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AN AUTOMATED EQUATIONAL LOGIC DERIVATION OF EXERCISE 60 (“I avoid kangaroos”) IN CARROLL’S *SYMBOLIC LOGIC*

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Abstract
Lewis Carroll’s introductory symbolic logic textbook, Symbolic Logic*, contains tens of exercises that require the reader to discover the consequent of a given set of sentences. Here, I first “manually” solve Exercise 60 in that textbook, then use the automated equational logic deduction system contained in Mathematica to corroborate the solution. The automated corroboration proof appears to be novel.*

1.0 Introduction

Symbolic Logic (Carroll 1896) contains tens of exercises that require the reader to discover “the” conclusion that follows from of a given set of (often whimsically formulated) sentences. Here, I first “manually” solve Exercise 60 of Carroll 1896 (*Symbolic Logic*, Book VIII, Chap 1, Section 9, p. 124), then use the automated equational logic deduction system contained in *Mathematica* (Wolfram Research 2021) to corroborate the solution.

Section 2 contains a statement of Exercise 60 and a manual solution of the same. Section 3 contains an overview of the algorithm implemented in the *Mathematica* (Wolfram Research 2021) function **FindEquationalProof**, which is the heart of Mathematica’s automated equational logic deduction framework. Section 4 contains the Mathematica code for an automated equational logic proof that the solution of Exercise 60 follows from the premises of the Exercise. The Appendix contains the detailed proof corresponding to the proof-summary in Section 4.

2.0 Statement, and “manual” solution, of Exercise 60

We are given:

(1) The only animals in this house are cats.
(2) Every animal is suitable for a pet, that loves to gaze at the moon.

(3) When I detest an animal, I avoid it.
(4) No animals are carnivorous, unless they prowl at night.
(5) No cat fails to kill mice.
(6) No animals ever take to me, except what are in this house.
(7) Kangaroos are not suitable for pets.
(8) None but carnivora kill mice.
(9) I detest animals that do not take to me.
(10) Animals, that prowl at night, always love to gaze at the moon.

Carroll 1896 suggests abbreviations for the predicates in (1)-(10). To the right of each of Carroll's suggestions, I have added a translation of the predicate to a form that facilitates its translation to Mathematica's predicate-calculus (see, for example, Church 1956) idiom:

N = avoided by me (is avoided by me)
B = carnivora (is a carnivore)
C = cats (is a cat)
D = detested by me (is detested by me)
E = in this house (is in this house)
H = kangaroos (is a kangaroo)
K = killing mice (kills mice)
L = loving to gaze at the moon (loves to gaze at the moon)
M = prowling at night (prowls at night)
N = suitable for pets (is suitable for a pet)
R = taking to me (takes to me)

Using these abbreviations, we can transform (1)-(10) to a set of implications. Informally put, these are:

(1') E --> C
(2') L --> N
(3') D --> A
(4') -M --> -B
(5') C --> K
(6') R --> E
(7') H --> -N
(8') K --> B
(9') -R --> D
(10') M --> L

where universal quantification is implicit, "-->" means "implies", and "-" means "not", in the sense of Russell and Whitehead 1910.

The objective of Exercise 60 is to find “the” conclusion that (1’) - (10’) collectively imply.

Exercise 60 is a *sorites* -- a set of premises that can be analyzed as a chain of syllogisms, with each syllogism’s major term forming the minor term of the next, until a final conclusion is attained (for more information on the theory of syllogism, see Aristotle 350 BCE; Łukasiewicz 1963, Chap. V). For example, a sorites might consist of the premises that some pets are lions, that no lions have leaves, and that only leafy things emit oxygen, yielding the conclusion that not all pets emit oxygen.

Let “X” and “Y” range over a set of predicates and x range over some domain to which the elements of X and Y are attributed. Each member of Exercise 60 is, or can be converted to, the form “For all x, if x is X, then x is Y”. Such a set forms a chain of implications whose conclusion is “if FH, then LC“, where FH is the hypothesis of the first element in the chain, and LC is the conclusion of the last element in the chain. Carroll 1896 provides a method/theory for solving soriteses (Carroll 1896, *Symbolic Logic*, Book VII). That method/theory is a submethod/theory (Chang and Keisler 2012, Section 1.4) of the theory of Whitehead and Russell 1910 (Sections *9(B) and *10). From here on, I will use the latter in the following.

To solve the Exercise, we first locate the sentence that contains a predicate that appears only on the right-hand side (RHS) of implication connective (“-->”) in all of (1’) - (10’). That sentence is

(3’) D --> N

Next, we find the sentence, or its transposition, that has D on RHS of its implication connective. That sentence is the transposition of (9’), which we will label (9’’), and write it below (3’), yielding

(3’) D --> N
(9’’) -R --> D

Similarly, we next find a sentence, or its transposition, in (1’) - (10’) that has -R on the RHS of its implication connective. That sentence is the transposition of (6’), which we will label (6’’), and write it below (9’’), yielding

(3’) D --> N
(9’’) -R --> D
(6’’) -E --> -R

We continue in this way until we have exhausted all the sentences in (1’) - (10’), yielding (where a double-prime denotes the transposition of an item in the original list of premises)

(3’) D --> N
(9’’) -R --> D

(6’’) -E --> -R
(1’’) -C --> -E
(5’’) -K --> -C
(8’’) -B --> -K
(4’) -M --> -B
(10’’) -K --> -M
(2’’) -N --> -L
(7’) H --> -N

We then note that this list, read bottom to top, is a transitive sequence of implications whose implication is

H --> N,

or in idiomatic English

“I avoid kangaroos”,

which is the solution of the Exercise.

3.0 An overview of Mathematica’s automated equational logic derivation framework

This section provides an overview of the algorithm implemented in the Mathematica function **FindEquationalProof**, which is the heart of Mathematica’s automated equational logic derivation framework.

3.1 Some terminology

Assume the definitions of *term, value of a term, variable,* and *constant* in Baader and Nipkov 1999, Chapter 3.

A *rewriting system* is a system of R rules that transforms expressions that satisfy some well-defined set of formation rules to another expression that satisfy those formation rules. For the purposes of this paper, I restrict “rewriting system” to a rewriting system that concerns only *identical terms.*

Two terms are said to be *identical* if the values of the terms are equal for all values of variables occurring in them.

A *reduction of a term T to a term T’* is a (typically recursive) rewriting of T to T’ using a set of rewriting rules R such that T' is "simpler than" T (given some definition of "simpler than"). A *reduction sequence of a term T to a term T'* is a sequence T0 = T, T1, T2, T3, ..., Tn = T', where each Ti is the result of applying R to Ti-1, i = 1, 2, ..., n.

If "simpler than" is a partial ordering (Suppes 1972, Df. 21, 72) on a reduction sequence that begins with T and ends with T' in a system with a set of rewriting rules R, "simpler than" induces a reduction order (Baader and Nipkov 1999, 102) on the reduction sequence that begins with T and ends with T'. A term *Tn is in normal form* if no application of R to Tn changes Tn.

A rewriting system is said to be *finitely terminating* if every reduction sequence of any term T produces, in a finite number of iterations, a normal form of T. A rewriting system is said to be *confluent* if the normal forms of all terms in the system are unique.

Some term rewriting systems are both finitely terminating and confluent (Baader and Nipkov 1999, esp. Chapter 9). Such rewriting systems have unique normal forms for all expressions. This permits us to use the the output of such a system to determine whether there is an identity between two terms T1 and T2 in the following manner. If T1 and T2 and have the same normal form, then there is an identity between T1 and T2. Otherwise, there is not an identity.

3.2 Mathematica's equational logic inference algorithm

The inference algorithm in Mathematica's equational logic automated deduction framework (ADF), invoked by the Mathematica function **FindEquationalProof**, is the Knuth-Bendix completion algorithm (Knuth and Bendix 1970). KBC attempts to transform a given finite set of identities (an "input" to KBC) to a finitely terminating, confluent term rewriting system that preserves identity. At initialization, KBC attempts to "orient" the identities supplied in its input according to the KnuthBendix reduction order (Baader and Nipkov 1999, Section 5.4.4). This results in an initial set of reduction rules. KBC then attempts to complete this initial set of rules with additional rules, obtaining their normal forms, and adding a new rule for every pair of the normal forms in accordance with the reduction order.

KBC may

1. Terminate with success, yielding a finitely terminating, confluent set of rules, or
2. Terminate with failure, or
3. Loop without terminating.

Part of Mathematica's equational logic ADF is a descendant of Waldmeister (Hillenbrand, Buch, Vogt , and Löchner 1997).

For further information on term rewriting, see Baader and Nipkov 1999. For additional information on KBC, see Knuth and Bendix 1970.

4.0 An automated equational logic derivation of "I avoid kangaroos" from the premises of Exercise 60

The *Mathematica* (Wolfram Research 2021) script shown in Sections 4.1 - 4. 3 was executed on a Dell Inspiron 545 with an Intel Core2 Quad CPU Q8200 @ 2.33 GHz and 8.00 GB RAM, running under Windows 10. x is a variable.

4.1 Statement of premises in Mathematica format

(Predicates N, C, E, and D have been renamed Nnew, Cnew, Enew, and Dnew, respectively, to avoid conflict with Mathematica reserved names.)

```
In[1]:= premise3prime = ForAll[x, Implies[H[x], Not[Nnew[x]]]]
Out[1]= ∀x (H[x] ⇒ ! Nnew[x])

In[2]:= premise9primeprime = ForAll[x, Implies[Not[Nnew[x]], Not[L[x]]]]
Out[2]= ∀x (! Nnew[x] ⇒ ! L[x])

In[3]:= premise6primeprime = ForAll[x, Implies[Not[L[x]], Not[M[x]]]]
Out[3]= ∀x (! L[x] ⇒ ! M[x])

In[4]:= premise1primeprime = ForAll[x, Implies[Not[M[x]], Not[B[x]]]]
Out[4]= ∀x (! M[x] ⇒ ! B[x])

In[5]:= premise5primeprime = ForAll[x, Implies[Not[B[x]], Not[K[x]]]]
Out[5]= ∀x (! B[x] ⇒ ! K[x])

In[6]:= premise8primeprime = ForAll[x, Implies[Not[K[x]], Not[Cnew[x]]]]
Out[6]= ∀x (! K[x] ⇒ ! Cnew[x])

In[7]:= premise4prime = ForAll[x, Implies[Not[Cnew[x]], Not[Enew[x]]]]
Out[7]= ∀x (! Cnew[x] ⇒ ! Enew[x])

In[8]:= premise10primeprime = ForAll[x, Implies[Not[Enew[x]], Not[R[x]]]]
Out[8]= ∀x (! Enew[x] ⇒ ! R[x])

In[9]:= premise2primeprime = ForAll[x, Implies[Not[R[x]], Dnew[x]]]
Out[9]= ∀x (! R[x] ⇒ Dnew[x])

In[10]:= premise7prime = ForAll[x, Implies[Dnew[x], Nnew[x]]]
Out[10]= ∀x (Dnew[x] ⇒ Nnew[x])
```

```
In[11]:= premises = {premise3prime, premise9primeprime, premise6primeprime,
           premise1primeprime, premise5primeprime, premise8primeprime,
           premise4prime, premise10primeprime, premise2primeprime, premise7prime}
```

```
Out[11]= {∀x (H[x] ⇒ ! Nnew[x]), ∀x (! Nnew[x] ⇒ ! L[x]), ∀x (! L[x] ⇒ ! M[x]), ∀x (! M[x] ⇒ ! B[x]),
          ∀x (! B[x] ⇒ ! K[x]), ∀x (! K[x] ⇒ ! Cnew[x]), ∀x (! Cnew[x] ⇒ ! Enew[x]),
          ∀x (! Enew[x] ⇒ ! R[x]), ∀x (! R[x] ⇒ Dnew[x]), ∀x (Dnew[x] ⇒ Nnew[x])}
```

4.2 Statement of conclusion to be derived

```
In[12]:= conclusion = ForAll[x, Implies[H[x], Nnew[x]]]
```

```
Out[12]= ∀x (H[x] ⇒ Nnew[x])
```

4.3 Proof that the premises imply the conclusion

```
In[13]:= proofEx60 = FindEquationalProof[conclusion, premises]
```

```
Out[13]= ProofObject[ Logic: Predicate/EquationalLogic   Steps: 206
                      Theorem: ∀x (H[x] ⇒ Nnew[x]) ]
```

The Appendix shows the proof in detail.

REFERENCES

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APPENDIX. Proof of Exercise 60.

In[14]:=
```
proofEx60["ProofNotebook"]
```

Axiom 1

We are given that:

$\forall_x (H[x] \Rightarrow !Nnew[x])$

Axiom 2

We are given that:

$\forall_x (!Nnew[x] \Rightarrow !L[x])$

Axiom 3

We are given that:

$\forall_x (!L[x] \Rightarrow !M[x])$

Axiom 4

We are given that:

$\forall_x (!M[x] \Rightarrow !B[x])$

Axiom 5

We are given that:

$\forall_x (!B[x] \Rightarrow !K[x])$

Axiom 6

We are given that:

$\forall_x (!K[x] \Rightarrow !Cnew[x])$

Axiom 7

We are given that:

$\forall_x (!Cnew[x] \Rightarrow !Enew[x])$

Axiom 8

We are given that:

$\forall_x (!Enew[x] \Rightarrow !R[x])$

Axiom 9

We are given that:

$\forall_x (!R[x] \Rightarrow Dnew[x])$

Axiom 10

We are given that:

$\forall_x (Dnew[x] \Rightarrow Nnew[x])$

Hypothesis 1

We would like to show that:

$\forall_x$ (H[x]⇒Nnew[x])

Equationalized Axiom 1

We generate the "equationalized" axiom:

x1==(x1||(x2&&!x2))

Equationalized Axiom 2

We generate the "equationalized" axiom:

x1==(x1&&(x2||!x2))

Equationalized Axiom 3

We generate the "equationalized" axiom:

(x1||x2)==(x2||x1)

Equationalized Axiom 4

We generate the "equationalized" axiom:

(x1||(x2&&x3))==((x1||x2)&&(x1||x3))

Equationalized Axiom 5

We generate the "equationalized" axiom:

(Nnew[x1]||!L[x1])==(a_0||!a_0)

Equationalized Axiom 6

We generate the "equationalized" axiom:

(R[x1]||Dnew[x1])==(a_0||!a_0)

Equationalized Axiom 7

We generate the "equationalized" axiom:

(M[x1]||!B[x1])==(a_0||!a_0)

Equationalized Axiom 8

We generate the "equationalized" axiom:

(K[x1]||!Cnew[x1])==(a_0||!a_0)

Equationalized Axiom 9

We generate the "equationalized" axiom:

(Enew[x1]||!R[x1])==(a_0||!a_0)

Equationalized Axiom 10

We generate the "equationalized" axiom:

(Cnew[x1]||!Enew[x1])==(a_0||!a_0)

Equationalized Axiom 11

We generate the "equationalized" axiom:

`(B[x1]||!K[x1])==(a0||!a0)`

Equationalized Axiom 12

We generate the "equationalized" axiom:

`(L[x1]||!M[x1])==(a0||!a0)`

Equationalized Axiom 13

We generate the "equationalized" axiom:

`(!Dnew[x1]||Nnew[x1])==(a0||!a0)`

Equationalized Axiom 14

We generate the "equationalized" axiom:

`((x1&&x2)||(x1&&x3))==(x1&&(x2||x3))`

Equationalized Axiom 15

We generate the "equationalized" axiom:

`(x1&&x2)==(x2&&x1)`

Equationalized Hypothesis 1

We generate the "equationalized" hypothesis:

`(!H[x0]||Nnew[x0])==(a0||!a0)`

Critical Pair Lemma 1

The following expressions are equivalent:

`((x1&&!x1)||x2)==x2`

Proof

Note that the input for the rule:

`x1_||x2_↔x2_||x1_`

contains a subpattern of the form:

`x1_||x2_`

which can be unified with the input for the rule:

`x1_||(x2_&&!x2_)→x1`

where these rules follow from Equationalized Axiom 3 and Equationalized Axiom 1 respectively.

Critical Pair Lemma 2

The following expressions are equivalent:

`(x1||(x2&&!x1))==(x1||x2)`

Proof

Note that the input for the rule:

`(x1_||x2_)&&(x1_||x3_)→x1||(x2&&x3)`

contains a subpattern of the form:

`(x1_||x2_)&&(x1_||x3_)`

`(x1_||x2_)&&(x1_||x3_)`

which can be unified with the input for the rule:

`x1_&&(x2_||!x2_)→x1`

where these rules follow from Equationalized Axiom 4 and Equationalized Axiom 2 respectively.

Critical Pair Lemma 3

The following expressions are equivalent:

`(x1||(x2&&x3))==((x2||x1)&&(x1||x3))`

Proof

Note that the input for the rule:

`(x1_||x2_)&&(x1_||x3_)→x1||(x2&&x3)`

contains a subpattern of the form:

`x1_||x2_`

which can be unified with the input for the rule:

`x1_||x2_↔x2_||x1_`

where these rules follow from Equationalized Axiom 4 and Equationalized Axiom 3 respectively.

Critical Pair Lemma 4

The following expressions are equivalent:

`(x1||(x2&&x3&&!x3))==((x1||x2)&&x1)`

Proof

Note that the input for the rule:

`(x1_||x2_)&&(x1_||x3_)→x1||(x2&&x3)`

contains a subpattern of the form:

`x1_||x3_`

which can be unified with the input for the rule:

`x1_||(x2_&&!x2_)→x1`

where these rules follow from Equationalized Axiom 4 and Equationalized Axiom 1 respectively.

Substitution Lemma 1

It can be shown that:

`(!L[x1]||Nnew[x1])==(a0||!a0)`

Proof

We start by taking Equationalized Axiom 5, and apply the substitution:

`x1_||x2_→x2||x1`

which follows from Equationalized Axiom 3.

Substitution Lemma 2

It can be shown that:

`(Dnew[x1]||R[x1])==(a0||!a0)`

Proof

We start by taking Equationalized Axiom 6, and apply the substitution:

`x1_||x2_→x2||x1`

which follows from Equationalized Axiom 3.

Substitution Lemma 3

It can be shown that:

`!L[x1_]||Nnew[x1_]→Dnew[x0]||R[x0]`

PROOF

We start by taking Substitution Lemma 1, and apply the substitution:

`a0||!a0→Dnew[x0]||R[x0]`

which follows from Substitution Lemma 2.

Substitution Lemma 4

It can be shown that:

`(!B[x1]||M[x1])==(a0||!a0)`

PROOF

We start by taking Equationalized Axiom 7, and apply the substitution:

`x1_||x2_→x2||x1`

which follows from Equationalized Axiom 3.

Substitution Lemma 5

It can be shown that:

`(!B[x1]||M[x1])==(Dnew[x0]||R[x0])`

PROOF

We start by taking Substitution Lemma 4, and apply the substitution:

`a0||!a0→Dnew[x0]||R[x0]`

which follows from Substitution Lemma 2.

Substitution Lemma 6

It can be shown that:

`(!Cnew[x1]||K[x1])==(a0||!a0)`

PROOF

We start by taking Equationalized Axiom 8, and apply the substitution:

`x1_||x2_→x2||x1`

which follows from Equationalized Axiom 3.

Substitution Lemma 7

It can be shown that:

`(!Cnew[x1]||K[x1])==(Dnew[x0]||R[x0])`

PROOF

We start by taking Substitution Lemma 6, and apply the substitution:

`a0||!a0→Dnew[x0]||R[x0]`

which follows from Substitution Lemma 2.

Substitution Lemma 8

It can be shown that:

`(!R[x1]||Enew[x1]) == (a0||!a0)`

Proof

We start by taking Equationalized Axiom 9, and apply the substitution:

`x1_||x2_→x2||x1`

which follows from Equationalized Axiom 3.

Substitution Lemma 9

It can be shown that:

`(!R[x1]||Enew[x1]) == (Dnew[x0]||R[x0])`

Proof

We start by taking Substitution Lemma 8, and apply the substitution:

`a0||!a0→Dnew[x0]||R[x0]`

which follows from Substitution Lemma 2.

Substitution Lemma 10

It can be shown that:

`(!Enew[x1]||Cnew[x1]) == (a0||!a0)`

Proof

We start by taking Equationalized Axiom 10, and apply the substitution:

`x1_||x2_→x2||x1`

which follows from Equationalized Axiom 3.

Substitution Lemma 11

It can be shown that:

`(!Enew[x1]||Cnew[x1]) == (Dnew[x0]||R[x0])`

Proof

We start by taking Substitution Lemma 10, and apply the substitution:

`a0||!a0→Dnew[x0]||R[x0]`

which follows from Substitution Lemma 2.

Substitution Lemma 12

It can be shown that:

`(!K[x1]||B[x1]) == (a0||!a0)`

Proof

PROOF

We start by taking Equationalized Axiom 11, and apply the substitution:

$x1\_||x2\_ \to x2||x1$

which follows from Equationalized Axiom 3.

Substitution Lemma 13

It can be shown that:

$(!K[x1]||B[x1]) == (Dnew[x_0]||R[x_0])$

PROOF

We start by taking Substitution Lemma 12, and apply the substitution:

$\underset{\cdot}{a}_0||!\underset{\cdot}{a}_0 \to Dnew[x_0]||R[x_0]$

which follows from Substitution Lemma 2.

Substitution Lemma 14

It can be shown that:

$(!M[x1]||L[x1]) == (\underset{\cdot}{a}_0||!\underset{\cdot}{a}_0)$

PROOF

We start by taking Equationalized Axiom 12, and apply the substitution:

$x1\_||x2\_ \to x2||x1$

which follows from Equationalized Axiom 3.

Substitution Lemma 15

It can be shown that:

$(!M[x1]||L[x1]) == (Dnew[x_0]||R[x_0])$

PROOF

We start by taking Substitution Lemma 14, and apply the substitution:

$\underset{\cdot}{a}_0||!\underset{\cdot}{a}_0 \to Dnew[x_0]||R[x_0]$

which follows from Substitution Lemma 2.

Substitution Lemma 16

It can be shown that:

$(Nnew[x1]||!Dnew[x1]) == (\underset{\cdot}{a}_0||!\underset{\cdot}{a}_0)$

PROOF

We start by taking Equationalized Axiom 13, and apply the substitution:

$x1\_||x2\_ \to x2||x1$

which follows from Equationalized Axiom 3.

Substitution Lemma 17

It can be shown that:

$(Nnew[x1]||!Dnew[x1]) == (Dnew[x_0]||R[x_0])$

PROOF

PROOF

We start by taking Substitution Lemma 16, and apply the substitution:

`a0||!a0→Dnew[x0]||R[x0]`

which follows from Substitution Lemma 2.

Substitution Lemma 18

It can be shown that:

`!L[x1_]||Nnew[x1_]→Nnew[x0]||!Dnew[x0]`

PROOF

We start by taking Substitution Lemma 3, and apply the substitution:

`Dnew[x0]||R[x0]→Nnew[x0]||!Dnew[x0]`

which follows from Substitution Lemma 17.

Substitution Lemma 19

It can be shown that:

`!M[x1_]||L[x1_]→Nnew[x0]||!Dnew[x0]`

PROOF

We start by taking Substitution Lemma 15, and apply the substitution:

`Dnew[x0]||R[x0]→Nnew[x0]||!Dnew[x0]`

which follows from Substitution Lemma 17.

Substitution Lemma 20

It can be shown that:

`!K[x1_]||B[x1_]→Nnew[x0]||!Dnew[x0]`

PROOF

We start by taking Substitution Lemma 13, and apply the substitution:

`Dnew[x0]||R[x0]→Nnew[x0]||!Dnew[x0]`

which follows from Substitution Lemma 17.

Substitution Lemma 21

It can be shown that:

`!Enew[x1_]||Cnew[x1_]→Nnew[x0]||!Dnew[x0]`

PROOF

We start by taking Substitution Lemma 11, and apply the substitution:

`Dnew[x0]||R[x0]→Nnew[x0]||!Dnew[x0]`

which follows from Substitution Lemma 17.

Substitution Lemma 22

It can be shown that:

`!R[x1_]||Enew[x1_]→Nnew[x0]||!Dnew[x0]`

PROOF

We start by taking Substitution Lemma 9, and apply the substitution:

`Dnew[x0]||R[x0]→Nnew[x0]||!Dnew[x0]`

which follows from Substitution Lemma 17.

Substitution Lemma 23

It can be shown that:

`!Cnew[x1_]||K[x1_]→Nnew[x0]||!Dnew[x0]`

Proof

We start by taking Substitution Lemma 7, and apply the substitution:

`Dnew[x0]||R[x0]→Nnew[x0]||!Dnew[x0]`

which follows from Substitution Lemma 17.

Substitution Lemma 24

It can be shown that:

`!B[x1_]||M[x1_]→Nnew[x0]||!Dnew[x0]`

Proof

We start by taking Substitution Lemma 5, and apply the substitution:

`Dnew[x0]||R[x0]→Nnew[x0]||!Dnew[x0]`

which follows from Substitution Lemma 17.

Critical Pair Lemma 5

The following expressions are equivalent:

`(x1&&(x2||!x1))==(x1&&x2)`

Proof

Note that the input for the rule:

`(x1_&&x2_)||(x1_&&x3_)→x1&&(x2||x3)`

contains a subpattern of the form:

`(x1_&&x2_)||(x1_&&x3_)`

which can be unified with the input for the rule:

`x1_||(x2_&&!x2_)→x1`

where these rules follow from Equationalized Axiom 14 and Equationalized Axiom 1 respectively.

Critical Pair Lemma 6

The following expressions are equivalent:

`((x1||!x1)&&x2)==x2`

Proof

Note that the input for the rule:

`x1_&&x2_↔x2_&&x1_`

contains a subpattern of the form:

`x1_&&x2_`

which can be unified with the input for the rule:

which can be unified with the input for the rule:

```
x1_&&(x2_||!x2_)→x1
```

where these rules follow from Equationalized Axiom 15 and Equationalized Axiom 2 respectively.

Critical Pair Lemma 7

The following expressions are equivalent:

```
(x1&&x2)==(x1&&(!x1||x2))
```

Proof

Note that the input for the rule:

```
(x1_&&!x1_)||x2_→x2
```

contains a subpattern of the form:

```
(x1_&&!x1_)||x2_
```

which can be unified with the input for the rule:

```
(x1_&&x2_)||(x1_&&x3_)→x1&&(x2||x3)
```

where these rules follow from Critical Pair Lemma 1 and Equationalized Axiom 14 respectively.

Critical Pair Lemma 8

The following expressions are equivalent:

```
(x1||x2)==(x1||(!x1&&x2))
```

Proof

Note that the input for the rule:

```
(x1_||!x1_)&&x2_→x2
```

contains a subpattern of the form:

```
(x1_||!x1_)&&x2_
```

which can be unified with the input for the rule:

```
(x1_||x2_)&&(x1_||x3_)→x1||(x2&&x3)
```

where these rules follow from Critical Pair Lemma 6 and Equationalized Axiom 4 respectively.

Critical Pair Lemma 9

The following expressions are equivalent:

```
(x1||!x1)==(x2||!x2)
```

Proof

Note that the input for the rule:

```
(x1_||!x1_)&&x2_→x2
```

contains a subpattern of the form:

```
(x1_||!x1_)&&x2_
```

which can be unified with the input for the rule:

```
x1_&&(x2_||!x2_)→x1
```

where these rules follow from Critical Pair Lemma 6 and Equationalized Axiom 2 respectively.

Critical Pair Lemma 10

The following expressions are equivalent:

The following expressions are equivalent:

`(x1||x1)==x1`

PROOF

Note that the input for the rule:

`x1_||(x2_&&!x1_)→x1||x2`

contains a subpattern of the form:

`x1_||(x2_&&!x1_)`

which can be unified with the input for the rule:

`x1_||(x2_&&!x2_)→x1`

where these rules follow from Critical Pair Lemma 2 and Equationalized Axiom 1 respectively.

Critical Pair Lemma 11

The following expressions are equivalent:

`(x1||(x1&&x2))==(x1&&(x1||x2))`

PROOF

Note that the input for the rule:

`(x1_||x2_)&&(x1_||x3_)→x1||(x2&&x3)`

contains a subpattern of the form:

`x1_||x2_`

which can be unified with the input for the rule:

`x1_||x1_→x1`

where these rules follow from Equationalized Axiom 4 and Critical Pair Lemma 10 respectively.

Critical Pair Lemma 12

The following expressions are equivalent:

`(x1||(x2&&x1))==((x1||x2)&&x1)`

PROOF

Note that the input for the rule:

`(x1_||x2_)&&(x1_||x3_)→x1||(x2&&x3)`

contains a subpattern of the form:

`x1_||x3_`

which can be unified with the input for the rule:

`x1_||x1_→x1`

where these rules follow from Equationalized Axiom 4 and Critical Pair Lemma 10 respectively.

Substitution Lemma 25

It can be shown that:

`(x1||(x2&&x1))==(x1&&(x1||x2))`

PROOF

We start by taking Critical Pair Lemma 12, and apply the substitution:

`x1_&&x2_→x2&&x1`

`x1_&&x2_→x2&&x1`

which follows from Equationalized Axiom 15.

Critical Pair Lemma 13

The following expressions are equivalent:

`(!R[x1]&&Dnew[x1])==(!R[x1]&&(a0||!a0))`

Proof

Note that the input for the rule:

`x1_&&(x2_||!x1_)→x1&&x2`

contains a subpattern of the form:

`x2_||!x1_`

which can be unified with the input for the rule:

`"0"`

where these rules follow from Critical Pair Lemma 5 and Substitution Lemma 2 respectively.

Substitution Lemma 26

It can be shown that:

`(!R[x1]&&Dnew[x1])==!R[x1]`

Proof

We start by taking Critical Pair Lemma 13, and apply the substitution:

`x1_&&(x2_||!x2_)→x1`

which follows from Equationalized Axiom 2.

Critical Pair Lemma 14

The following expressions are equivalent:

`(x1&&x1)==x1`

Proof

Note that the input for the rule:

`x1_&&(!x1_||x2_)→x1&&x2`

contains a subpattern of the form:

`x1_&&(!x1_||x2_)`

which can be unified with the input for the rule:

`x1_&&(x2_||!x2_)→x1`

where these rules follow from Critical Pair Lemma 7 and Equationalized Axiom 2 respectively.

Critical Pair Lemma 15

The following expressions are equivalent:

`True`

Proof

Note that the input for the rule:

`x1_||(!x1_&&x2_)→x1||x2`

contains a subpattern of the form:

`!x1_&&x2_`

which can be unified with the input for the rule:

`x1_&&x1_→x1`

where these rules follow from Critical Pair Lemma 8 and Critical Pair Lemma 14 respectively.

Substitution Lemma 27

It can be shown that:

`(Dnew[x1]&&!R[x1]) == !R[x1]`

Proof

We start by taking Substitution Lemma 26, and apply the substitution:

`x1_&&x2_→x2&&x1`

which follows from Equationalized Axiom 15.

Critical Pair Lemma 16

The following expressions are equivalent:

`(R[x1]||Dnew[x1]) == (R[x1]||!R[x1])`

Proof

Note that the input for the rule:

`x1_||(x2_&&!x1_)→x1||x2`

contains a subpattern of the form:

`x2_&&!x1_`

which can be unified with the input for the rule:

`Dnew[x1_]&&!R[x1_]→!R[x1]`

where these rules follow from Critical Pair Lemma 2 and Substitution Lemma 27 respectively.

Critical Pair Lemma 17

The following expressions are equivalent:

`(x1&&x1) == (x1&&(x1||!x1))`

Proof

Note that the input for the rule:

`x1_&&(x2_||!x1_)→x1&&x2`

contains a subpattern of the form:

`x2_||!x1_`

which can be unified with the input for the rule:

`x1_||!x1_→x1||!x1`

where these rules follow from Critical Pair Lemma 5 and Critical Pair Lemma 15 respectively.

Substitution Lemma 28

It can be shown that:

`(x1&&x1) == x1`

PROOF

We start by taking Critical Pair Lemma 17, and apply the substitution:

```
x1_&&(x2_||!x2_)→x1
```

which follows from Equationalized Axiom 2.

Substitution Lemma 29

It can be shown that:

```
(x1&&x1)==x1
```

PROOF

We start by taking Substitution Lemma 28, and apply the substitution:

```
x1_&&x2_→x2&&x1
```

which follows from Equationalized Axiom 15.

Substitution Lemma 30

It can be shown that:

```
True
```

PROOF

We start by taking Substitution Lemma 29, and apply the substitution:

```
x1_&&x1_→x1
```

which follows from Critical Pair Lemma 14.

Critical Pair Lemma 18

The following expressions are equivalent:

```
(!x1||x2)==(!x1||(x1&&x2))
```

PROOF

Note that the input for the rule:

```
x1_||(!x1_&&x2_)→x1||x2
```

contains a subpattern of the form:

```
!x1_
```

which can be unified with the input for the rule:

```
x1_→x1
```

where these rules follow from Critical Pair Lemma 8 and Substitution Lemma 30 respectively.

Critical Pair Lemma 19

The following expressions are equivalent:

```
(!x1&&x2)==(!x1&&(x2||x1))
```

PROOF

Note that the input for the rule:

```
x1_&&(x2_||!x1_)→x1&&x2
```

contains a subpattern of the form:

`!x1_`

which can be unified with the input for the rule:

`x1_→x1`

where these rules follow from Critical Pair Lemma 5 and Substitution Lemma 30 respectively.

Substitution Lemma 31

It can be shown that:

`(Nnew[x1] || !Dnew[x1]) == (Dnew[x0] || R[x0])`

Proof

We start by taking Substitution Lemma 17, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 30.

Substitution Lemma 32

It can be shown that:

`(!L[x1] || Nnew[x1]) == (Nnew[x0] || !Dnew[x0])`

Proof

We start by taking Substitution Lemma 18, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 30.

Substitution Lemma 33

It can be shown that:

`(!M[x1] || L[x1]) == (Nnew[x0] || !Dnew[x0])`

Proof

We start by taking Substitution Lemma 19, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 30.

Substitution Lemma 34

It can be shown that:

`(!K[x1] || B[x1]) == (Nnew[x0] || !Dnew[x0])`

Proof

We start by taking Substitution Lemma 20, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 30.

Substitution Lemma 35

It can be shown that:

`(!Enew[x1] || Cnew[x1]) == (Nnew[x0] || !Dnew[x0])`

Proof

We start by taking Substitution Lemma 21, and apply the substitution:

```
x1_→x1
```

which follows from Substitution Lemma 30.

Substitution Lemma 36

It can be shown that:

```
(!R[x1]||Enew[x1])==(Nnew[x0]||!Dnew[x0])
```

Proof

We start by taking Substitution Lemma 22, and apply the substitution:

```
x1_→x1
```

which follows from Substitution Lemma 30.

Substitution Lemma 37

It can be shown that:

```
(!Cnew[x1]||K[x1])==(Nnew[x0]||!Dnew[x0])
```

Proof

We start by taking Substitution Lemma 23, and apply the substitution:

```
x1_→x1
```

which follows from Substitution Lemma 30.

Substitution Lemma 38

It can be shown that:

```
(!B[x1]||M[x1])==(Nnew[x0]||!Dnew[x0])
```

Proof

We start by taking Substitution Lemma 24, and apply the substitution:

```
x1_→x1
```

which follows from Substitution Lemma 30.

Critical Pair Lemma 20

The following expressions are equivalent:

```
(x1&&x1&&x2)==(x1&&(!x1||x2))
```

Proof

Note that the input for the rule:

```
x1_&&(!x1_||x2_)→x1&&x2
```

contains a subpattern of the form:

```
!x1_||x2_
```

which can be unified with the input for the rule:

```
!x1_||(x1_&&x2_)→!x1||x2
```

where these rules follow from Critical Pair Lemma 7 and Critical Pair Lemma 18 respectively.

Substitution Lemma 39

It can be shown that:

```
(x1&&x1&&x2) == (x1&&x2)
```

PROOF

We start by taking Critical Pair Lemma 20, and apply the substitution:

```
x1_&&(!x1_||x2_)→x1&&x2
```

which follows from Critical Pair Lemma 7.

Critical Pair Lemma 21

The following expressions are equivalent:

```
(x1&&x2) == (x1&&x2&&x1)
```

PROOF

Note that the input for the rule:

```
x1_&&x1_&&x2_→x1&&x2
```

contains a subpattern of the form:

```
x1_&&x2_
```

which can be unified with the input for the rule:

```
x1_&&x2_↔x2_&&x1_
```

where these rules follow from Substitution Lemma 39 and Equationalized Axiom 15 respectively.

Substitution Lemma 40

It can be shown that:

```
(R[x1]||Dnew[x1]) == (x0||!x0)
```

PROOF

We start by taking Critical Pair Lemma 16, and apply the substitution:

```
x1_||!x1_→x0||!x0
```

which follows from Critical Pair Lemma 9.

Critical Pair Lemma 22

The following expressions are equivalent:

```
(R[x1]||(Dnew[x1]&&x2)) == ((x0||!x0)&&(R[x1]||x2))
```

PROOF

Note that the input for the rule:

```
(x1_||x2_)&&(x1_||x3_)→x1||(x2&&x3)
```

contains a subpattern of the form:

```
x1_||x2_
```

which can be unified with the input for the rule:

```
R[x1_]||Dnew[x1_]→x0||!x0
```

where these rules follow from Equationalized Axiom 4 and Substitution Lemma 40 respectively.

Substitution Lemma 41

It can be shown that:

It can be shown that:

`(R[x1]||(Dnew[x1]&&x2))==(R[x1]||x2)`

PROOF

We start by taking Critical Pair Lemma 22, and apply the substitution:

`(x1_||!x1_)&&x2_→x2`

which follows from Critical Pair Lemma 6.

Substitution Lemma 42

It can be shown that:

`(Nnew[x1]||!Dnew[x1])==(R[x0]||Dnew[x0])`

PROOF

We start by taking Substitution Lemma 31, and apply the substitution:

`x1_||x2_→x2||x1`

which follows from Equationalized Axiom 3.

Substitution Lemma 43

It can be shown that:

`(Nnew[x1]||!Dnew[x1])==(x0||!x0)`

PROOF

We start by taking Substitution Lemma 42, and apply the substitution:

`R[x1_]||Dnew[x1_]→x0||!x0`

which follows from Substitution Lemma 40.

Critical Pair Lemma 23

The following expressions are equivalent:

`(Dnew[x1]&&Nnew[x1])==(Dnew[x1]&&(x0||!x0))`

PROOF

Note that the input for the rule:

`!x1_&&(x2_||x1_)→!x1&&x2`

contains a subpattern of the form:

`x2_||x1_`

which can be unified with the input for the rule:

`Nnew[x1_]||!Dnew[x1_]→x0||!x0`

where these rules follow from Critical Pair Lemma 19 and Substitution Lemma 43 respectively.

Substitution Lemma 44

It can be shown that:

`(Dnew[x1]&&Nnew[x1])==(Dnew[x1]&&(x0||!x0))`

PROOF

We start by taking Critical Pair Lemma 23, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 30.

Substitution Lemma 45

It can be shown that:

```
(Dnew[x1]&&Nnew[x1])==Dnew[x1]
```

Proof

We start by taking Substitution Lemma 44, and apply the substitution:

```
x1_&&(x2_||!x2_)→x1
```

which follows from Equationalized Axiom 2.

Substitution Lemma 46

It can be shown that:

```
(Dnew[x1]&&Nnew[x1])==Dnew[x1]
```

Proof

We start by taking Substitution Lemma 45, and apply the substitution:

```
x1_→x1
```

which follows from Substitution Lemma 30.

Substitution Lemma 47

It can be shown that:

```
(Nnew[x1]||!L[x1])==(Nnew[x0]||!Dnew[x0])
```

Proof

We start by taking Substitution Lemma 32, and apply the substitution:

```
x1_||x2_→x2||x1
```

which follows from Equationalized Axiom 3.

Substitution Lemma 48

It can be shown that:

```
(Nnew[x1]||!L[x1])==(x0||!x0)
```

Proof

We start by taking Substitution Lemma 47, and apply the substitution:

```
Nnew[x1_]||!Dnew[x1_]→x0||!x0
```

which follows from Substitution Lemma 43.

Critical Pair Lemma 24

The following expressions are equivalent:

```
(L[x1]&&Nnew[x1])==(L[x1]&&(x0||!x0))
```

Proof

Note that the input for the rule:

```
!x1_&&(x2_||x1_)→!x1&&x2
```

contains a subpattern of the form:

`x2_||x1_`

which can be unified with the input for the rule:

`Nnew[x1_]||!L[x1_]→x0||!x0`

where these rules follow from Critical Pair Lemma 19 and Substitution Lemma 48 respectively.

Substitution Lemma 49

It can be shown that:

`(L[x1]&&Nnew[x1])==(L[x1]&&(x0||!x0))`

Proof

We start by taking Critical Pair Lemma 24, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 30.

Substitution Lemma 50

It can be shown that:

`(L[x1]&&Nnew[x1])==L[x1]`

Proof

We start by taking Substitution Lemma 49, and apply the substitution:

`x1_&&(x2_||!x2_)→x1`

which follows from Equationalized Axiom 2.

Substitution Lemma 51

It can be shown that:

`(L[x1]&&Nnew[x1])==L[x1]`

Proof

We start by taking Substitution Lemma 50, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 30.

Substitution Lemma 52

It can be shown that:

`(L[x1]||!M[x1])==(Nnew[x0]||!Dnew[x0])`

Proof

We start by taking Substitution Lemma 33, and apply the substitution:

`x1_||x2_→x2||x1`

which follows from Equationalized Axiom 3.

Substitution Lemma 53

It can be shown that:

`(L[x1]||!M[x1])==(x0||!x0)`

PROOF

We start by taking Substitution Lemma 52, and apply the substitution:

`Nnew[x1_]||!Dnew[x1_]→x0||!x0`

which follows from Substitution Lemma 43.

Critical Pair Lemma 25

The following expressions are equivalent:

`(M[x1]&&L[x1])==(M[x1]&&(x0||!x0))`

PROOF

Note that the input for the rule:

`!x1_&&(x2_||x1_)→!x1&&x2`

contains a subpattern of the form:

`x2_||x1_`

which can be unified with the input for the rule:

`L[x1_]||!M[x1_]→x0||!x0`

where these rules follow from Critical Pair Lemma 19 and Substitution Lemma 53 respectively.

Substitution Lemma 54

It can be shown that:

`(M[x1]&&L[x1])==(M[x1]&&(x0||!x0))`

PROOF

We start by taking Critical Pair Lemma 25, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 30.

Substitution Lemma 55

It can be shown that:

`(M[x1]&&L[x1])==M[x1]`

PROOF

We start by taking Substitution Lemma 54, and apply the substitution:

`x1_&&(x2_||!x2_)→x1`

which follows from Equationalized Axiom 2.

Substitution Lemma 56

It can be shown that:

`(M[x1]&&L[x1])==M[x1]`

PROOF

We start by taking Substitution Lemma 55, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 30.

Substitution Lemma 57

Substitution Lemma 57

It can be shown that:

`(L[x1]&&M[x1])==M[x1]`

Proof

We start by taking Substitution Lemma 56, and apply the substitution:

`x1_&&x2_→x2&&x1`

which follows from Equationalized Axiom 15.

Substitution Lemma 58

It can be shown that:

`(B[x1]||!K[x1])==(Nnew[x0]||!Dnew[x0])`

Proof

We start by taking Substitution Lemma 34, and apply the substitution:

`x1_||x2_→x2||x1`

which follows from Equationalized Axiom 3.

Substitution Lemma 59

It can be shown that:

`(B[x1]||!K[x1])==(x0||!x0)`

Proof

We start by taking Substitution Lemma 58, and apply the substitution:

`Nnew[x1_]||!Dnew[x1_]→x0||!x0`

which follows from Substitution Lemma 43.

Critical Pair Lemma 26

The following expressions are equivalent:

`(K[x1]&&B[x1])==(K[x1]&&(x0||!x0))`

Proof

Note that the input for the rule:

`!x1_&&(x2_||x1_)→!x1&&x2`

contains a subpattern of the form:

`x2_||x1_`

which can be unified with the input for the rule:

`B[x1_]||!K[x1_]→x0||!x0`

where these rules follow from Critical Pair Lemma 19 and Substitution Lemma 59 respectively.

Substitution Lemma 60

It can be shown that:

`(K[x1]&&B[x1])==(K[x1]&&(x0||!x0))`

Out[14]=

Proof

We start by taking Critical Pair Lemma 26, and apply the substitution:

We start by taking Critical Pair Lemma 26, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 30.

Substitution Lemma 61

It can be shown that:

`(K[x1]&&B[x1])==K[x1]`

Proof

We start by taking Substitution Lemma 60, and apply the substitution:

`x1_&&(x2_||!x2_)→x1`

which follows from Equationalized Axiom 2.

Substitution Lemma 62

It can be shown that:

`(K[x1]&&B[x1])==K[x1]`

Proof

We start by taking Substitution Lemma 61, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 30.

Substitution Lemma 63

It can be shown that:

`(B[x1]&&K[x1])==K[x1]`

Proof

We start by taking Substitution Lemma 62, and apply the substitution:

`x1_&&x2_→x2&&x1`

which follows from Equationalized Axiom 15.

Substitution Lemma 64

It can be shown that:

(Cnew[x1]||!Enew[x1])==(Nnew[x_0]||!Dnew[x_0])

Proof

We start by taking Substitution Lemma 35, and apply the substitution:

`x1_||x2_→x2||x1`

which follows from Equationalized Axiom 3.

Substitution Lemma 65

It can be shown that:

(Cnew[x1]||!Enew[x1])==(x_0||!x_0)

Proof

We start by taking Substitution Lemma 64, and apply the substitution:

`Nnew[x1_]||!Dnew[x1_]→x0||!x0`

which follows from Substitution Lemma 43.

Critical Pair Lemma 27

The following expressions are equivalent:

`(Enew[x1]&&Cnew[x1])==(Enew[x1]&&(x0||!x0))`

Proof

Note that the input for the rule:

`!x1_&&(x2_||x1_)→!x1&&x2`

contains a subpattern of the form:

`x2_||x1_`

which can be unified with the input for the rule:

`Cnew[x1_]||!Enew[x1_]→x0||!x0`

where these rules follow from Critical Pair Lemma 19 and Substitution Lemma 65 respectively.

Substitution Lemma 66

It can be shown that:

`(Enew[x1]&&Cnew[x1])==(Enew[x1]&&(x0||!x0))`

Proof

We start by taking Critical Pair Lemma 27, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 30.

Substitution Lemma 67

It can be shown that:

`(Enew[x1]&&Cnew[x1])==Enew[x1]`

Proof

We start by taking Substitution Lemma 66, and apply the substitution:

`x1_&&(x2_||!x2_)→x1`

which follows from Equationalized Axiom 2.

Substitution Lemma 68

It can be shown that:

`(Enew[x1]&&Cnew[x1])==Enew[x1]`

Proof

We start by taking Substitution Lemma 67, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 30.

Substitution Lemma 69

It can be shown that:

`(Cnew[x1]&&Enew[x1])==Enew[x1]`

Proof

We start by taking Substitution Lemma 68, and apply the substitution:

`x1_&&x2_→x2&&x1`

which follows from Equationalized Axiom 15.

Substitution Lemma 70

It can be shown that:

`(Enew[x1]||!R[x1])==(Nnew[x0]||!Dnew[x0])`

Proof

We start by taking Substitution Lemma 36, and apply the substitution:

`x1_||x2_→x2||x1`

which follows from Equationalized Axiom 3.

Substitution Lemma 71

It can be shown that:

`(Enew[x1]||!R[x1])==(x0||!x0)`

Proof

We start by taking Substitution Lemma 70, and apply the substitution:

`Nnew[x1_]||!Dnew[x1_]→x0||!x0`

which follows from Substitution Lemma 43.

Critical Pair Lemma 28

The following expressions are equivalent:

`(R[x1]&&Enew[x1])==(R[x1]&&(x0||!x0))`

Proof

Note that the input for the rule:

`!x1_&&(x2_||x1_)→!x1&&x2`

contains a subpattern of the form:

`x2_||x1_`

which can be unified with the input for the rule:

`Enew[x1_]||!R[x1_]→x0||!x0`

where these rules follow from Critical Pair Lemma 19 and Substitution Lemma 71 respectively.

Substitution Lemma 72

It can be shown that:

`(R[x1]&&Enew[x1])==(R[x1]&&(x0||!x0))`

Proof

We start by taking Critical Pair Lemma 28, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 30.

which follows from Substitution Lemma 30.

Substitution Lemma 73

It can be shown that:

`(R[x1]&&Enew[x1])==R[x1]`

Proof

We start by taking Substitution Lemma 72, and apply the substitution:

`x1_&&(x2_||!x2_)→x1`

which follows from Equationalized Axiom 2.

Substitution Lemma 74

It can be shown that:

`(R[x1]&&Enew[x1])==R[x1]`

Proof

We start by taking Substitution Lemma 73, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 30.

Substitution Lemma 75

It can be shown that:

`(Enew[x1]&&R[x1])==R[x1]`

Proof

We start by taking Substitution Lemma 74, and apply the substitution:

`x1_&&x2_→x2&&x1`

which follows from Equationalized Axiom 15.

Substitution Lemma 76

It can be shown that:

`(K[x1]||!Cnew[x1])==(Nnew[x0]||!Dnew[x0])`

Proof

We start by taking Substitution Lemma 37, and apply the substitution:

`x1_||x2_→x2||x1`

which follows from Equationalized Axiom 3.

Substitution Lemma 77

It can be shown that:

`(K[x1]||!Cnew[x1])==(x0||!x0)`

Proof

We start by taking Substitution Lemma 76, and apply the substitution:

`Nnew[x1_]||!Dnew[x1_]→x0||!x0`

which follows from Substitution Lemma 43.

Critical Pair Lemma 29

The following expressions are equivalent:

```
(Cnew[x1]&&K[x1]) == (Cnew[x1]&&(x0||!x0))
```

Proof

Note that the input for the rule:

```
!x1_&&(x2_||x1_)→!x1&&x2
```

contains a subpattern of the form:

```
x2_||x1_
```

which can be unified with the input for the rule:

```
K[x1_]||!Cnew[x1_]→x0||!x0
```

where these rules follow from Critical Pair Lemma 19 and Substitution Lemma 77 respectively.

Substitution Lemma 78

It can be shown that:

```
(Cnew[x1]&&K[x1]) == (Cnew[x1]&&(x0||!x0))
```

Proof

We start by taking Critical Pair Lemma 29, and apply the substitution:

```
x1_→x1
```

which follows from Substitution Lemma 30.

Substitution Lemma 79

It can be shown that:

```
(Cnew[x1]&&K[x1]) == Cnew[x1]
```

Proof

We start by taking Substitution Lemma 78, and apply the substitution:

```
x1_&&(x2_||!x2_)→x1
```

which follows from Equationalized Axiom 2.

Substitution Lemma 80

It can be shown that:

```
(Cnew[x1]&&K[x1]) == Cnew[x1]
```

Proof

We start by taking Substitution Lemma 79, and apply the substitution:

```
x1_→x1
```

which follows from Substitution Lemma 30.

Substitution Lemma 81

It can be shown that:

```
(M[x1]||!B[x1]) == (Nnew[x0]||!Dnew[x0])
```

Proof

We start by taking Substitution Lemma 38, and apply the substitution:

`x1_||x2_→x2||x1`

which follows from Equationalized Axiom 3.

Substitution Lemma 82

It can be shown that:

`(M[x1]||!B[x1])==(x0||!x0)`

Proof

We start by taking Substitution Lemma 81, and apply the substitution:

`Nnew[x1_]||!Dnew[x1_]→x0||!x0`

which follows from Substitution Lemma 43.

Critical Pair Lemma 30

The following expressions are equivalent:

`(B[x1]&&M[x1])==(B[x1]&&(x0||!x0))`

Proof

Note that the input for the rule:

`!x1_&&(x2_||x1_)→!x1&&x2`

contains a subpattern of the form:

`x2_||x1_`

which can be unified with the input for the rule:

`M[x1_]||!B[x1_]→x0||!x0`

where these rules follow from Critical Pair Lemma 19 and Substitution Lemma 82 respectively.

Substitution Lemma 83

It can be shown that:

`(B[x1]&&M[x1])==(B[x1]&&(x0||!x0))`

Proof

We start by taking Critical Pair Lemma 30, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 30.

Substitution Lemma 84

It can be shown that:

`(B[x1]&&M[x1])==B[x1]`

Proof

We start by taking Substitution Lemma 83, and apply the substitution:

`x1_&&(x2_||!x2_)→x1`

which follows from Equationalized Axiom 2.

Substitution Lemma 85

It can be shown that:

`(B[x1]&&M[x1])==B[x1]`

Proof

We start by taking Substitution Lemma 84, and apply the substitution:

`x1_→x1`

which follows from Substitution Lemma 30.

Critical Pair Lemma 31

The following expressions are equivalent:

`(R[x1]||Nnew[x1])==(R[x1]||Dnew[x1])`

Proof

Note that the input for the rule:

`R[x1_]||(Dnew[x1_]&&x2_)→R[x1]||x2`

contains a subpattern of the form:

`Dnew[x1_]&&x2_`

which can be unified with the input for the rule:

`Dnew[x1_]&&Nnew[x1_]→Dnew[x1]`

where these rules follow from Substitution Lemma 41 and Substitution Lemma 46 respectively.

Substitution Lemma 86

It can be shown that:

`(R[x1]||Nnew[x1])==(x0||!x0)`

Proof

We start by taking Critical Pair Lemma 31, and apply the substitution:

`R[x1_]||Dnew[x1_]→x0||!x0`

which follows from Substitution Lemma 40.

Critical Pair Lemma 32

The following expressions are equivalent:

`(Nnew[x1]||(R[x1]&&x2))==((x0||!x0)&&(Nnew[x1]||x2))`

Proof

Note that the input for the rule:

`(x1_||x2_)&&(x2_||x3_)→x2||(x1&&x3)`

contains a subpattern of the form:

`x1_||x2_`

which can be unified with the input for the rule:

`R[x1_]||Nnew[x1_]→x0||!x0`

where these rules follow from Critical Pair Lemma 3 and Substitution Lemma 86 respectively.

Substitution Lemma 87

It can be shown that:

It can be shown that:

`(Nnew[x1]||(R[x1]&&x2))==(Nnew[x1]||x2)`

PROOF

We start by taking Critical Pair Lemma 32, and apply the substitution:

`(x1_||!x1_)&&x2_→x2`

which follows from Critical Pair Lemma 6.

Critical Pair Lemma 33

The following expressions are equivalent:

`(Nnew[x1]||(x2&&R[x1]))==(Nnew[x1]||(R[x1]&&x2))`

PROOF

Note that the input for the rule:

`Nnew[x1_]||(R[x1_]&&x2_)→Nnew[x1]||x2`

contains a subpattern of the form:

`R[x1_]&&x2_`

which can be unified with the input for the rule:

`x1_&&x2_&&x1_→x1&&x2`

where these rules follow from Substitution Lemma 87 and Critical Pair Lemma 21 respectively.

Substitution Lemma 88

It can be shown that:

`(Nnew[x1]||(x2&&R[x1]))==(Nnew[x1]||x2)`

PROOF

We start by taking Critical Pair Lemma 33, and apply the substitution:

`Nnew[x1_]||(R[x1_]&&x2_)→Nnew[x1]||x2`

which follows from Substitution Lemma 87.

Critical Pair Lemma 34

The following expressions are equivalent:

`(Nnew[x1]||Enew[x1])==(Nnew[x1]||R[x1])`

PROOF

Note that the input for the rule:

`Nnew[x1_]||(x2_&&R[x1_])→Nnew[x1]||x2`

contains a subpattern of the form:

`x2_&&R[x1_]`

which can be unified with the input for the rule:

`Enew[x1_]&&R[x1_]→R[x1]`

where these rules follow from Substitution Lemma 88 and Substitution Lemma 75 respectively.

Substitution Lemma 89

It can be shown that:

`(Enew[x1]||Nnew[x1])==(Nnew[x1]||R[x1])`

PROOF

We start by taking Critical Pair Lemma 34, and apply the substitution:

`x1_||x2_→x2||x1`

which follows from Equationalized Axiom 3.

Substitution Lemma 90

It can be shown that:

`(Enew[x1]||Nnew[x1])==(R[x1]||Nnew[x1])`

PROOF

We start by taking Substitution Lemma 89, and apply the substitution:

`x1_||x2_→x2||x1`

which follows from Equationalized Axiom 3.

Substitution Lemma 91

It can be shown that:

`(Enew[x1]||Nnew[x1])==(x0||!x0)`

PROOF

We start by taking Substitution Lemma 90, and apply the substitution:

`R[x1_]||Nnew[x1_]→x0||!x0`

which follows from Substitution Lemma 86.

Critical Pair Lemma 35

The following expressions are equivalent:

`(Nnew[x1]||(Enew[x1]&&x2))==((x0||!x0)&&(Nnew[x1]||x2))`

PROOF

Note that the input for the rule:

`(x1_||x2_)&&(x2_||x3_)→x2||(x1&&x3)`

contains a subpattern of the form:

`x1_||x2_`

which can be unified with the input for the rule:

`Enew[x1_]||Nnew[x1_]→x0||!x0`

where these rules follow from Critical Pair Lemma 3 and Substitution Lemma 91 respectively.

Substitution Lemma 92

It can be shown that:

`(Nnew[x1]||(Enew[x1]&&x2))==(Nnew[x1]||x2)`

PROOF

We start by taking Critical Pair Lemma 35, and apply the substitution:

`(x1_||!x1_)&&x2_→x2`

which follows from Critical Pair Lemma 6.

which follows from Critical Pair Lemma 6.

Critical Pair Lemma 36

The following expressions are equivalent:

```
(Nnew[x1]||(x2&&Enew[x1]))==(Nnew[x1]||(Enew[x1]&&x2))
```

PROOF

Note that the input for the rule:

```
Nnew[x1_]||(Enew[x1_]&&x2_)→Nnew[x1]||x2
```

contains a subpattern of the form:

```
Enew[x1_]&&x2_
```

which can be unified with the input for the rule:

```
x1_&&x2_&&x1_→x1&&x2
```

where these rules follow from Substitution Lemma 92 and Critical Pair Lemma 21 respectively.

Substitution Lemma 93

It can be shown that:

```
(Nnew[x1]||(x2&&Enew[x1]))==(Nnew[x1]||x2)
```

PROOF

We start by taking Critical Pair Lemma 36, and apply the substitution:

```
Nnew[x1_]||(Enew[x1_]&&x2_)→Nnew[x1]||x2
```

which follows from Substitution Lemma 92.

Critical Pair Lemma 37

The following expressions are equivalent:

```
(Nnew[x1]||Cnew[x1])==(Nnew[x1]||Enew[x1])
```

PROOF

Note that the input for the rule:

```
Nnew[x1_]||(x2_&&Enew[x1_])→Nnew[x1]||x2
```

contains a subpattern of the form:

```
x2_&&Enew[x1_]
```

which can be unified with the input for the rule:

```
Cnew[x1_]&&Enew[x1_]→Enew[x1]
```

where these rules follow from Substitution Lemma 93 and Substitution Lemma 69 respectively.

Substitution Lemma 94

It can be shown that:

```
(Cnew[x1]||Nnew[x1])==(Nnew[x1]||Enew[x1])
```

PROOF

We start by taking Critical Pair Lemma 37, and apply the substitution:

```
x1_||x2_→x2||x1
```

which follows from Equationalized Axiom 3.

Substitution Lemma 95

It can be shown that:

`(Cnew[x1]||Nnew[x1])==(Enew[x1]||Nnew[x1])`

Proof

We start by taking Substitution Lemma 94, and apply the substitution:

`x1_||x2_→x2||x1`

which follows from Equationalized Axiom 3.

Substitution Lemma 96

It can be shown that:

`(Cnew[x1]||Nnew[x1])==(x0||!x0)`

Proof

We start by taking Substitution Lemma 95, and apply the substitution:

`Enew[x1_]||Nnew[x1_]→x0||!x0`

which follows from Substitution Lemma 91.

Critical Pair Lemma 38

The following expressions are equivalent:

`(Nnew[x1]||(Cnew[x1]&&x2))==((x0||!x0)&&(Nnew[x1]||x2))`

Proof

Note that the input for the rule:

`(x1_||x2_)&&(x2_||x3_)→x2||(x1&&x3)`

contains a subpattern of the form:

`x1_||x2_`

which can be unified with the input for the rule:

`Cnew[x1_]||Nnew[x1_]→x0||!x0`

where these rules follow from Critical Pair Lemma 3 and Substitution Lemma 96 respectively.

Substitution Lemma 97

It can be shown that:

`(Nnew[x1]||(Cnew[x1]&&x2))==(Nnew[x1]||x2)`

Proof

We start by taking Critical Pair Lemma 38, and apply the substitution:

`(x1_||!x1_)&&x2_→x2`

which follows from Critical Pair Lemma 6.

Substitution Lemma 98

It can be shown that:

`(x1||(x2&&x3&&!x3))==(x1&&(x1||x2))`

Proof

We start by taking Critical Pair Lemma 4, and apply the substitution:

```
x1_&&x2_→x2&&x1
```

which follows from Equationalized Axiom 15.

Critical Pair Lemma 39

The following expressions are equivalent:

```
(x1&&(x1||x2))⩵(x1||(x2&&!x2))
```

Proof

Note that the input for the rule:

```
x1_||(x2_&&x3_&&!x3_)↔x1_&&(x1_||x2_)
```

contains a subpattern of the form:

```
x2_&&x3_&&!x3_
```

which can be unified with the input for the rule:

```
x1_&&x1_&&x2_→x1&&x2
```

where these rules follow from Substitution Lemma 98 and Substitution Lemma 39 respectively.

Substitution Lemma 99

It can be shown that:

```
(x1&&(x1||x2))⩵x1
```

Proof

We start by taking Critical Pair Lemma 39, and apply the substitution:

```
x1_||(x2_&&!x2_)→x1
```

which follows from Equationalized Axiom 1.

Substitution Lemma 100

It can be shown that:

```
x1_||(x2_&&x1_)→x1
```

Proof

We start by taking Substitution Lemma 25, and apply the substitution:

```
x1_&&(x1_||x2_)→x1
```

which follows from Substitution Lemma 99.

Substitution Lemma 101

It can be shown that:

```
x1_||(x1_&&x2_)→x1
```

Proof

We start by taking Critical Pair Lemma 11, and apply the substitution:

```
x1_&&(x1_||x2_)→x1
```

which follows from Substitution Lemma 99.

Critical Pair Lemma 40

The following expressions are equivalent:

`(x1||!x1)==(x1||!x1||x2)`

PROOF

Note that the input for the rule:

`x1_&&(x1_||x2_)→x1`

contains a subpattern of the form:

`x1_&&(x1_||x2_)`

which can be unified with the input for the rule:

`(x1_||!x1_)&&x2_→x2`

where these rules follow from Substitution Lemma 99 and Critical Pair Lemma 6 respectively.

Critical Pair Lemma 41

The following expressions are equivalent:

`x1==(x1&&(x2||x1))`

PROOF

Note that the input for the rule:

`x1_&&(x1_||x2_)→x1`

contains a subpattern of the form:

`x1_||x2_`

which can be unified with the input for the rule:

`x1_||x2_↔x2_||x1_`

where these rules follow from Substitution Lemma 99 and Equationalized Axiom 3 respectively.

Critical Pair Lemma 42

The following expressions are equivalent:

`(x1&&(x2||x1||x3))==(x1||(x1&&x3))`

PROOF

Note that the input for the rule:

`(x1_&&x2_)||(x1_&&x3_)→x1&&(x2||x3)`

contains a subpattern of the form:

`x1_&&x2_`

which can be unified with the input for the rule:

`x1_&&(x2_||x1_)→x1`

where these rules follow from Equationalized Axiom 14 and Critical Pair Lemma 41 respectively.

Substitution Lemma 102

It can be shown that:

`(x1&&(x2||x1||x3))==x1`

PROOF

We start by taking Critical Pair Lemma 42, and apply the substitution:

```
x1_||(x1_&&x2_)→x1
```

which follows from Substitution Lemma 101.

Critical Pair Lemma 43

The following expressions are equivalent:

```
x1⩵(x1&&(x2||x3||x1))
```

Proof

Note that the input for the rule:

```
x1_&&(x2_||x1_||x3_)→x1
```

contains a subpattern of the form:

```
x2_||x1_||x3_
```

which can be unified with the input for the rule:

```
x1_||x2_↔x2_||x1_
```

where these rules follow from Substitution Lemma 102 and Equationalized Axiom 3 respectively.

Critical Pair Lemma 44

The following expressions are equivalent:

```
(x1&&x2)⩵(x1&&x2&&(x2||x3))
```

Proof

Note that the input for the rule:

```
x1_&&(x2_||x1_||x3_)→x1
```

contains a subpattern of the form:

```
x2_||x1_
```

which can be unified with the input for the rule:

```
Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[2470,x1_&&(x1_|
```

where these rules follow from Substitution Lemma 102 and Substitution Lemma 100 respectively.

Critical Pair Lemma 45

The following expressions are equivalent:

```
(x1&&x2)⩵(x1&&x2&&(x1||x3))
```

Proof

Note that the input for the rule:

```
x1_&&(x2_||x1_||x3_)→x1
```

contains a subpattern of the form:

```
x2_||x1_
```

which can be unified with the input for the rule:

```
Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[2471,x1_&&(x1_|
```

where these rules follow from Substitution Lemma 102 and Substitution Lemma 101 respectively.

Critical Pair Lemma 46

The following expressions are equivalent:

```
(x1&&x2) == (x1&&x2&&(x3||x2))
```

PROOF

Note that the input for the rule:

```
x1_&&(x2_||x3_||x1_)→x1
```

contains a subpattern of the form:

```
x3_||x1_
```

which can be unified with the input for the rule:

```
Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[2470,x1_&&(x1_|
```

where these rules follow from Critical Pair Lemma 43 and Substitution Lemma 100 respectively.

Critical Pair Lemma 47

The following expressions are equivalent:

```
(Cnew[x1]&&K[x1]) == (Cnew[x1]&&(K[x1]||x2))
```

PROOF

Note that the input for the rule:

```
x1_&&x2_&&(x2_||x3_)→x1&&x2
```

contains a subpattern of the form:

```
x1_&&x2_
```

which can be unified with the input for the rule:

```
Cnew[x1_]&&K[x1_]→Cnew[x1]
```

where these rules follow from Critical Pair Lemma 44 and Substitution Lemma 80 respectively.

Substitution Lemma 103

It can be shown that:

```
Cnew[x1] == (Cnew[x1]&&(K[x1]||x2))
```

PROOF

We start by taking Critical Pair Lemma 47, and apply the substitution:

```
Cnew[x1_]&&K[x1_]→Cnew[x1]
```

which follows from Substitution Lemma 80.

Critical Pair Lemma 48

The following expressions are equivalent:

```
(B[x1]&&K[x1]) == (K[x1]&&(B[x1]||x2))
```

PROOF

Note that the input for the rule:

```
x1_&&x2_&&(x1_||x3_)→x1&&x2
```

contains a subpattern of the form:

```
x1_&&x2_
```

which can be unified with the input for the rule:

```
B[x1_]&&K[x1_]→K[x1]
```

where these rules follow from Critical Pair Lemma 45 and Substitution Lemma 63 respectively.

Substitution Lemma 104

It can be shown that:

```
K[x1]==(K[x1]&&(B[x1]||x2))
```

Proof

We start by taking Critical Pair Lemma 48, and apply the substitution:

```
B[x1_]&&K[x1_]→K[x1]
```

which follows from Substitution Lemma 63.

Critical Pair Lemma 49

The following expressions are equivalent:

```
(L[x1]&&M[x1])==(M[x1]&&(L[x1]||x2))
```

Proof

Note that the input for the rule:

```
x1_&&x2_&&(x1_||x3_)→x1&&x2
```

contains a subpattern of the form:

```
x1_&&x2_
```

which can be unified with the input for the rule:

```
L[x1_]&&M[x1_]→M[x1]
```

where these rules follow from Critical Pair Lemma 45 and Substitution Lemma 57 respectively.

Substitution Lemma 105

It can be shown that:

```
M[x1]==(M[x1]&&(L[x1]||x2))
```

Proof

We start by taking Critical Pair Lemma 49, and apply the substitution:

```
L[x1_]&&M[x1_]→M[x1]
```

which follows from Substitution Lemma 57.

Critical Pair Lemma 50

The following expressions are equivalent:

```
(x1||x2)==(x1||x2||(x3&&x2))
```

Proof

Note that the input for the rule:

```
Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[2470,x1_&&(x1_|
```

contains a subpattern of the form:

```
x2_&&x1_
```

which can be unified with the input for the rule:

```
x1_&&x2_&&(x3_||x2_)→x1&&x2
```

where these rules follow from Substitution Lemma 100 and Critical Pair Lemma 46 respectively.

Critical Pair Lemma 51

The following expressions are equivalent:

`(x1||(x2&&x1))==(x1||(x3&&x2&&x1))`

Proof

Note that the input for the rule:

`x1_||x2_||(x3_&&x2_)→x1||x2`

contains a subpattern of the form:

`x1_||x2_`

which can be unified with the input for the rule:

`Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[2470,x1_&&(x1_|`

where these rules follow from Critical Pair Lemma 50 and Substitution Lemma 100 respectively.

Substitution Lemma 106

It can be shown that:

`x1==(x1||(x2&&x3&&x1))`

Proof

We start by taking Critical Pair Lemma 51, and apply the substitution:

`x1_||(x2_&&x1_)→x1`

which follows from Substitution Lemma 100.

Critical Pair Lemma 52

The following expressions are equivalent:

`M[x1]==(M[x1]||(x2&&B[x1]))`

Proof

Note that the input for the rule:

`x1_||(x2_&&x3_&&x1_)→x1`

contains a subpattern of the form:

`x3_&&x1_`

which can be unified with the input for the rule:

`B[x1_]&&M[x1_]→B[x1]`

where these rules follow from Substitution Lemma 106 and Substitution Lemma 85 respectively.

Critical Pair Lemma 53

The following expressions are equivalent:

`Nnew[x1]==(Nnew[x1]||(x2&&L[x1]))`

Proof

Note that the input for the rule:

`x1_||(x2_&&x3_&&x1_)→x1`

contains a subpattern of the form:

`x3_&&x1_`

which can be unified with the input for the rule:

`L[x1_]&&Nnew[x1_]→L[x1]`

where these rules follow from Substitution Lemma 106 and Substitution Lemma 51 respectively.

Critical Pair Lemma 54

The following expressions are equivalent:

`M[x1]==(M[x1]||B[x1])`

Proof

Note that the input for the rule:

`M[x1_]||(x2_&&B[x1_])→M[x1]`

contains a subpattern of the form:

`M[x1_]||(x2_&&B[x1_])`

which can be unified with the input for the rule:

`x1_||(!x1_&&x2_)→x1||x2`

where these rules follow from Critical Pair Lemma 52 and Critical Pair Lemma 8 respectively.

Substitution Lemma 107

It can be shown that:

`M[x1]==(B[x1]||M[x1])`

Proof

We start by taking Critical Pair Lemma 54, and apply the substitution:

`x1_||x2_→x2||x1`

which follows from Equationalized Axiom 3.

Critical Pair Lemma 55

The following expressions are equivalent:

`K[x1]==(K[x1]&&M[x1])`

Proof

Note that the input for the rule:

`K[x1_]&&(B[x1_]||x2_)→K[x1]`

contains a subpattern of the form:

`B[x1_]||x2_`

which can be unified with the input for the rule:

`B[x1_]||M[x1_]→M[x1]`

where these rules follow from Substitution Lemma 104 and Substitution Lemma 107 respectively.

Critical Pair Lemma 56

The following expressions are equivalent:

`M[x1]==(M[x1]||K[x1])`

Proof

Note that the input for the rule:

```
Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[2470,x1_&&(x1_|
```

contains a subpattern of the form:

```
x2_&&x1_
```

which can be unified with the input for the rule:

```
K[x1_]&&M[x1_]→K[x1]
```

where these rules follow from Substitution Lemma 100 and Critical Pair Lemma 55 respectively.

Substitution Lemma 108

It can be shown that:

```
M[x1]==(K[x1]||M[x1])
```

Proof

We start by taking Critical Pair Lemma 56, and apply the substitution:

```
x1_||x2_→x2||x1
```

which follows from Equationalized Axiom 3.

Critical Pair Lemma 57

The following expressions are equivalent:

```
Cnew[x1]==(Cnew[x1]&&M[x1])
```

Proof

Note that the input for the rule:

```
Cnew[x1_]&&(K[x1_]||x2_)→Cnew[x1]
```

contains a subpattern of the form:

```
K[x1_]||x2_
```

which can be unified with the input for the rule:

```
K[x1_]||M[x1_]→M[x1]
```

where these rules follow from Substitution Lemma 103 and Substitution Lemma 108 respectively.

Critical Pair Lemma 58

The following expressions are equivalent:

```
(Nnew[x1]||M[x1])==(Nnew[x1]||Cnew[x1])
```

Proof

Note that the input for the rule:

```
Nnew[x1_]||(Cnew[x1_]&&x2_)→Nnew[x1]||x2
```

contains a subpattern of the form:

```
Cnew[x1_]&&x2_
```

which can be unified with the input for the rule:

```
Cnew[x1_]&&M[x1_]→Cnew[x1]
```

where these rules follow from Substitution Lemma 97 and Critical Pair Lemma 57 respectively.

Substitution Lemma 109

It can be shown that:

`(M[x1]||Nnew[x1])==(Nnew[x1]||Cnew[x1])`

PROOF

We start by taking Critical Pair Lemma 58, and apply the substitution:

`x1_||x2_→x2||x1`

which follows from Equationalized Axiom 3.

Substitution Lemma 110

It can be shown that:

`(M[x1]||Nnew[x1])==(Cnew[x1]||Nnew[x1])`

PROOF

We start by taking Substitution Lemma 109, and apply the substitution:

`x1_||x2_→x2||x1`

which follows from Equationalized Axiom 3.

Substitution Lemma 111

It can be shown that:

`(M[x1]||Nnew[x1])==(x0||!x0)`

PROOF

We start by taking Substitution Lemma 110, and apply the substitution:

`Cnew[x1_]||Nnew[x1_]→x0||!x0`

which follows from Substitution Lemma 96.

Critical Pair Lemma 59

The following expressions are equivalent:

`Nnew[x1]==(Nnew[x1]||L[x1])`

PROOF

Note that the input for the rule:

`Nnew[x1_]||(x2_&&L[x1_])→Nnew[x1]`

contains a subpattern of the form:

`Nnew[x1_]||(x2_&&L[x1_])`

which can be unified with the input for the rule:

`Nnew[x1_]||(Cnew[x1_]&&x2_)→Nnew[x1]||x2`

where these rules follow from Critical Pair Lemma 53 and Substitution Lemma 97 respectively.

Substitution Lemma 112

It can be shown that:

`Nnew[x1]==(L[x1]||Nnew[x1])`

PROOF

We start by taking Critical Pair Lemma 59, and apply the substitution:

We start by taking Critical Pair Lemma 59, and apply the substitution:

`x1_||x2_→x2||x1`

which follows from Equationalized Axiom 3.

Critical Pair Lemma 60

The following expressions are equivalent:

`M[x1]==(M[x1]&&Nnew[x1])`

Proof

Note that the input for the rule:

`M[x1_]&&(L[x1_]||x2_)→M[x1]`

contains a subpattern of the form:

`L[x1_]||x2_`

which can be unified with the input for the rule:

`L[x1_]||Nnew[x1_]→Nnew[x1]`

where these rules follow from Substitution Lemma 105 and Substitution Lemma 112 respectively.

Critical Pair Lemma 61

The following expressions are equivalent:

`Nnew[x1]==(Nnew[x1]||M[x1])`

Proof

Note that the input for the rule:

`` Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[2470,x1_&&(x1_| ``

contains a subpattern of the form:

`x2_&&x1_`

which can be unified with the input for the rule:

`M[x1_]&&Nnew[x1_]→M[x1]`

where these rules follow from Substitution Lemma 100 and Critical Pair Lemma 60 respectively.

Substitution Lemma 113

It can be shown that:

`Nnew[x1]==(M[x1]||Nnew[x1])`

Proof

We start by taking Critical Pair Lemma 61, and apply the substitution:

`x1_||x2_→x2||x1`

which follows from Equationalized Axiom 3.

Substitution Lemma 114

It can be shown that:

Nnew[x1]==(x_0||!x_0)

Proof

We start by taking Substitution Lemma 113, and apply the substitution:

`M[x1_]||Nnew[x1_]→x0||!x0`

which follows from Substitution Lemma 111.

Substitution Lemma 115

It can be shown that:

`(Nnew[x0]||!H[x0])==(a0||!a0)`

Proof

We start by taking Equationalized Hypothesis 1, and apply the substitution:

`x1_||x2_→x2||x1`

which follows from Equationalized Axiom 3.

Substitution Lemma 116

It can be shown that:

`(Nnew[x0]||!H[x0])==(x0||!x0)`

Proof

We start by taking Substitution Lemma 115, and apply the substitution:

`x1_||!x1_→x0||!x0`

which follows from Critical Pair Lemma 9.

Substitution Lemma 117

It can be shown that:

`(x0||!x0||!H[x0])==(x0||!x0)`

Proof

We start by taking Substitution Lemma 116, and apply the substitution:

`Nnew[x1_]→x0||!x0`

which follows from Substitution Lemma 114.

Conclusion 1

We obtain the conclusion:

`True`

Proof

Take Substitution Lemma 117, and apply the substitution:

`x1_||!x1_||x2_→x1||!x1`

which follows from Critical Pair Lemma 40.