattern of the form:

```
x2_&&x3_
```

which can be unified with the input for the rule:

```
baby[x1_]&&logical[x1_]→x0&&!x0
```

where these rules follow from Critical Pair Lemma 7 and Substitution Lemma 31 respectively.

Substitution Lemma 32

It can be shown that:

```
(baby[x1]&&(x2||logical[x1]))==(x2&&baby[x1])
```

Proof

We start by taking Critical Pair Lemma 39, and apply the substitution:

```
x1_||(x2_&&!x2_)→x1
```

which follows from Equationalized Axiom 1.

Substitution Lemma 33

It can be shown that:

```
(!manageCrocodile[x1]||(!despised[x1]&&x2))==((despised[x0]||logical[x0])&&(!manageCroco
```

Proof

We start by taking Critical Pair Lemma 4, and apply the substitution:

```
x1_→x1
```

which follows from Substitution Lemma 8.

Substitution Lemma 34

It can be shown that:

```
(!manageCrocodile[x1]||(!despised[x1]&&x2))==((logical[x0]||despised[x0])&&(!manageCroco
```

Proof

We start by taking Substitution Lemma 33, and apply the substitution:

```
x1_||x2_→x2||x1
```

which follows from Equationalized Axiom 3.

Substitution Lemma 35

It can be shown that:

```
(!manageCrocodile[x1]||(!despised[x1]&&x2))==((x0||!x0)&&(!manageCrocodile[x1]||x2))
```

PROOF

We start by taking Substitution Lemma 34, and apply the substitution:

logical[x1_]||despised[x1_]→x_0||!x_0

which follows from Substitution Lemma 11.

Substitution Lemma 36

It can be shown that:

`(!manageCrocodile[x1]||(!despised[x1]&&x2))==(!manageCrocodile[x1]||x2)`

PROOF

We start by taking Substitution Lemma 35, and apply the substitution:

`(x1_||!x1_)&&x2_→x2`

which follows from Critical Pair Lemma 6.

Critical Pair Lemma 40

The following expressions are equivalent:

`(!manageCrocodile[x1]||despised[x1])==!manageCrocodile[x1]`

PROOF

Note that the input for the rule:

`!manageCrocodile[x1_]||(!despised[x1_]&&x2_)→!manageCrocodile[x1]||x2`

contains a subpattern of the form:

`!manageCrocodile[x1_]||(!despised[x1_]&&x2_)`

which can be unified with the input for the rule:

`x1_||(x2_&&!x2_)→x1`

where these rules follow from Substitution Lemma 36 and Equationalized Axiom 1 respectively.

Substitution Lemma 37

It can be shown that:

`(!manageCrocodile[x1]||despised[x1])==!manageCrocodile[x1]`

PROOF

We start by taking Critical Pair Lemma 40, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 8.

Substitution Lemma 38

It can be shown that:

`(despised[x1]||!manageCrocodile[x1])==!manageCrocodile[x1]`

PROOF

We start by taking Substitution Lemma 37, and apply the substitution:

`x1_||x2_→x2||x1`

which follows from Equationalized Axiom 3.

Critical Pair Lemma 41

The following expressions are equivalent:

`logical[x1] == (logical[x1] || manageCrocodile[x1])`

Proof

Note that the input for the rule:

`logical[x1_] || !(despised[x1_] || x2_) → logical[x1]`

contains a subpattern of the form:

`despised[x1_] || x2_`

which can be unified with the input for the rule:

`despised[x1_] || !manageCrocodile[x1_] → !manageCrocodile[x1]`

where these rules follow from Substitution Lemma 29 and Substitution Lemma 38 respectively.

Substitution Lemma 39

It can be shown that:

`logical[x1] == (logical[x1] || manageCrocodile[x1])`

Proof

We start by taking Critical Pair Lemma 41, and apply the substitution:

`x1_ → x1`

which follows from Substitution Lemma 8.

Substitution Lemma 40

It can be shown that:

`logical[x1] == (manageCrocodile[x1] || logical[x1])`

Proof

We start by taking Substitution Lemma 39, and apply the substitution:

`x1_ || x2_ → x2 || x1`

which follows from Equationalized Axiom 3.

Critical Pair Lemma 42

The following expressions are equivalent:

`(manageCrocodile[x1] && baby[x1]) == (baby[x1] && logical[x1])`

Proof

Note that the input for the rule:

`baby[x1_] && (x2_ || logical[x1_]) → x2 && baby[x1]`

contains a subpattern of the form:

`x2_ || logical[x1_]`

which can be unified with the input for the rule:

`manageCrocodile[x1_] || logical[x1_] → logical[x1]`

where these rules follow from Substitution Lemma 32 and Substitution Lemma 40 respectively.

Substitution Lemma 41

Substitution Lemma 41

It can be shown that:

`(manageCrocodile[x1]&&baby[x1]) == (x0&&!x0)`

Proof

We start by taking Critical Pair Lemma 42, and apply the substitution:

`baby[x1_]&&logical[x1_]→x0&&!x0`

which follows from Substitution Lemma 31.

Substitution Lemma 42

It can be shown that:

`(a0||!a0) == !(manageCrocodile[x0]&&baby[x0])`

Proof

We start by taking Equationalized Hypothesis 1, and apply the substitution:

`x1_&&x2_→x2&&x1`

which follows from Equationalized Axiom 9.

Substitution Lemma 43

It can be shown that:

`(!(x0&&!x0)) == !(manageCrocodile[x0]&&baby[x0])`

Proof

We start by taking Substitution Lemma 42, and apply the substitution:

`x1_||!x1_→!(x0&&!x0)`

which follows from Critical Pair Lemma 19.

Conclusion 1

We obtain the conclusion:

`True`

Proof

Take Substitution Lemma 43, and apply the substitution:

`manageCrocodile[x1_]&&baby[x1_]→x0&&!x0`

which follows from Substitution Lemma 41.