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AN AUTOMATED DERIVATION OF SOME IMPLICATIONAL EQUIVALENTS OF EQUATIONAL BOOLEAN LOGIC

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Abstract
This paper provides extensively automated derivations of the implicational equivalence of Huntington, Robbins, Sheffer, Wolfram "short", Wolfram "single-axiom", and equational Boolean, logic. These derivations include an apparently novel proof of the Robbins Conjecture ("Robbins logic implies Boolean logic").

1.0 Introduction

Section 1 of this paper states the axioms of the theories of interest. Section 2 provides an overview of the methodology used in this study. Section 3 summarizes the derivations obtained. The Appendices contain detailed listings and graphs of the derivations.

This paper is maintained as a *Mathematica* notebook (with filename of the form *yyyymmdd*_**All-Boolean_eqvs.nb**, where "yyyymmdd" is a date of the form yyyy = year, mm = month, and dd = day of month, and "nb" designates "*Mathematica* notebook format"). A *Mathematica* notebook file is a binary file executable under [1].

A *Mathematica* notebook can, and typically does, contain instructions that execute in the *Mathematica* runtime. Unless a *Mathematica* instruction is terminated with a semicolon, *Mathematica* reports, in summary form, the results of attempting to execute that instruction. Those reports are prefixed with an expression of the form "ln[k] = ", followed by an informational message, where k = 1, 2, 3, ... In this file, default reporting is "on".

In the following, unless otherwise noted, *a, b, c* ,and *d*, where they occur in *Mathematica* expressions, are variables.

In Sections 1 and 3, "B" will occasionally be used to denote equational Boolean logic, "H" will occasionally be used to denote Huntington logic , "R" will occasionally be used to denote Robbins logic, and "SHE" will occasionally be used to denote Sheffer logic.

1.1 Equational Boolean logic

The language of equational Boolean logic consists of two binary function symbols, ∪ and ∩, one unary function symbol, ~, and two (optional) constants, 0 and 1. The axioms of equational Boolean logic can be formulated as:

(B1)	$x \cup (y \cup z) = (x \cup y) \cup z$	Associativity of ∪
(B1')	$x \cap (y \cap z) = (x \cup y) \cap z$	Associativity of ∩
(B2)	$x \cup y = y \cup x$	Commutativity of ∪
(B2')	$x \cap y = y \cap x$	Commutativity of ∩
(B3)	$x \cup (x \cap y) = x$	Absorption
(B3')	$x \cap (x \cup y) = x$	Absorption
(B4)	$x \cap (y \cup z) = (x \cap y) \cup (x \cap z)$	Distribution of ∩ over ∪
(B4')	$x \cup (y \cap z) = (x \cup y) \cap (x \cup z)$	Distribution of ∪ over ∩
(B5)	$x \cup \sim x = 1$	Complementation
(B5')	$x \cap \sim x = 0$	Complementation

Figure 1. The axioms of equational Boolean logic.

Associativity (corresponding to B1 and B1' in Figure 1) is implemented by default in *Mathematica*'s equational logic functions, so does not explicitly appear in the *Mathematica* formulation of any of the axioms of the theories that follow.

In *Mathematica* notation, B2-B5, B2'-B5' can be expressed as:

```
In[]:= AxB1 = ForAll[{a, b}, and[a, b] == and[b, a]]          (* comm. of "and" *)
Out[]= ∀{a,b} and[a, b] == and[b, a]
```

```
In[]:= AxB2 = ForAll[{a, b}, or[a, b] == or[b, a]]                    (* comm. of "or" *)
Out[]= ∀{a,b} or[a, b] == or[b, a]

In[]:= AxB3 = ForAll[{a, b}, and[a, or[b, not[b]]] == a]              (* absorption *)
Out[]= ∀{a,b} and[a, or[b, not[b]]] == a

In[]:= AxB4 = ForAll[{a, b}, or[a, and[b, not[b]]] == a]              (* absorption *)
Out[]= ∀{a,b} or[a, and[b, not[b]]] == a

In[]:= AxB5 = ForAll[{a, b, c}, and[a, or[b, c]] == or[and[a, b], and[a, c]]]   (* distribution *)
Out[]= ∀{a,b,c} and[a, or[b, c]] == or[and[a, b], and[a, c]]

In[]:= AxB6 = ForAll[{a, b, c}, or[a, and[b, c]] == and[or[a, b], or[a, c]]]    (* distribution *)
Out[]= ∀{a,b,c} or[a, and[b, c]] == and[or[a, b], or[a, c]]

In[]:= AxB7 = ForAll[a, or[a, not[a]] == 1]
        (* complementation *)
Out[]= ∀a or[a, not[a]] == 1

In[]:= AxB8 = ForAll[a, and[a, not[a]] == 0]
        (* complementation *)
Out[]= ∀a and[a, not[a]] == 0

In[]:= booleanLogic = {AxB1, AxB2, AxB3, AxB4, AxB5, AxB6, AxB7, AxB8}
Out[]= {∀{a,b} and[a, b] == and[b, a], ∀{a,b} or[a, b] == or[b, a], ∀{a,b} and[a, or[b, not[b]]] == a,
  ∀{a,b} or[a, and[b, not[b]]] == a, ∀{a,b,c} and[a, or[b, c]] == or[and[a, b], and[a, c]],
  ∀{a,b,c} or[a, and[b, c]] == and[or[a, b], or[a, c]],
  ∀a or[a, not[a]] == 1, ∀a and[a, not[a]] == 0}
```

Axioms AxB1-AxB6 are sufficient to characterize what is typically called a Boolean logic. From that perspective, Axioms AxB7-AxB8 can be thought of as definitions. I have included these definitions in booleanLogic in order facilitate the automated proofs of some theorems below.

Equational Boolean logic in the sense of Figure 1 is isomorphic to a Boolean algebra ([12]; [2], pp. 7-8).

1.2 Huntington logic

In 1933, Huntington discovered ([5],[6]) a basis for (a set of mutually independent axioms implicationally equivalent to) Boolean logic which requires only a single binary relation, “or” (and the unary operator “not”). The axioms of Huntington logic in *Mathematica* notation are:

```
In[]:= AxH1 = ForAll[{a, b}, or[a, b] == or[b, a]]
Out[]= ∀{a,b} or[a, b] == or[b, a]

In[]:= AxH2 = ForAll[{a, b, c}, or[a, or[b, c]] == or[or[a, b], c]]
Out[]= ∀{a,b,c} or[a, or[b, c]] == or[or[a, b], c]

In[]:= AxH3 = ForAll[{a, b}, or[not[or[not[a], b]], not[or[not[a], not[b]]]] == a]
Out[]= ∀{a,b} or[not[or[not[a], b]], not[or[not[a], not[b]]]] == a

In[]:= huntingtonLogic = {AxH1, AxH2, AxH3}
Out[]= {∀{a,b} or[a, b] == or[b, a], ∀{a,b,c} or[a, or[b, c]] == or[or[a, b], c],
  ∀{a,b} or[not[or[not[a], b]], not[or[not[a], not[b]]]] == a}
```

AxH3 is called the *Huntington equation.*

1.3 Robbins logic

If we replace AxH3 (in Huntington logic) with

ForAll[{a,b},not[or[not[or[a,b]],not[or[a,not[b]]]]]==a],

called the *Robbins equation*, we obtain *Robbins logic*

```
In[]:= AxR1 = ForAll[{a, b}, or[a, b] == or[b, a]]
Out[]= ∀{a,b} or[a, b] == or[b, a]

In[]:= AxR2 = ForAll[{a, b, c}, or[a, or[b, c]] == or[or[a, b], c]]
Out[]= ∀{a,b,c} or[a, or[b, c]] == or[or[a, b], c]

In[]:= AxR3 = ForAll[{a, b}, not[or[not[or[a, b]], not[or[a, not[b]]]]] == a]
Out[]= ∀{a,b} not[or[not[or[a, b]], not[or[a, not[b]]]]] == a

In[]:= robbinsLogic = {AxR1, AxR2, AxR3}

Out[]= {∀{a,b} or[a, b] == or[b, a], ∀{a,b,c} or[a, or[b, c]] == or[or[a, b], c],
  ∀{a,b} not[or[not[or[a, b]], not[or[a, not[b]]]]] == a}
```

Note that axiom AxH1 of Huntington logic is the same as axiom AxR1 of Robbins logic, and AxH2 is the same as AxR2. AxR3 is nominally “simpler” than AxH3, in the sense that AxR3 has one less “not” than AxH3.

The claim that equational Boolean logic is implied by Robbins logic is called the *Robbins conjecture*

([4]).

Robbins and Huntington could not find a proof (or a counterexample to) the Robbins conjecture ([10]). Whether the Robbins conjecture is a theorem was extensively studied, but not definitively answered, by Tarski and his students (see, for example, [9], Problem 1.1).

In the early 1990s, Winker ([7],[8]) identified five propositions, any one of which, conjoined with Robbins logic, implies (with the Boolean operator for "and" defined in terms of "not" and "or") equational Boolean logic. Let's call these propositions *Winker conditions*. One of these, which I denote by "W1" or "winker1", can be used to derive Huntington logic from Robbins logic (see Section 3.2.2 of this paper):

W1 := ∃c ∃d ((c or d) ≡ d)

In 1996, McCune ([4]) proved the Robbins conjecture by refuting the union of

the Robbins equation (AxR3)
the denial of W1

using the automated equational deduction framework EQP ([13]). An excellent exposition of [4] can be found in ([10]).

In 2003, with the assistance of the automated deduction framework Otter ([21]) and the equational deduction framework EQP ([13]), Mann ([14], Chap. 1) generated a proof of the Robbins conjecture. (It's unclear from [14] how much of Chap. 1 of [14] is derived using [13] and [21]. No specific source code written in [13] or [21] is contained in or referenced by [14].)

1.4 Wolfram single-axiom logic

Wolfram logic is a single-connective (nand), single-axiom equational logic, closely related to [20] and [21]. The axioms of Wolfram single-axiom logic in Mathematica notation are:

```
In[]:= wolframLogic = ForAll[{a, b, c}, nand[nand[nand[a, b], c], nand[a, nand[nand[a, c], a]]] == c]
Out[]= ∀{a,b,c} nand[nand[nand[a, b], c], nand[a, nand[nand[a, c], a]]] == c
```

1.5 Wolfram “short” logic

Wolfram “short” logic is a three-axiom, single-connective (nand) equational logic. The axioms of “short” logic in Mathematica notation are:

```
In[∘]:= shortLogic = {ForAll[{a, b}, nand[nand[a, a], nand[a, b]] == a],
          ForAll[{a, b}, nand[a, nand[a, b]] == nand[a, nand[b, b]]],
          ForAll[{a, b, c}, nand[a, nand[a, nand[b, c]]] == nand[b, nand[b, nand[a, c]]]]}
```

Out[∘]= $\{\forall_{\{a,b\}}$ nand[nand[a, a], nand[a, b]] == a, $\forall_{\{a,b\}}$ nand[a, nand[a, b]] == nand[a, nand[b, b]],
$\forall_{\{a,b,c\}}$ nand[a, nand[a, nand[b, c]]] == nand[b, nand[b, nand[a, c]]]$\}$

1.6 Sheffer logic

Sheffer logic is a single-connective (nand), three-axiom equational logic based on ([19]). In Mathematica notation, the axioms of Sheffer logic are:

```
In[∘]:= AxSHE1 = ForAll[a, nand[nand[a, a], nand[a, a]] == a]
```

Out[∘]= $\forall_{a}$ nand[nand[a, a], nand[a, a]] == a

```
In[∘]:= AxSHE2 = ForAll[{a, b}, nand[a, nand[b, nand[b, b]]] == nand[a, a]]
```

Out[∘]= $\forall_{\{a,b\}}$ nand[a, nand[b, nand[b, b]]] == nand[a, a]

```
In[∘]:= AxSHE3 = ForAll[{a, b, c}, nand[nand[a, nand[b, c]], nand[a, nand[b, c]]] ==
          nand[nand[nand[b, b], a], nand[nand[c, c], a]]]
```

Out[∘]= $\forall_{\{a,b,c\}}$ nand[nand[a, nand[b, c]], nand[a, nand[b, c]]] ==
nand[nand[nand[b, b], a], nand[nand[c, c], a]]

```
In[∘]:= shefferLogic = {AxSHE1, AxSHE2, AxSHE3}
```

Out[∘]= $\{\forall_{a}$ nand[nand[a, a], nand[a, a]] == a, $\forall_{\{a,b\}}$ nand[a, nand[b, nand[b, b]]] == nand[a, a],
$\forall_{\{a,b,c\}}$ nand[nand[a, nand[b, c]], nand[a, nand[b, c]]] ==
nand[nand[nand[b, b], a], nand[nand[c, c], a]]$\}$

2.0 Method

The *Mathematica* ([1]) equational automated deduction framework (ADF) was used to assist the derivation of the implicational equivalence of equational Boolean, Robbins, Huntington, Sheffer, Wolfram single-axiom, and Wolfram “short” logics.

2.1 Some definitions

Mathematica’s equational ADF is based on a *rewriting system* that can be formulated in the nomenclature of *universal algebra*.

A universal algebra is a pair <A; F>, where A is a non-void set and F is a family of finitary operations defined on A. F is not necessarily finite, and it may be void ([15], p. 8).

We assume the definitions of *term, value of a term, variable,* and *constant* contained in [3], Chapter 3.

A rewriting system is a system of R rules that transforms expressions that satisfy some well-defined set of formation rules to another expression that satisfy those formation rules. For the purposes of this paper, we will restrict our interest in a rewriting system that concerns identities of terms. In an identity of two terms, the values of the terms are equal for all values of variables occurring in them.

A *reduction* of a term T to a term T’ is a (typically recursive) rewriting of T to T’ using a set of rewriting rules R such that T’ is “simpler than” T (given some definition of “simpler than”).

A *reduction sequence* of a term T to a term T’ is a sequence T0 = T, T1, T2, T3, ..., Tn = T’, where each Ti is the result of applying R to Ti-1, i = 1, 2, ..., n.

If “simpler than” is a partial ordering ([16], p. 72) on a reduction sequence that begins with T and ends with T’ in a system with a set of rewriting rules R, “simpler than” induces a *reduction orde*r ([3], p. 102) on the reduction sequence that begins with T and ends with T’.

A term Tn is in *normal form* if no application of R to Tn changes Tn.

A rewriting system is said to be *finitely terminating* if every reduction sequence of any term T produces, in a finite number of iterations, a normal form of T.

A rewriting system is said to be *confluent* if the normal forms of all terms in the system are unique.

Some term rewriting systems are both finitely terminating and confluent ([3], esp. Chapter 9). Such rewriting systems have unique normal forms for all expressions. This permits us to use the the output of such a system to determine whether there is an identity between two terms T1 and T2 in the following manner. If T1 and T2 and have the same normal form, then there is an identity between T1 and T2. Otherwise, there is not an identity.

2.2 *Mathematica*'s equational logic inference algorithm

The inference algorithm in *Mathematica*'s ADF is the *Knuth-Bendix completion algorithm* (KBC, [11]). KBC attempts to transform a given finite set of identities (an "input" to KBC) to a finitely terminating, confluent term rewriting system that preserves identity.

At initialization, KBC attempts to "orient" the identities supplied in its input according to the *Knuth-Bendix reduction order* ([3], Section 5.4.4). This results in an initial set of reduction rules. KBC then attempts to complete this initial set of rules with additional rules, obtaining their normal forms, and adding a new rule for every pair of the normal forms in accordance with the reduction order.

KBC may

1. Terminate with success, yielding a finitely terminating, confluent set of rules, or
2. Terminate with failure, or
3. Loop without terminating.

For further details of KBC, see [11].

Mathematica's ADF is implemented in the function **FindEquationalProof**.

2.3 Platform

This notebook was executed under [1], with "Enable Dynamics" on, and with all other *Mathematica* configuration options set to their installation default values, running under Windows 10, on a Dell Inspiron 545 containing an Intel Q8200 quadprocessor clocked at 2.33 GHz and containing 8 GB memory.

This notebook was also converted to a PDF file. I found I could not directly **Save** this notebook **As** a **PDF** file (this is a bug in [1]): the content of the output file was only about 5% of the content of the input file. I found a workaround. I first executed the file *yyyymmdd*_**AllBoolean_eqvs.nb**, block-marked the content of that file with a Mathematica editing session, copied that marked block to memory, pasted that copy to a new notebook, then **Save**(d) the new notebook **As** a **PDF** file.

The resulting PDF file contains page breaks in the middle of some printed lines (this is a bug in [1]). (I'm still investigating what can be done about this.   Using [1] to directly print the notebook (i.e., printing the notebook from the default ".nb" format) produces a correctly formatted output (i.e., no page breaks occur in the middle of a line of output)).

3.0 Results

This section contains the results of the method described in Section 2.0.

Mathematica's ADF can produce, without human intervention, many of the interderivability proofs that are the core of this paper. But that ADF, like all ADFs as of 2018, cannot produce proofs of *arbitrary* interderivability theorems automatically. The derivations of some of these theorems require some human guidance ("adult supervision"). In the following, I accordingly distinguish derivations that:

a. require no insight on the part of the ADF user in order to succeed, or
b. succeed by invoking the *Mathematica* function **FindEquationalProof** once per axiom to be derived, or
c. employ non-trivial proof strategies (e.g., as in the proof of the Robbins Conjecture in Section 3.2.2 of this paper)

Derivations of type (a) can be regarded as a "brute-force"/mechanical application of *Mathematica*'s ADF; of type (b), somewhat less so. Derivations of type (c) typically trade on knowledge of proof strategies for, and the limitations of, the KBC inference algorithm in *Mathematica*'s equational ADF.

3.1 Implicational equivalence of equational Boolean and Huntington logic

This section summarizes the implicational equivalence of equational Boolean and Huntington logic.

3.1.1 Derivation of Huntington logic from equational Boolean logic

```
In[]:= proofHuntfromBool = FindEquationalProof[huntingtonLogic, booleanLogic]
Out[]= ProofObject[
  Logic: EquationalLogic    Steps: 123
  Theorem: ∀{a,b} or[a, b] == or[b, a] &&
      ∀{a,b,c} or[a, or[b, c]] == or[≪1≫] && ∀{a,b} or[not[or[not[a], b]], not[or[not[a], not[b]]]] == a
]
```

See Appendix 1 for a listing and a graph of the proof.

3.1.2 Derivation of equational Boolean logic from Huntington logic

This section summarizes the derivation of equational Boolean logic from Huntington logic. Huntington logic does not contain the Boolean expressions "and", "1", or "0", so we first define those Boolean expressions in terms of Huntington expressions.

```
In[∘]:= dfandHunt[a, b] = ForAll[{a, b}, and[a, b] == not[or[not[a], not[b]]]]
Out[∘]= ∀{a,b} and[a, b] == not[or[not[a], not[b]]]

In[∘]:= one = ForAll[a, or[a, not[a]] == 1]
Out[∘]= ∀a or[a, not[a]] == 1

In[∘]:= zero = ForAll[a, and[a, not[a]] == 0]
Out[∘]= ∀a and[a, not[a]] == 0
```

[1] cannot derive, in a single invocation of the *Mathematica* function FindEquationalProof, the conjunction of all the axioms of equational Boolean logic from Huntington logic, but fortunately, can derive the Boolean axioms one at a time from Huntington logic:

```
In[∘]:= proofAxB1fromHunt = FindEquationalProof[
          ForAll[{a, b}, and[a, b] == and[b, a]], {huntingtonLogic, dfandHunt[a, b]}]
Out[∘]= ProofObject[ Logic: EquationalLogic   Steps: 6
                     Theorem: ∀{a,b} and[a, b] == and[b, a] ]
```

AxB2 is identical to AxH1, so the derivation of AxB2 from AxH1 is trivial.

```
In[∘]:= proofAxB2fromHunt =
         FindEquationalProof[ForAll[{a, b}, or[a, b] == or[b, a]], huntingtonLogic]
Out[∘]= ProofObject[ Logic: EquationalLogic   Steps: 3
                     Theorem: ∀{a,b} or[a, b] == or[b, a] ]

In[∘]:= proofAxB3fromHunt = FindEquationalProof[
          ForAll[{a, b}, and[a, or[b, not[b]]] == a], {dfandHunt[a, b], huntingtonLogic}]
Out[∘]= ProofObject[ Logic: EquationalLogic   Steps: 127
                     Theorem: ∀{a,b} and[a, or[b, not[b]]] == a ]

In[∘]:= proofAxB4fromHunt = FindEquationalProof[
          ForAll[{a, b}, or[a, and[b, not[b]]] == a], {dfandHunt[a, b], huntingtonLogic}]
Out[∘]= ProofObject[ Logic: EquationalLogic   Steps: 124
                     Theorem: ∀{a,b} or[a, and[b, not[b]]] == a ]

In[∘]:= proofAxB5fromHunt =
         FindEquationalProof[ForAll[{a, b, c}, and[a, or[b, c]] == or[and[a, b], and[a, c]]],
          {dfandHunt[a, b], huntingtonLogic}]
Out[∘]= ProofObject[ Logic: EquationalLogic   Steps: 198
                     Theorem: ∀{a,b,c} and[a, or[b, c]] == or[and[a, b], and[a, c]] ]
```

In[ ]:=
```
proofAxB6fromHunt =
  FindEquationalProof[ForAll[{a, b, c}, or[a, and[b, c]] == and[or[a, b], or[a, c]]],
   {dfandHunt[a, b], huntingtonLogic}]
```

Out[ ]= ProofObject[Logic: EquationalLogic Steps: 182 Theorem: $\forall_{\{a,b,c\}}$ or[a, and[b, c]] == and[or[a, b], or[a, c]]]

The derivations of the Boolean axioms for “1” and “0” (AxB7 and AxB8) from Huntington logic are trivial.

In[ ]:=
```
proofAxB7fromHunt =
  FindEquationalProof[ForAll[a, or[a, not[a]] == 1], {one, huntingtonLogic}]
```

Out[ ]= ProofObject[Logic: EquationalLogic Steps: 3 Theorem: $\forall_{a}$ or[a, not[a]] == 1]

In[ ]:=
```
proofAxB8fromHunt =
  FindEquationalProof[ForAll[a, and[a, not[a]] == 0], {zero, huntingtonLogic}]
```

Out[ ]= ProofObject[Logic: EquationalLogic Steps: 3 Theorem: $\forall_{a}$ and[a, not[a]] == 0]

The conjunction of the derivations proofAxB*n*fromHunt, where $n = 1, 2, 3, \ldots, 8$, constitutes a derivation of equational Boolean logic from Huntington logic.

See Appendix 4 for a listing and graphs of the derivations in this section.

3.2 Implicational equivalence of equational Boolean and Robbins logic

This section summarizes the derivation of the implicational equivalence of Boolean and Robbins logic.

3.2.1 Derivation of Robbins logic from Boolean logic

In[ ]:=
```
proofRobbinsfromBool = FindEquationalProof[robbinsLogic, booleanLogic]
```

Out[ ]= ProofObject[Logic: EquationalLogic Steps: 128 Theorem: $\forall_{\{a,b\}}$ or[a, b] == or[b, a] &&
$\forall_{\{a,b,c\}}$ or[a, or[b, c]] == ≪1≫ && $\forall_{\{a,b\}}$ not[or[not[or[a, b]], not[or[a, not[b]]]]] == a]

See Appendix 2 for a listing and graph of the proof.

3.2.2 Derivation of equational Boolean logic from Robbins logic (the Robbins conjecture)

The “and” of equational Boolean logic is not defined in Robbins logic, so we define:

```
In[ ]:= dfandRobbins[a, b] = ForAll[{a, b}, and[a, b] == not[or[not[a], not[b]]]]
Out[ ]= ∀{a,b} and[a, b] == not[or[not[a], not[b]]]
```

(dfandRobbins is the same as dfandHuntington.)

Let P and Q be sets of propositions and P ⊢ Q denote “Q is derivable from P”. Section 3.1.2 shows that H ⊢ B. AxH1 is the same as AxR1, and AxH2 is the same as AxR2 (and thus, trivially, R ⊢ AxH1 and R ⊢ AxH2). Thus, to show that R ⊢ B, it suffices to show that R ⊢ AxH3.

In the early 1990s, Winker ([7],[8]) found five conditions, here called the *Winker conditions,* any one of which, when conjoined with the axioms of the Robbins logic, implies AxH3.

The proof strategy of this section is informed by, but not equivalent to, the approach of [4].

I first show that R + winker1 ⊢ AxH3, then show that R ⊢ winker1, from which it follows that R ⊢ AxH3. From that result (and the fact that AxH1 is AxR1, and AxH2 is AxR2) it would follow that R ⊢ H. Because by Section 3.1.2, H ⊢ B, it would follow that R ⊢ B.

Expressed in *Mathematica* notation, one of the Winker conditions, “winker1”, is

```
In[ ]:= winker1 = or[c, d] == d
Out[ ]= or[c, d] == d
```

A summary of the derivation of AxH3 from R + winker1 follows. See Appendix 5 for details.

```
In[ ]:= proofAxH3fromRobbinsw1 = FindEquationalProof[ForAll[{a, b},
           or[not[or[not[a], b]], not[or[not[a], not[b]]]] == a], {winker1, robbinsLogic}]
Out[ ]= ProofObject[ Logic: EquationalLogic   Steps: 349
                     Theorem: ∀{a,b} or[not[or[not[a], b]], not[or[not[a], not[b]]]] == a ]
                     Data not in notebook; Store now »
```

I now show that R ⊢ winker1. [1] (more precisely, the platform described in Section 2.3) cannot generate a proof of this proposition (*Mathematica* generates an error message and aborts), but a proof by contradiction, it turns out, will work. To do this, I conjoin the negation of winker1 to Robbins logic, then show that such a conjunction implies the negation of a theorem of Robbins logic. Assuming R is consistent (it is), it would follow that the negation of winker1 is not the case, so as desired, R ⊢ winker1.

To begin this proof, I first show that ForAll[a,or[a,0]==a] is a theorem of Robbins logic (when that logic is conjoined with two definitions (of “zero” and “and”) that are formulated solely within the resources of

Robbins logic).

In[ ]:=
```
proofzeropropRobbins =
 FindEquationalProof[ForAll[a, or[a, 0] == a], {dfandRobbins[a, b], zero, robbinsLogic}]
```

Out[ ]= ProofObject[Logic: EquationalLogic Steps: 49 Theorem: ∀a or[a, 0] == a]

Details of the proof of this proposition are in Appendix 8.

The negation right-hand-side (RHS) of winker1 can be rewritten in Mathematica equational form as

ForAll[{c,d}, or[c,d] == not[d]] OR ForAll[{c,d}, not[or[c,d]] == d]   Exp. 2

Note that the two disjuncts in Exp. 2,

ForAll[{c,d}, or[c,d] == not[d]]

and

ForAll[{c,d}, not[or[c,d]] == d]

are equivalent by the Law of Double Negation.

[1] cannot derive the Law of Double Negation from robbinsLogic alone, but if we conjoin the definition of "1", above, to the axioms of Robbins logic, we obtain a proof

In[ ]:=
```
proofDNfromRobbins = FindEquationalProof[ForAll[a, not[not[a]] == a], {one, robbinsLogic}]
```

Out[ ]= ProofObject[Logic: EquationalLogic Steps: 53 Theorem: ∀a not[not[a]] == a]

A listing and graph of this derivation is in Appendix 13.

To use Exp. 2 in a proof by contradiction, it suffices to use the rightmost disjunct of Exp. 2. We accordingly define

In[ ]:=
```
Notwinker1 = ForAll[{a, b}, or[a, b] == not[b]]
```

Out[ ]= $\forall_{\{a,b\}}$ or[a, b] == not[b]

We now show that

In[ ]:=
```
proofWinker1fromRobbins = FindEquationalProof[
  ForAll[a, or[a, 0] == not[0]], {dfandRobbins[a, b], Notwinker1, robbinsLogic}]
```

Out[ ]= ProofObject[Logic: EquationalLogic Steps: 3 Theorem: $\forall_a$ or[a, 0] == not[0]]

See Appendix 7 for a listing and graph of this proof

ForAll[a,or[a,0]==not[0]] contradicts ForAll[a,or[a,0]==a], and the latter is, as shown above, a theorem of R. Thus, assuming the consistency and mutual independence of the axioms of of Robbins logic , Notwinker1 contradicts R, so by proof by contradiction, R ⊢ winker1.

From the above, it follows that R ⊢ H. This result, conjoined with the results of Section 3.1 (H ⟺ B), implies that R ⟺ B.

3.3 Derivation of Robbins logic from Huntington logic

This section summarizes the derivation of Robbins logic from Huntington logic.

Because the first two axioms of Huntington logic are identical to the first two axioms of Robbins logic, to show that Robbins logic is implied by Huntington logic,to show that Huntington logic implies Robbins logic, it suffices to show that Huntington logic implies the third axiom (AxR3) of Robbins logic:

In[ ]:=
```
proofRobbinsfromHuntington = FindEquationalProof[
  ForAll[{a, b}, not[or[not[or[a, b]], not[or[a, not[b]]]]] == a], huntingtonLogic]
```

Out[ ]= ProofObject[Logic: EquationalLogic Steps: 27 Theorem: $\forall_{\{a,b\}}$ not[or[not[or[a, b]], not[or[a, not[b]]]]] == a]

See Appendix 3 for a listing and graph of the proof.

3.5 Implicational equivalence of Wolfram and Wolfram “short” logic

The following two proofs of the derivation of the implicational equivalence of Wolfram single-axiom and Wolfram “short” logic are summarized in the documentation for [1]. As expected, the derivation of Wolfram short logic from Wolfram single-axiom logic is longer than the converse.

```
In[ ]:= proofWolframfromShort = FindEquationalProof[wolframLogic, shortLogic]

Out[ ]= ProofObject[ Logic: EquationalLogic   Steps: 63
          Theorem: ∀{a,b,c} nand[nand[nand[a, b], c], nand[a, nand[nand[a, c], a]]] == c ]

In[ ]:= proofShortfromWolfram = FindEquationalProof[shortLogic, wolframLogic]

Out[ ]= ProofObject[ Logic: EquationalLogic   Steps: 250
          Theorem: ∀{a,b} nand[nand[a, a], nand[a, b]] == a && ∀≪1≫ ≪1≫ == ≪1≫ &&
            ∀{a,b,c} nand[a, nand[a, nand[b, c]]] == nand[b, nand[b, nand[a, c]]]
        Data not in notebook; Store now » ]
```

See Appendix 6 for a listing and graphs of these proofs.

3.6 Derivation of Wolfram "short" from Sheffer logic

This section summarizes the derivation of Wolfram "short" logic from Sheffer logic.

```
In[ ]:= proofShortfromSheffer = FindEquationalProof[shortLogic, shefferLogic]

Out[ ]= ProofObject[
          Logic: EquationalLogic   Steps: 159
          Theorem: ∀{a,b} nand[nand[a, a], nand[a, b]] == a &&
            ∀≪1≫ ≪1≫ == ≪1≫ && ∀{a,b,c} nand[a, nand[a, nand[b, c]]] == nand[b, nand[b, nand[a, c]]] ]
```

See Appendix 9 for a listing and graph of this proof.

Note that from Sections 3.5 and 3.6 we can infer Sheffer logic ⇒ Wolfram single-axiom logic.

From Sections 3.4 and 3.5 we can infer equational Boolean logic ⇒ Wolfram "short" logic.

3.7 Derivation of Sheffer logic from equational Boolean logic

This section summarizes the derivation of Sheffer logic from equational Boolean logic.

In[∘]:=
```
proofShefferfromBoolean = FindEquationalProof[shefferLogic,
  {ForAll[{a, b}, nand[a, b] == not[and[a, b]]], booleanLogic}]
```

Out[∘]= ProofObject[Logic: EquationalLogic Steps: 191
Theorem: $\forall_{a}$ nand[nand[a, a], nand[a, a]] == a && ≪1≫ ≪1≫ &&
$\forall_{\{a,b,c\}}$ nand[nand[a, nand[b, c]], nand[a, nand[b, c]]] == nand[nand[≪1≫, a], ≪1≫]]

See Appendix 10 for a listing and graph of this proof.

3.8 Derivation of equational Boolean logic from Sheffer logic

This section summarizes the derivation of equational Boolean logic from Sheffer logic.

Sheffer logic does not contain “and”, “or”, or “not”, so we must define them in terms of “nand”. [1] cannot derive the entire set of equational Boolean logic axioms from a single invocation of *Mathematica*’s FindEquationalProof, but [1] can derive the Boolean axioms, one axiom per invocation.

In[∘]:=
```
proofAxB1fromSheffer = FindEquationalProof[AxB1,
  {ForAll[a, not[a] == nand[a, a]], ForAll[{a, b}, and[a, b] == not[nand[a, b]]],
   ForAll[{a, b}, or[a, b] == nand[nand[a, a], nand[b, b]]], shefferLogic}]
```

Out[∘]= ProofObject[Logic: EquationalLogic Steps: 26
Theorem: $\forall_{\{a,b\}}$ and[a, b] == and[b, a]]

In[∘]:=
```
proofAxB2fromSheffer = FindEquationalProof[AxB2,
  {ForAll[a, not[a] == nand[a, a]], ForAll[{a, b}, and[a, b] == not[nand[a, b]]],
   ForAll[{a, b}, or[a, b] == nand[nand[a, a], nand[b, b]]], shefferLogic}]
```

Out[∘]= ProofObject[Logic: EquationalLogic Steps: 32
Theorem: $\forall_{\{a,b\}}$ or[a, b] == or[b, a]]

In[∘]:=
```
proofAxB3fromSheffer = FindEquationalProof[AxB3,
  {ForAll[a, not[a] == nand[a, a]], ForAll[{a, b}, and[a, b] == not[nand[a, b]]],
   ForAll[{a, b}, or[a, b] == nand[nand[a, a], nand[b, b]]], shefferLogic}]
```

Out[∘]= ProofObject[Logic: EquationalLogic Steps: 21
Theorem: $\forall_{\{a,b\}}$ and[a, or[b, not[b]]] == a]

In[∘]:=
```
proofAxB4fromSheffer = FindEquationalProof[AxB4,
  {ForAll[a, not[a] == nand[a, a]], ForAll[{a, b}, and[a, b] == not[nand[a, b]]],
   ForAll[{a, b}, or[a, b] == nand[nand[a, a], nand[b, b]]], shefferLogic}]
```

Out[∘]=
```
ProofObject[ Logic: EquationalLogic   Steps: 21
             Theorem: ∀{a,b} or[a, and[b, not[b]]] == a ]
```

In[∘]:=
```
proofAxB5fromSheffer = FindEquationalProof[AxB5,
  {ForAll[a, not[a] == nand[a, a]], ForAll[{a, b}, and[a, b] == not[nand[a, b]]],
   ForAll[{a, b}, or[a, b] == nand[nand[a, a], nand[b, b]]], shefferLogic}]
```

Out[∘]=
```
ProofObject[ Logic: EquationalLogic   Steps: 137
             Theorem: ∀{a,b,c} and[a, or[b, c]] == or[and[a, b], and[a, c]] ]
```

In[∘]:=
```
proofAxB6fromSheffer = FindEquationalProof[AxB6,
  {ForAll[a, not[a] == nand[a, a]], ForAll[{a, b}, and[a, b] == not[nand[a, b]]],
   ForAll[{a, b}, or[a, b] == nand[nand[a, a], nand[b, b]]], shefferLogic}]
```

Out[∘]=
```
ProofObject[ Logic: EquationalLogic   Steps: 43
             Theorem: ∀{a,b,c} or[a, and[b, c]] == and[or[a, b], or[a, c]] ]
```

Listings and graphs of these proofs are contained in Appendix 11.

The proof of AxB7 and AxB8 from Sheffer logic are trivial and left as an exercise (hint: just rewrite AxB7 and AxB8 in terms of the resources of Sheffer logic).

3.9 Derivation of equational Boolean logic from Wolfram “short” logic

This section summarizes the derivation of equational Boolean logic from Wolfram “short” logic.

[1] cannot derive the set of equational Boolean logic axioms from “short” logic from a single invocation of FindEquationalProof, but [1] can derive each of the axioms of equational Boolean logic, one axiom per invocation of FindEquational Proof.

In[∘]:=
```
proofAxB1fromShort = FindEquationalProof[AxB1,
  {ForAll[a, not[a] == nand[a, a]], ForAll[{a, b}, and[a, b] == not[nand[a, b]]],
   ForAll[{a, b}, or[a, b] == nand[nand[a, a], nand[b, b]]], shortLogic}]
```

Out[∘]=
```
ProofObject[ Logic: EquationalLogic   Steps: 71
             Theorem: ∀{a,b} and[a, b] == and[b, a] ]
```

```
In[]:= proofAxB2fromShort = FindEquationalProof[AxB2,
   {ForAll[a, not[a] == nand[a, a]], ForAll[{a, b}, and[a, b] == not[nand[a, b]]],
    ForAll[{a, b}, or[a, b] == nand[nand[a, a], nand[b, b]]], shortLogic}]

Out[]= ProofObject[ Logic: EquationalLogic  Steps: 85
                    Theorem: ∀{a,b} or[a, b] == or[b, a] ]

In[]:= proofAxB3fromShort = FindEquationalProof[AxB3,
   {ForAll[a, not[a] == nand[a, a]], ForAll[{a, b}, and[a, b] == not[nand[a, b]]],
    ForAll[{a, b}, or[a, b] == nand[nand[a, a], nand[b, b]]], shortLogic}]

Out[]= ProofObject[ Logic: EquationalLogic  Steps: 67
                    Theorem: ∀{a,b} and[a, or[b, not[b]]] == a ]

In[]:= proofAxB4fromShort = FindEquationalProof[AxB4,
   {ForAll[a, not[a] == nand[a, a]], ForAll[{a, b}, and[a, b] == not[nand[a, b]]],
    ForAll[{a, b}, or[a, b] == nand[nand[a, a], nand[b, b]]], shortLogic}]

Out[]= ProofObject[ Logic: EquationalLogic  Steps: 92
                    Theorem: ∀{a,b} or[a, and[b, not[b]]] == a ]

In[]:= proofAxB5fromShort = FindEquationalProof[AxB5,
   {ForAll[a, not[a] == nand[a, a]], ForAll[{a, b}, and[a, b] == not[nand[a, b]]],
    ForAll[{a, b}, or[a, b] == nand[nand[a, a], nand[b, b]]], shortLogic}]

Out[]= ProofObject[ Logic: EquationalLogic  Steps: 192
                    Theorem: ∀{a,b,c} and[a, or[b, c]] == or[and[a, b], and[a, c]] ]

In[]:= proofAxB6fromShort = FindEquationalProof[AxB6,
   {ForAll[a, not[a] == nand[a, a]], ForAll[{a, b}, and[a, b] == not[nand[a, b]]],
    ForAll[{a, b}, or[a, b] == nand[nand[a, a], nand[b, b]]], shortLogic}]

Out[]= ProofObject[ Logic: EquationalLogic  Steps: 251
                    Theorem: ∀{a,b,c} or[a, and[b, c]] == and[or[a, b], or[a, c]] ]
```

Listings and graphs of these proofs are contained in Appendix 12.

The derivations of AxB7 and AxB8 are simple and are left as an exercise.

Note that Sections 3.9, 3.8, 3.7, and 3.5 collectively imply that “short” logic implies Sheffer logic, i.e.

Wolfram “short” logic ⇒ equational Boolean logic ⇒ Sheffer logic

4.0 Conclusions and discussion

Mathematica can produce extensively automated derivations of the following implicational equivalences:

Robbins logic ⇔ Huntington logic ⇔ equational Boolean logic ⇔ Sheffer logic ⇔ Wolfram short logic ⇔ Wolfram logic

About 80% of the proofs in Section 3 require only a mechanical application of FindEquationalLogic. About 15% of the proofs in Section 3 require axiom-by-axiom proofs. The remaining 5% of the theorems in Section 3 require inspiration and luck (e.g., in Section 3.2.2 (the proof of the Robbins conjecture), and in the choice of the order in which the equivalences are proven.)

The derivations in Section 3 executed in about five minutes on the platform described in Section 2.3. Based on the system monitor of that platform, these derivations (including operating system and disk overhead) required, at peak, ~30% of the available CPU power and ~40% of the available physical memory.

5.0 Acknowledgements

This paper benefited from discussions with Ed Zalta and Paul Oppenheimer, and from [18], about the art of automated deduction. I am also indebted to Alberto Coffa and Art Skidmore, whose passion for formal methods in philosophy was an inspiration for all who were privileged to have known them. For any errors that remain, I am solely responsible.

APPENDICES

The following appendices contain listings and graphs of the proofs that are summarized in Section 3.0.

In file *yyyymmdd*_**AllBoolean_eqvs.nb**, the proposition in the textual representation of the proof that corresponds to a given node in the graph below can be obtained interactively by placing the cursor over the node symbol in the graph. The pdf translation of the notebook file does not support this kind of interactive translation.

The symbology of the proof graphs in these Appendices is:

a turquoise pentagon represents an axiom
a olive-green diamond represents a hypothesis
a red triangle represents a critical pair lemma
an ochre circle represents a substitution lemma
a line represents a dependency in an inference

Appendix 1. Derivation of Huntington logic from Boolean logic

In[ ]:= proofHuntfromBool["ProofNotebook"]

Axiom 1

We are given that:

`x1==and[x1,or[x2,not[x2]]]`

Axiom 2

We are given that:

`x1==or[x1,and[x2,not[x2]]]`

Axiom 3

We are given that:

`and[x1,x2]==and[x2,x1]`

Axiom 4

We are given that:

`and[x1,or[x2,x3]]==or[and[x1,x2],and[x1,x3]]`

Axiom 5

We are given that:

`and[x1,not[x1]]==0`

Axiom 6

We are given that:

`and[or[x1,x2],or[x1,x3]]==or[x1,and[x2,x3]]`

Axiom 7

We are given that:

`or[x1,x2]==or[x2,x1]`

Axiom 8

We are given that:

`or[x1,not[x1]]==1`

Hypothesis 1

We would like to show that:

`or[or[a,b],c]==or[a,or[b,c]]`

Hypothesis 2

We would like to show that:

`or[not[or[not[a],b]],not[or[not[a],not[b]]]]==a`

Hypothesis 3

We would like to show that:

`or[a,b]==or[b,a]`

Critical Pair Lemma 1

The following expressions are equivalent:

`and[x1,or[x2,x3]]==or[and[x2,x1],and[x1,x3]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],and[x1_,x3_]]→and[x1,or[x2,x3]]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Axiom 4 and Axiom 3 respectively.

Critical Pair Lemma 2

The following expressions are equivalent:

`and[x1,or[not[x1],x2]]==or[0,and[x1,x2]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],and[x1_,x3_]]→and[x1,or[x2,x3]]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,not[x1_]]→0`

where these rules follow from Axiom 4 and Axiom 5 respectively.

Substitution Lemma 1

It can be shown that:

`or[x1,0]==x1`

Proof

We start by taking Axiom 2, and apply the substitution:

`and[x1_,not[x1_]]→0`

which follows from Axiom 5.

Conclusion 1

We obtain the conclusion:

`True`

Proof

Take Hypothesis 3, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 7.

Critical Pair Lemma 3

The following expressions are equivalent:

The following expressions are equivalent:

`or[x1,and[x2,x3]]==and[or[x2,x1],or[x1,x3]]`

Proof

Note that the input for the rule:

`and[or[x1_,x2_],or[x1_,x3_]]→or[x1,and[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Axiom 6 and Axiom 7 respectively.

Critical Pair Lemma 4

The following expressions are equivalent:

`or[x1,and[not[x1],x2]]==and[1,or[x1,x2]]`

Proof

Note that the input for the rule:

`and[or[x1_,x2_],or[x1_,x3_]]→or[x1,and[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,not[x1_]]→1`

where these rules follow from Axiom 6 and Axiom 8 respectively.

Substitution Lemma 2

It can be shown that:

`and[x1,1]==x1`

Proof

We start by taking Axiom 1, and apply the substitution:

`or[x1_,not[x1_]]→1`

which follows from Axiom 8.

Critical Pair Lemma 5

The following expressions are equivalent:

`x1==or[0,x1]`

Proof

Note that the input for the rule:

`or[x1_,0]→x1`

contains a subpattern of the form:

`or[x1_,0]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Substitution Lemma 1 and Axiom 7 respectively.

Critical Pair Lemma 6

The following expressions are equivalent:

`or[x1,and[0,x2]]==and[x1,or[x1,x2]]`

Proof

Note that the input for the rule:

`and[or[x1_,x2_],or[x1_,x3_]]→or[x1,and[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,0]→x1`

where these rules follow from Axiom 6 and Substitution Lemma 1 respectively.

Critical Pair Lemma 7

The following expressions are equivalent:

`x1==and[1,x1]`

Proof

Note that the input for the rule:

`and[x1_,1]→x1`

contains a subpattern of the form:

`and[x1_,1]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Substitution Lemma 2 and Axiom 3 respectively.

Critical Pair Lemma 8

The following expressions are equivalent:

`not[0]==1`

Proof

Note that the input for the rule:

`or[0,x1_]→x1`

contains a subpattern of the form:

`or[0,x1_]`

which can be unified with the input for the rule:

`or[x1_,not[x1_]]→1`

where these rules follow from Critical Pair Lemma 5 and Axiom 8 respectively.

Critical Pair Lemma 9

The following expressions are equivalent:

`or[x1,and[0,0]]==and[x1,x1]`

Proof

Note that the input for the rule:

`and[x1_,or[x1_,x2_]]→or[x1,and[0,x2]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,0]→x1`

where these rules follow from Critical Pair Lemma 6 and Substitution Lemma 1 respectively.

Critical Pair Lemma 10

The following expressions are equivalent:

`or[x1,and[0,x2]]==and[x1,or[x2,x1]]`

Proof

Note that the input for the rule:

`and[x1_,or[x1_,x2_]]→or[x1,and[0,x2]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Critical Pair Lemma 6 and Axiom 7 respectively.

Critical Pair Lemma 11

The following expressions are equivalent:

`or[not[x1],and[0,x1]]==and[not[x1],1]`

Proof

Note that the input for the rule:

`and[x1_,or[x2_,x1_]]→or[x1,and[0,x2]]`

contains a subpattern of the form:

`or[x2_,x1_]`

which can be unified with the input for the rule:

`or[x1_,not[x1_]]→1`

where these rules follow from Critical Pair Lemma 10 and Axiom 8 respectively.

Substitution Lemma 3

It can be shown that:

`or[not[x1],and[0,x1]]==not[x1]`

Proof

We start by taking Critical Pair Lemma 11, and apply the substitution:

`and[x1_,1]→x1`

which follows from Substitution Lemma 2.

Critical Pair Lemma 12

The following expressions are equivalent:

`or[and[0,x1],and[0,not[x1]]]==and[and[0,x1],not[x1]]`

Proof

Note that the input for the rule:

`and[x1_,or[x2_,x1_]]→or[x1,and[0,x2]]`

contains a subpattern of the form:

`or[x2_,x1_]`

which can be unified with the input for the rule:

`or[not[x1_],and[0,x1_]]→not[x1]`

where these rules follow from Critical Pair Lemma 10 and Substitution Lemma 3 respectively.

Substitution Lemma 4

It can be shown that:

`and[0,or[x1,not[x1]]]==and[and[0,x1],not[x1]]`

Proof

We start by taking Critical Pair Lemma 12, and apply the substitution:

`or[and[x1_,x2_],and[x1_,x3_]]→and[x1,or[x2,x3]]`

which follows from Axiom 4.

Substitution Lemma 5

It can be shown that:

`and[0,1]==and[and[0,x1],not[x1]]`

Proof

We start by taking Substitution Lemma 4, and apply the substitution:

`or[x1_,not[x1_]]→1`

which follows from Axiom 8.

Substitution Lemma 6

It can be shown that:

`0==and[and[0,x1],not[x1]]`

Proof

We start by taking Substitution Lemma 5, and apply the substitution:

`and[x1_,1]→x1`

which follows from Substitution Lemma 2.

Substitution Lemma 7

It can be shown that:

`0==and[not[x1],and[0,x1]]`

Proof

We start by taking Substitution Lemma 6, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Axiom 3.

Critical Pair Lemma 13

The following expressions are equivalent:

`0==and[1,and[0,0]]`

PROOF

Note that the input for the rule:

`and[not[x1_],and[0,x1_]]→0`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[0]→1`

where these rules follow from Substitution Lemma 7 and Critical Pair Lemma 8 respectively.

Substitution Lemma 8

It can be shown that:

`0==and[0,0]`

PROOF

We start by taking Critical Pair Lemma 13, and apply the substitution:

`and[1,x1_]→x1`

which follows from Critical Pair Lemma 7.

Substitution Lemma 9

It can be shown that:

`or[x1,0]==and[x1,x1]`

PROOF

We start by taking Critical Pair Lemma 9, and apply the substitution:

`and[0,0]→0`

which follows from Substitution Lemma 8.

Substitution Lemma 10

It can be shown that:

`x1==and[x1,x1]`

PROOF

We start by taking Substitution Lemma 9, and apply the substitution:

`or[x1_,0]→x1`

which follows from Substitution Lemma 1.

Critical Pair Lemma 14

The following expressions are equivalent:

`and[x1,or[x2,x1]]==or[and[x1,x2],x1]`

PROOF

Note that the input for the rule:

`or[and[x1_,x2_],and[x1_,x3_]]→and[x1,or[x2,x3]]`

contains a subpattern of the form:

`and[x1_,x3_]`

which can be unified with the input for the rule:

```
and[x1_,x1_]→x1
```

where these rules follow from Axiom 4 and Substitution Lemma 10 respectively.

Substitution Lemma 11

It can be shown that:

```
or[x1,and[0,x2]]⩵or[and[x1,x2],x1]
```

Proof

We start by taking Critical Pair Lemma 14, and apply the substitution:

```
and[x1_,or[x2_,x1_]]→or[x1,and[0,x2]]
```

which follows from Critical Pair Lemma 10.

Substitution Lemma 12

It can be shown that:

```
or[x1,and[0,x2]]⩵or[x1,and[x1,x2]]
```

Proof

We start by taking Substitution Lemma 11, and apply the substitution:

```
or[x1_,x2_]→or[x2,x1]
```

which follows from Axiom 7.

Critical Pair Lemma 15

The following expressions are equivalent:

```
or[x1,and[x1,not[0]]]⩵or[x1,0]
```

Proof

Note that the input for the rule:

```
or[x1_,and[0,x2_]]↔or[x1_,and[x1_,x2_]]
```

contains a subpattern of the form:

```
and[0,x2_]
```

which can be unified with the input for the rule:

```
and[x1_,not[x1_]]→0
```

where these rules follow from Substitution Lemma 12 and Axiom 5 respectively.

Substitution Lemma 13

It can be shown that:

```
or[x1,and[x1,1]]⩵or[x1,0]
```

Proof

We start by taking Critical Pair Lemma 15, and apply the substitution:

```
not[0]→1
```

which follows from Critical Pair Lemma 8.

Substitution Lemma 14

It can be shown that:

```
or[x1,x1]⩵or[x1,0]
```

Proof

PROOF

We start by taking Substitution Lemma 13, and apply the substitution:

`and[x1_,1]→x1`

which follows from Substitution Lemma 2.

Substitution Lemma 15

It can be shown that:

`or[x1,x1]==x1`

PROOF

We start by taking Substitution Lemma 14, and apply the substitution:

`or[x1_,0]→x1`

which follows from Substitution Lemma 1.

Critical Pair Lemma 16

The following expressions are equivalent:

`or[x1,and[x2,x1]]==and[or[x1,x2],x1]`

PROOF

Note that the input for the rule:

`and[or[x1_,x2_],or[x1_,x3_]]→or[x1,and[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x3_]`

which can be unified with the input for the rule:

`or[x1_,x1_]→x1`

where these rules follow from Axiom 6 and Substitution Lemma 15 respectively.

Substitution Lemma 16

It can be shown that:

`or[x1,and[x2,x1]]==and[x1,or[x1,x2]]`

PROOF

We start by taking Critical Pair Lemma 16, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Axiom 3.

Substitution Lemma 17

It can be shown that:

`or[x1,and[x2,x1]]==or[x1,and[0,x2]]`

PROOF

We start by taking Substitution Lemma 16, and apply the substitution:

`and[x1_,or[x1_,x2_]]→or[x1,and[0,x2]]`

which follows from Critical Pair Lemma 6.

Critical Pair Lemma 17

The following expressions are equivalent:

`or[and[0,x1],and[0,not[x1]]]==or[and[0,x1],0]`

Proof

Note that the input for the rule:

`or[x1_,and[x2_,x1_]]↔or[x1_,and[0,x2_]]`

contains a subpattern of the form:

`and[x2_,x1_]`

which can be unified with the input for the rule:

`and[not[x1_],and[0,x1_]]→0`

where these rules follow from Substitution Lemma 17 and Substitution Lemma 7 respectively.

Substitution Lemma 18

It can be shown that:

`and[0,or[x1,not[x1]]]==or[and[0,x1],0]`

Proof

We start by taking Critical Pair Lemma 17, and apply the substitution:

`or[and[x1_,x2_],and[x1_,x3_]]→and[x1,or[x2,x3]]`

which follows from Axiom 4.

Substitution Lemma 19

It can be shown that:

`and[0,1]==or[and[0,x1],0]`

Proof

We start by taking Substitution Lemma 18, and apply the substitution:

`or[x1_,not[x1_]]→1`

which follows from Axiom 8.

Substitution Lemma 20

It can be shown that:

`0==or[and[0,x1],0]`

Proof

We start by taking Substitution Lemma 19, and apply the substitution:

`and[x1_,1]→x1`

which follows from Substitution Lemma 2.

Substitution Lemma 21

It can be shown that:

`0==and[0,x1]`

Proof

We start by taking Substitution Lemma 20, and apply the substitution:

`or[x1_,0]→x1`

which follows from Substitution Lemma 1.

Substitution Lemma 22

Substitution Lemma 22

It can be shown that:

`and[x1_,or[x2_,x1_]]→or[x1,0]`

Proof

We start by taking Critical Pair Lemma 10, and apply the substitution:

`and[0,x1_]→0`

which follows from Substitution Lemma 21.

Substitution Lemma 23

It can be shown that:

`and[x1_,or[x2_,x1_]]→x1`

Proof

We start by taking Substitution Lemma 22, and apply the substitution:

`or[x1_,0]→x1`

which follows from Substitution Lemma 1.

Substitution Lemma 24

It can be shown that:

`and[x1_,or[x1_,x2_]]→or[x1,0]`

Proof

We start by taking Critical Pair Lemma 6, and apply the substitution:

`and[0,x1_]→0`

which follows from Substitution Lemma 21.

Substitution Lemma 25

It can be shown that:

`and[x1_,or[x1_,x2_]]→x1`

Proof

We start by taking Substitution Lemma 24, and apply the substitution:

`or[x1_,0]→x1`

which follows from Substitution Lemma 1.

Substitution Lemma 26

It can be shown that:

`or[x1,and[x2,x1]]==or[x1,0]`

Proof

We start by taking Substitution Lemma 17, and apply the substitution:

`and[0,x1_]→0`

which follows from Substitution Lemma 21.

Substitution Lemma 27

It can be shown that:

`or[x1,and[x2,x1]]==x1`

PROOF

We start by taking Substitution Lemma 26, and apply the substitution:

`or[x1_,0]→x1`

which follows from Substitution Lemma 1.

Critical Pair Lemma 18

The following expressions are equivalent:

`or[x1,x2]==or[or[x1,x2],x1]`

PROOF

Note that the input for the rule:

`or[x1_,and[x2_,x1_]]→x1`

contains a subpattern of the form:

`and[x2_,x1_]`

which can be unified with the input for the rule:

`Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[283,or[x1_,0]→x1,and[x1`

where these rules follow from Substitution Lemma 27 and Substitution Lemma 25 respectively.

Critical Pair Lemma 19

The following expressions are equivalent:

`and[x1,x2]==and[and[x1,x2],x2]`

PROOF

Note that the input for the rule:

`Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[281,or[x1_,0]→x1,and[x1`

contains a subpattern of the form:

`or[x2_,x1_]`

which can be unified with the input for the rule:

`or[x1_,and[x2_,x1_]]→x1`

where these rules follow from Substitution Lemma 23 and Substitution Lemma 27 respectively.

Substitution Lemma 28

It can be shown that:

`or[x1,x2]==or[x1,or[x1,x2]]`

PROOF

We start by taking Critical Pair Lemma 18, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 7.

Critical Pair Lemma 20

The following expressions are equivalent:

`or[x1,and[or[x1,x2],x3]]==and[or[x1,x2],or[x1,x3]]`

PROOF

Note that the input for the rule:

Note that the input for the rule:

`and[or[x1_,x2_],or[x1_,x3_]]→or[x1,and[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,or[x1_,x2_]]→or[x1,x2]`

where these rules follow from Axiom 6 and Substitution Lemma 28 respectively.

Substitution Lemma 29

It can be shown that:

`or[x1,and[or[x1,x2],x3]]==or[x1,and[x2,x3]]`

Proof

We start by taking Critical Pair Lemma 20, and apply the substitution:

`and[or[x1_,x2_],or[x1_,x3_]]→or[x1,and[x2,x3]]`

which follows from Axiom 6.

Critical Pair Lemma 21

The following expressions are equivalent:

`or[x1,and[x2,or[x1,x3]]]==and[or[x1,x2],or[x1,x3]]`

Proof

Note that the input for the rule:

`and[or[x1_,x2_],or[x1_,x3_]]→or[x1,and[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x3_]`

which can be unified with the input for the rule:

`or[x1_,or[x1_,x2_]]→or[x1,x2]`

where these rules follow from Axiom 6 and Substitution Lemma 28 respectively.

Substitution Lemma 30

It can be shown that:

`or[x1,and[x2,or[x1,x3]]]==or[x1,and[x2,x3]]`

Proof

We start by taking Critical Pair Lemma 21, and apply the substitution:

`and[or[x1_,x2_],or[x1_,x3_]]→or[x1,and[x2,x3]]`

which follows from Axiom 6.

Substitution Lemma 31

It can be shown that:

`and[x1,x2]==and[x2,and[x1,x2]]`

Proof

We start by taking Critical Pair Lemma 19, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Axiom 3.

which follows from Axiom 8.

Substitution Lemma 32

It can be shown that:

`and[x1,or[not[x1],x2]]==and[x1,x2]`

Proof

We start by taking Critical Pair Lemma 2, and apply the substitution:

`or[0,x1_]→x1`

Out[ ]=

which follows from Critical Pair Lemma 5.

Critical Pair Lemma 22

The following expressions are equivalent:

`and[x1,not[not[x1]]]==and[x1,1]`

Proof

Note that the input for the rule:

`and[x1_,or[not[x1_],x2_]]→and[x1,x2]`

contains a subpattern of the form:

`or[not[x1_],x2_]`

which can be unified with the input for the rule:

`or[x1_,not[x1_]]→1`

where these rules follow from Substitution Lemma 32 and Axiom 8 respectively.

Substitution Lemma 33

It can be shown that:

`and[x1,not[not[x1]]]==x1`

Proof

We start by taking Critical Pair Lemma 22, and apply the substitution:

`and[x1_,1]→x1`

which follows from Substitution Lemma 2.

Critical Pair Lemma 23

The following expressions are equivalent:

`and[not[x1],or[x1,x2]]==or[0,and[not[x1],x2]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],and[x2_,x3_]]→and[x2,or[x1,x3]]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,not[x1_]]→0`

where these rules follow from Critical Pair Lemma 1 and Axiom 5 respectively.

Substitution Lemma 34

It can be shown that:

It can be shown that:

`and[not[x1],or[x1,x2]]==and[not[x1],x2]`

Proof

We start by taking Critical Pair Lemma 23, and apply the substitution:

`or[0,x1_]→x1`

which follows from Critical Pair Lemma 5.

Critical Pair Lemma 24

The following expressions are equivalent:

`and[x1,or[x2,x3]]==or[and[x2,x1],and[x3,x1]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],and[x2_,x3_]]→and[x2,or[x1,x3]]`

contains a subpattern of the form:

`and[x2_,x3_]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Critical Pair Lemma 1 and Axiom 3 respectively.

Critical Pair Lemma 25

The following expressions are equivalent:

`and[not[x1],x2]==and[not[x1],or[x2,x1]]`

Proof

Note that the input for the rule:

`and[not[x1_],or[x1_,x2_]]→and[not[x1],x2]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Substitution Lemma 34 and Axiom 7 respectively.

Critical Pair Lemma 26

The following expressions are equivalent:

`and[not[not[x1]],x1]==and[not[not[x1]],1]`

Proof

Note that the input for the rule:

`and[not[x1_],or[x2_,x1_]]→and[not[x1],x2]`

contains a subpattern of the form:

`or[x2_,x1_]`

which can be unified with the input for the rule:

`or[x1_,not[x1_]]→1`

where these rules follow from Critical Pair Lemma 25 and Axiom 8 respectively.

Substitution Lemma 35

It can be shown that:

`and[not[not[x1]],x1]==not[not[x1]]`

Proof

We start by taking Critical Pair Lemma 26, and apply the substitution:

`and[x1_,1]→x1`

which follows from Substitution Lemma 2.

Substitution Lemma 36

It can be shown that:

`and[x1,not[not[x1]]]==not[not[x1]]`

Proof

We start by taking Substitution Lemma 35, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Axiom 3.

Substitution Lemma 37

It can be shown that:

`x1==not[not[x1]]`

Proof

We start by taking Substitution Lemma 36, and apply the substitution:

`and[x1_,not[not[x1_]]]→x1`

which follows from Substitution Lemma 33.

Substitution Lemma 38

It can be shown that:

`or[x1,and[not[x1],x2]]==or[x1,x2]`

Proof

We start by taking Critical Pair Lemma 4, and apply the substitution:

`and[1,x1_]→x1`

which follows from Critical Pair Lemma 7.

Critical Pair Lemma 27

The following expressions are equivalent:

`or[x1,or[not[x1],x2]]==or[x1,not[x1]]`

Proof

Note that the input for the rule:

`or[x1_,and[not[x1_],x2_]]→or[x1,x2]`

contains a subpattern of the form:

`and[not[x1_],x2_]`

which can be unified with the input for the rule:

`Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[283,or[x1_,0]→x1,and[x1`

where these rules follow from Substitution Lemma 38 and Substitution Lemma 25 respectively.

Substitution Lemma 39

It can be shown that:

`or[x1,or[not[x1],x2]]==1`

Proof

We start by taking Critical Pair Lemma 27, and apply the substitution:

`or[x1_,not[x1_]]→1`

which follows from Axiom 8.

Critical Pair Lemma 28

The following expressions are equivalent:

`or[x1,or[x2,not[x1]]]==or[x1,not[x1]]`

Proof

Note that the input for the rule:

`or[x1_,and[not[x1_],x2_]]→or[x1,x2]`

contains a subpattern of the form:

`and[not[x1_],x2_]`

which can be unified with the input for the rule:

`` Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[281,or[x1_,0]→x1,and[x1 ``

where these rules follow from Substitution Lemma 38 and Substitution Lemma 23 respectively.

Substitution Lemma 40

It can be shown that:

`or[x1,or[x2,not[x1]]]==1`

Proof

We start by taking Critical Pair Lemma 28, and apply the substitution:

`or[x1_,not[x1_]]→1`

which follows from Axiom 8.

Critical Pair Lemma 29

The following expressions are equivalent:

`and[not[or[not[x1],x2]],x1]==and[not[or[not[x1],x2]],1]`

Proof

Note that the input for the rule:

`and[not[x1_],or[x2_,x1_]]→and[not[x1],x2]`

contains a subpattern of the form:

`or[x2_,x1_]`

which can be unified with the input for the rule:

`or[x1_,or[not[x1_],x2_]]→1`

where these rules follow from Critical Pair Lemma 25 and Substitution Lemma 39 respectively.

Substitution Lemma 41

Substitution Lemma 41

It can be shown that:

```
and[not[or[not[x1],x2]],x1]==not[or[not[x1],x2]]
```

Proof

We start by taking Critical Pair Lemma 29, and apply the substitution:

```
and[x1_,1]→x1
```

which follows from Substitution Lemma 2.

Critical Pair Lemma 30

The following expressions are equivalent:

```
1==or[not[x1],or[x2,x1]]
```

Proof

Note that the input for the rule:

```
or[x1_,or[x2_,not[x1_]]]→1
```

contains a subpattern of the form:

```
not[x1_]
```

which can be unified with the input for the rule:

```
not[not[x1_]]→x1
```

where these rules follow from Substitution Lemma 40 and Substitution Lemma 37 respectively.

Critical Pair Lemma 31

The following expressions are equivalent:

```
and[not[or[x1,x2]],not[x2]]==and[not[or[x1,x2]],1]
```

Proof

Note that the input for the rule:

```
and[not[x1_],or[x2_,x1_]]→and[not[x1],x2]
```

contains a subpattern of the form:

```
or[x2_,x1_]
```

which can be unified with the input for the rule:

```
or[not[x1_],or[x2_,x1_]]→1
```

where these rules follow from Critical Pair Lemma 25 and Critical Pair Lemma 30 respectively.

Substitution Lemma 42

It can be shown that:

```
and[not[or[x1,x2]],not[x2]]==not[or[x1,x2]]
```

Proof

We start by taking Critical Pair Lemma 31, and apply the substitution:

```
and[x1_,1]→x1
```

which follows from Substitution Lemma 2.

Substitution Lemma 43

It can be shown that:

```
and[not[x1],not[or[x2,x1]]]==not[or[x2,x1]]
```

`and[not[x1],not[or[x2,x1]]]==not[or[x2,x1]]`

PROOF

We start by taking Substitution Lemma 42, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Axiom 3.

Critical Pair Lemma 32

The following expressions are equivalent:

`or[x1,and[x2,or[x2,x3]]]==or[x1,or[x2,and[x1,x3]]]`

PROOF

Note that the input for the rule:

`or[x1_,and[or[x1_,x2_],x3_]]→or[x1,and[x2,x3]]`

contains a subpattern of the form:

`and[or[x1_,x2_],x3_]`

which can be unified with the input for the rule:

`and[or[x1_,x2_],or[x2_,x3_]]→or[x2,and[x1,x3]]`

where these rules follow from Substitution Lemma 29 and Critical Pair Lemma 3 respectively.

Substitution Lemma 44

It can be shown that:

`or[x1,x2]==or[x1,or[x2,and[x1,x3]]]`

PROOF

We start by taking Critical Pair Lemma 32, and apply the substitution:

`and[x1_,or[x1_,x2_]]→x1`

which follows from Substitution Lemma 25.

Critical Pair Lemma 33

The following expressions are equivalent:

`or[x1,x2]==or[x1,or[and[x1,x3],x2]]`

PROOF

Note that the input for the rule:

`or[x1_,or[x2_,and[x1_,x3_]]]→or[x1,x2]`

contains a subpattern of the form:

`or[x2_,and[x1_,x3_]]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Substitution Lemma 44 and Axiom 7 respectively.

Critical Pair Lemma 34

The following expressions are equivalent:

`or[x1,x2]==or[x1,or[x2,and[x3,x1]]]`

PROOF

Note that the input for the rule:

Note that the input for the rule:

`or[x1_,or[x2_,and[x1_,x3_]]]→or[x1,x2]`

contains a subpattern of the form:

`and[x1_,x3_]`

which can be unified with the input for the rule:

`and[x1_,and[x2_,x1_]]→and[x2,x1]`

where these rules follow from Substitution Lemma 44 and Substitution Lemma 31 respectively.

Critical Pair Lemma 35

The following expressions are equivalent:

`or[x1,x2]⩵or[x1,or[and[x3,x1],x2]]`

Proof

Note that the input for the rule:

`or[x1_,or[and[x1_,x2_],x3_]]→or[x1,x3]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,and[x2_,x1_]]→and[x2,x1]`

where these rules follow from Critical Pair Lemma 33 and Substitution Lemma 31 respectively.

Critical Pair Lemma 36

The following expressions are equivalent:

`or[or[x1,x2],x3]⩵or[or[x1,x2],or[x3,x1]]`

Proof

Note that the input for the rule:

`or[x1_,or[x2_,and[x3_,x1_]]]→or[x1,x2]`

contains a subpattern of the form:

`and[x3_,x1_]`

which can be unified with the input for the rule:

`Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[283,or[x1_,0]→x1,and[x1`

where these rules follow from Critical Pair Lemma 34 and Substitution Lemma 25 respectively.

Critical Pair Lemma 37

The following expressions are equivalent:

`or[or[x1,x2],x3]⩵or[or[x1,x2],or[x2,x3]]`

Proof

Note that the input for the rule:

`or[x1_,or[and[x2_,x1_],x3_]]→or[x1,x3]`

contains a subpattern of the form:

`and[x2_,x1_]`

which can be unified with the input for the rule:

`Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[281,or[x1_,0]→x1,and[x1`

where these rules follow from Critical Pair Lemma 35 and Substitution Lemma 23 respectively.

Substitution Lemma 45

It can be shown that:

`and[x1,not[or[not[x1],x2]]]==not[or[not[x1],x2]]`

Proof

We start by taking Substitution Lemma 41, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Axiom 3.

Critical Pair Lemma 38

The following expressions are equivalent:

`or[or[x1,x2],x3]==or[or[x3,x1],or[x1,x2]]`

Proof

Note that the input for the rule:

`or[or[x1_,x2_],or[x3_,x1_]]→or[or[x1,x2],x3]`

contains a subpattern of the form:

`or[or[x1_,x2_],or[x3_,x1_]]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Critical Pair Lemma 36 and Axiom 7 respectively.

Substitution Lemma 46

It can be shown that:

`or[or[x1,x2],x3]==or[or[x3,x1],x2]`

Proof

We start by taking Critical Pair Lemma 38, and apply the substitution:

`or[or[x1_,x2_],or[x2_,x3_]]→or[or[x1,x2],x3]`

which follows from Critical Pair Lemma 37.

Substitution Lemma 47

It can be shown that:

`or[or[b,a],c]==or[a,or[b,c]]`

Proof

We start by taking Hypothesis 1, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 7.

Substitution Lemma 48

It can be shown that:

`or[or[b,a],c]==or[a,or[c,b]]`

Proof

We start by taking Substitution Lemma 47, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 7.

Substitution Lemma 49

It can be shown that:

`or[or[b,a],c]==or[or[c,b],a]`

Proof

We start by taking Substitution Lemma 48, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 7.

Conclusion 2

We obtain the conclusion:

`True`

Proof

Take Substitution Lemma 49, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[or[x3,x1],x2]`

which follows from Substitution Lemma 46.

Critical Pair Lemma 39

The following expressions are equivalent:

`or[or[x1,x2],x3]==or[x1,or[x2,x3]]`

Proof

Note that the input for the rule:

`or[or[x1_,x2_],x3_]↔or[or[x3_,x1_],x2_]`

contains a subpattern of the form:

`or[or[x1_,x2_],x3_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Substitution Lemma 46 and Axiom 7 respectively.

Critical Pair Lemma 40

The following expressions are equivalent:

`or[x1,or[x2,not[or[x1,x2]]]]==1`

Proof

Note that the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

contains a subpattern of the form:

`or[or[x1_,x2_],x3_]`

which can be unified with the input for the rule:

`or[x1_,not[x1_]]→1`

where these rules follow from Critical Pair Lemma 39 and Axiom 8 respectively.

Critical Pair Lemma 41

The following expressions are equivalent:

`and[x1,or[x2,not[or[not[x1],x2]]]]==and[x1,1]`

Proof

Note that the input for the rule:

`and[x1_,or[not[x1_],x2_]]→and[x1,x2]`

contains a subpattern of the form:

`or[not[x1_],x2_]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,not[or[x1_,x2_]]]]→1`

where these rules follow from Substitution Lemma 32 and Critical Pair Lemma 40 respectively.

Substitution Lemma 50

It can be shown that:

`and[x1,or[x2,not[or[not[x1],x2]]]]==x1`

Proof

We start by taking Critical Pair Lemma 41, and apply the substitution:

`and[x1_,1]→x1`

which follows from Substitution Lemma 2.

Critical Pair Lemma 42

The following expressions are equivalent:

`or[x1,and[x2,not[or[not[x2],x1]]]]==or[x1,x2]`

Proof

Note that the input for the rule:

`or[x1_,and[x2_,or[x1_,x3_]]]→or[x1,and[x2,x3]]`

contains a subpattern of the form:

`and[x2_,or[x1_,x3_]]`

which can be unified with the input for the rule:

`and[x1_,or[x2_,not[or[not[x1_],x2_]]]]→x1`

where these rules follow from Substitution Lemma 30 and Substitution Lemma 50 respectively.

Substitution Lemma 51

It can be shown that:

`or[x1,not[or[not[x2],x1]]]==or[x1,x2]`

Proof

We start by taking Critical Pair Lemma 42, and apply the substitution:

`and[x1_,not[or[not[x1_],x2_]]]→not[or[not[x1],x2]]`

which follows from Substitution Lemma 45.

Critical Pair Lemma 43

The following expressions are equivalent:

`and[not[x1],not[or[not[x2],x1]]]==and[not[x1],or[x1,x2]]`

PROOF

Note that the input for the rule:

`and[not[x1_],or[x1_,x2_]]→and[not[x1],x2]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,not[or[not[x2_],x1_]]]→or[x1,x2]`

where these rules follow from Substitution Lemma 34 and Substitution Lemma 51 respectively.

Substitution Lemma 52

It can be shown that:

`not[or[not[x1],x2]]==and[not[x2],or[x2,x1]]`

PROOF

We start by taking Critical Pair Lemma 43, and apply the substitution:

`and[not[x1_],not[or[x2_,x1_]]]→not[or[x2,x1]]`

which follows from Substitution Lemma 43.

Substitution Lemma 53

It can be shown that:

`not[or[not[x1],x2]]==and[not[x2],x1]`

PROOF

We start by taking Substitution Lemma 52, and apply the substitution:

`and[not[x1_],or[x1_,x2_]]→and[not[x1],x2]`

which follows from Substitution Lemma 34.

Substitution Lemma 54

It can be shown that:

`or[not[or[b,not[a]]],not[or[not[a],not[b]]]]==a`

PROOF

We start by taking Hypothesis 2, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 7.

Substitution Lemma 55

It can be shown that:

`or[not[or[not[a],not[b]]],not[or[b,not[a]]]]==a`

PROOF

We start by taking Substitution Lemma 54, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 7.

Substitution Lemma 56

Substitution Lemma 56

It can be shown that:

`or[and[not[not[b]],a],not[or[b,not[a]]]]==a`

Proof

We start by taking Substitution Lemma 55, and apply the substitution:

`not[or[not[x1_],x2_]]→and[not[x2],x1]`

which follows from Substitution Lemma 53.

Substitution Lemma 57

It can be shown that:

`or[and[a,not[not[b]]],not[or[b,not[a]]]]==a`

Proof

We start by taking Substitution Lemma 56, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Axiom 3.

Substitution Lemma 58

It can be shown that:

`or[and[a,not[not[b]]],not[or[not[a],b]]]==a`

Proof

We start by taking Substitution Lemma 57, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 7.

Substitution Lemma 59

It can be shown that:

`or[not[or[not[a],b]],and[a,not[not[b]]]]==a`

Proof

We start by taking Substitution Lemma 58, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 7.

Substitution Lemma 60

It can be shown that:

`or[not[or[not[a],b]],and[a,b]]==a`

Proof

We start by taking Substitution Lemma 59, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 37.

Substitution Lemma 61

It can be shown that:

`or[not[or[not[a],b]],and[b,a]]==a`

`or[not[or[not[a],b]],and[b,a]]==a`

Proof

We start by taking Substitution Lemma 60, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Axiom 3.

Substitution Lemma 62

It can be shown that:

`or[and[b,a],not[or[not[a],b]]]==a`

Proof

We start by taking Substitution Lemma 61, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 7.

Substitution Lemma 63

It can be shown that:

`or[and[b,a],and[not[b],a]]==a`

Proof

We start by taking Substitution Lemma 62, and apply the substitution:

`not[or[not[x1_],x2_]]→and[not[x2],x1]`

which follows from Substitution Lemma 53.

Substitution Lemma 64

It can be shown that:

`and[a,or[b,not[b]]]==a`

Proof

We start by taking Substitution Lemma 63, and apply the substitution:

`or[and[x1_,x2_],and[x3_,x2_]]→and[x2,or[x1,x3]]`

which follows from Critical Pair Lemma 24.

Substitution Lemma 65

It can be shown that:

`and[or[b,not[b]],a]==a`

Proof

We start by taking Substitution Lemma 64, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Axiom 3.

Substitution Lemma 66

It can be shown that:

`and[1,a]==a`

Proof

We start by taking Substitution Lemma 65, and apply the substitution:

```
or[x1_,not[x1_]]→1
```

which follows from Axiom 8.

Conclusion 3

We obtain the conclusion:

```
True
```

Proof

Take Substitution Lemma 66, and apply the substitution:

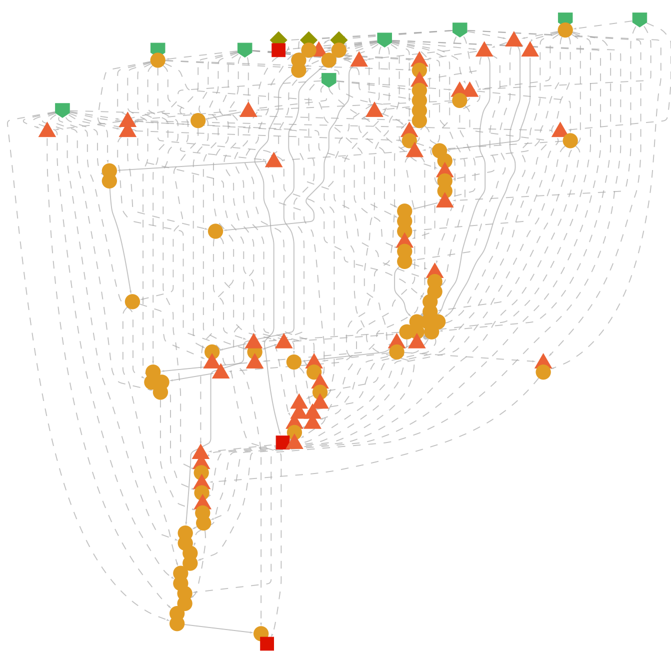
```
and[1,x1_]→x1
```

which follows from Critical Pair Lemma 7.

Represent the derivation of Huntington, from Boolean equational, logic as a graph.

In[∘]:=
```
proofHuntfromBool["ProofGraph"]
```

Out[∘]=



In[∘]:=
```
Clear[proofHuntfromBoolean]
```

Appendix 2. Derivation of Robbins logic from Boolean logic

In[ ]:=
```
proofRobbinsfromBool["ProofNotebook"]
```

Axiom 1

We are given that:

```
x1==and[x1,or[x2,not[x2]]]
```

Axiom 2

We are given that:

```
x1==or[x1,and[x2,not[x2]]]
```

Axiom 3

We are given that:

```
and[x1,x2]==and[x2,x1]
```

Axiom 4

We are given that:

```
and[x1,or[x2,x3]]==or[and[x1,x2],and[x1,x3]]
```

Axiom 5

We are given that:

```
and[x1,not[x1]]==0
```

Axiom 6

We are given that:

```
and[or[x1,x2],or[x1,x3]]==or[x1,and[x2,x3]]
```

Axiom 7

We are given that:

```
or[x1,x2]==or[x2,x1]
```

Axiom 8

We are given that:

```
or[x1,not[x1]]==1
```

Hypothesis 1

We would like to show that:

```
or[or[a,b],c]==or[a,or[b,c]]
```

Hypothesis 2

We would like to show that:

```
or[a,b]==or[b,a]
```

Hypothesis 3

Hypothesis 3

We would like to show that:

`not[or[not[or[a,b]],not[or[a,not[b]]]]]==a`

Critical Pair Lemma 1

The following expressions are equivalent:

`and[x1,or[x2,x3]]==or[and[x2,x1],and[x1,x3]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],and[x1_,x3_]]→and[x1,or[x2,x3]]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Axiom 4 and Axiom 3 respectively.

Substitution Lemma 1

It can be shown that:

`or[x1,0]==x1`

Proof

We start by taking Axiom 2, and apply the substitution:

`and[x1_,not[x1_]]→0`

which follows from Axiom 5.

Conclusion 1

We obtain the conclusion:

`True`

Proof

Take Hypothesis 2, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 7.

Critical Pair Lemma 2

The following expressions are equivalent:

`or[x1,and[x2,x3]]==and[or[x2,x1],or[x1,x3]]`

Proof

Note that the input for the rule:

`and[or[x1_,x2_],or[x1_,x3_]]→or[x1,and[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Axiom 6 and Axiom 7 respectively.

Critical Pair Lemma 3

The following expressions are equivalent:

`or[x1,and[not[x1],x2]]==and[1,or[x1,x2]]`

Proof

Note that the input for the rule:

`and[or[x1_,x2_],or[x1_,x3_]]→or[x1,and[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,not[x1_]]→1`

where these rules follow from Axiom 6 and Axiom 8 respectively.

Substitution Lemma 2

It can be shown that:

`and[x1,1]==x1`

Proof

We start by taking Axiom 1, and apply the substitution:

`or[x1_,not[x1_]]→1`

which follows from Axiom 8.

Critical Pair Lemma 4

The following expressions are equivalent:

`x1==or[0,x1]`

Proof

Note that the input for the rule:

`or[x1_,0]→x1`

contains a subpattern of the form:

`or[x1_,0]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Substitution Lemma 1 and Axiom 7 respectively.

Critical Pair Lemma 5

The following expressions are equivalent:

`or[x1,and[0,x2]]==and[x1,or[x1,x2]]`

Proof

Note that the input for the rule:

`and[or[x1_,x2_],or[x1_,x3_]]→or[x1,and[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,0]→x1`

where these rules follow from Axiom 6 and Substitution Lemma 1 respectively.

Critical Pair Lemma 6

The following expressions are equivalent:

`x1==and[1,x1]`

Proof

Note that the input for the rule:

`and[x1_,1]→x1`

contains a subpattern of the form:

`and[x1_,1]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Substitution Lemma 2 and Axiom 3 respectively.

Critical Pair Lemma 7

The following expressions are equivalent:

`or[x1,and[0,not[x1]]]==and[x1,1]`

Proof

Note that the input for the rule:

`and[x1_,or[x1_,x2_]]→or[x1,and[0,x2]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,not[x1_]]→1`

where these rules follow from Critical Pair Lemma 5 and Axiom 8 respectively.

Substitution Lemma 3

It can be shown that:

`or[x1,and[0,not[x1]]]==x1`

Proof

We start by taking Critical Pair Lemma 7, and apply the substitution:

`and[x1_,1]→x1`

which follows from Substitution Lemma 2.

Critical Pair Lemma 8

The following expressions are equivalent:

`or[x1,and[0,x2]]==and[x1,or[x2,x1]]`

Proof

Note that the input for the rule:

`and[x1_,or[x1_,x2_]]→or[x1,and[0,x2]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Critical Pair Lemma 5 and Axiom 7 respectively.

Critical Pair Lemma 9

The following expressions are equivalent:

`and[not[x1],or[x1,x2]]==or[0,and[not[x1],x2]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],and[x2_,x3_]]→and[x2,or[x1,x3]]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,not[x1_]]→0`

where these rules follow from Critical Pair Lemma 1 and Axiom 5 respectively.

Substitution Lemma 4

It can be shown that:

`and[not[x1],or[x1,x2]]==and[not[x1],x2]`

Proof

We start by taking Critical Pair Lemma 9, and apply the substitution:

`or[0,x1_]→x1`

which follows from Critical Pair Lemma 4.

Critical Pair Lemma 10

The following expressions are equivalent:

`and[x1,or[x2,not[x1]]]==or[and[x2,x1],0]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],and[x2_,x3_]]→and[x2,or[x1,x3]]`

contains a subpattern of the form:

`and[x2_,x3_]`

which can be unified with the input for the rule:

`and[x1_,not[x1_]]→0`

where these rules follow from Critical Pair Lemma 1 and Axiom 5 respectively.

Substitution Lemma 5

It can be shown that:

`and[x1,or[x2,not[x1]]]==and[x2,x1]`

PROOF

We start by taking Critical Pair Lemma 10, and apply the substitution:

`or[x1_,0]→x1`

which follows from Substitution Lemma 1.

Critical Pair Lemma 11

The following expressions are equivalent:

`and[x1,or[x2,x3]]⩵or[and[x2,x1],and[x3,x1]]`

PROOF

Note that the input for the rule:

`or[and[x1_,x2_],and[x2_,x3_]]→and[x2,or[x1,x3]]`

contains a subpattern of the form:

`and[x2_,x3_]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Critical Pair Lemma 1 and Axiom 3 respectively.

Critical Pair Lemma 12

The following expressions are equivalent:

`and[x1,x1]⩵or[x1,and[0,not[x1]]]`

PROOF

Note that the input for the rule:

`and[x1_,or[x2_,not[x1_]]]→and[x2,x1]`

contains a subpattern of the form:

`and[x1_,or[x2_,not[x1_]]]`

which can be unified with the input for the rule:

`and[x1_,or[x1_,x2_]]→or[x1,and[0,x2]]`

where these rules follow from Substitution Lemma 5 and Critical Pair Lemma 5 respectively.

Substitution Lemma 6

It can be shown that:

`and[x1,x1]⩵x1`

PROOF

We start by taking Critical Pair Lemma 12, and apply the substitution:

`or[x1_,and[0,not[x1_]]]→x1`

which follows from Substitution Lemma 3.

Critical Pair Lemma 13

The following expressions are equivalent:

`and[0,x1]⩵and[x1,not[x1]]`

PROOF

Note that the input for the rule:

Note that the input for the rule:

`and[x1_,or[x2_,not[x1_]]]→and[x2,x1]`

contains a subpattern of the form:

`or[x2_,not[x1_]]`

which can be unified with the input for the rule:

`or[0,x1_]→x1`

where these rules follow from Substitution Lemma 5 and Critical Pair Lemma 4 respectively.

Substitution Lemma 7

It can be shown that:

`and[0,x1]==0`

Proof

We start by taking Critical Pair Lemma 13, and apply the substitution:

`and[x1_,not[x1_]]→0`

which follows from Axiom 5.

Critical Pair Lemma 14

The following expressions are equivalent:

`and[x1,x2]==and[x2,or[not[x2],x1]]`

Proof

Note that the input for the rule:

`and[x1_,or[x2_,not[x1_]]]→and[x2,x1]`

contains a subpattern of the form:

`or[x2_,not[x1_]]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Substitution Lemma 5 and Axiom 7 respectively.

Critical Pair Lemma 15

The following expressions are equivalent:

`and[x1,or[x2,x1]]==or[and[x1,x2],x1]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],and[x1_,x3_]]→and[x1,or[x2,x3]]`

contains a subpattern of the form:

`and[x1_,x3_]`

which can be unified with the input for the rule:

`and[x1_,x1_]→x1`

where these rules follow from Axiom 4 and Substitution Lemma 6 respectively.

Substitution Lemma 8

It can be shown that:

`or[x1,and[0,x2]]==or[and[x1,x2],x1]`

`or[x1,and[0,x2]]==or[and[x1,x2],x1]`

PROOF

We start by taking Critical Pair Lemma 15, and apply the substitution:

`and[x1_,or[x2_,x1_]]→or[x1,and[0,x2]]`

which follows from Critical Pair Lemma 8.

Substitution Lemma 9

It can be shown that:

`and[x1_,or[x2_,x1_]]→or[x1,0]`

PROOF

We start by taking Critical Pair Lemma 8, and apply the substitution:

`and[0,x1_]→0`

which follows from Substitution Lemma 7.

Substitution Lemma 10

It can be shown that:

`and[x1_,or[x2_,x1_]]→x1`

PROOF

We start by taking Substitution Lemma 9, and apply the substitution:

`or[x1_,0]→x1`

which follows from Substitution Lemma 1.

Substitution Lemma 11

It can be shown that:

`and[x1_,or[x1_,x2_]]→or[x1,0]`

PROOF

We start by taking Critical Pair Lemma 5, and apply the substitution:

`and[0,x1_]→0`

which follows from Substitution Lemma 7.

Substitution Lemma 12

It can be shown that:

`and[x1_,or[x1_,x2_]]→x1`

PROOF

We start by taking Substitution Lemma 11, and apply the substitution:

`or[x1_,0]→x1`

which follows from Substitution Lemma 1.

Substitution Lemma 13

It can be shown that:

`or[x1,0]==or[and[x1,x2],x1]`

PROOF

We start by taking Substitution Lemma 8, and apply the substitution:

`and[0,x1_]→0`

which follows from Substitution Lemma 7.

Substitution Lemma 14

It can be shown that:

`x1==or[and[x1,x2],x1]`

Proof

We start by taking Substitution Lemma 13, and apply the substitution:

`or[x1_,0]→x1`

which follows from Substitution Lemma 1.

Substitution Lemma 15

It can be shown that:

`x1==or[x1,and[x1,x2]]`

Proof

We start by taking Substitution Lemma 14, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 7.

Critical Pair Lemma 16

The following expressions are equivalent:

`x1==or[x1,and[x2,x1]]`

Proof

Note that the input for the rule:

`or[x1_,and[x1_,x2_]]→x1`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,or[x2_,not[x1_]]]→and[x2,x1]`

where these rules follow from Substitution Lemma 15 and Substitution Lemma 5 respectively.

Critical Pair Lemma 17

The following expressions are equivalent:

`and[x1,x2]==and[and[x1,x2],x1]`

Proof

Note that the input for the rule:

`Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[218,or[x1_,0]→x`

contains a subpattern of the form:

`or[x2_,x1_]`

which can be unified with the input for the rule:

`or[x1_,and[x1_,x2_]]→x1`

```
or[x1_,and[x1_,x2_]]→x1
```

where these rules follow from Substitution Lemma 10 and Substitution Lemma 15 respectively.

Critical Pair Lemma 18

The following expressions are equivalent:

```
or[x1,x2]==or[or[x1,x2],x1]
```

Proof

Note that the input for the rule:

```
or[x1_,and[x2_,x1_]]→x1
```

contains a subpattern of the form:

```
and[x2_,x1_]
```

which can be unified with the input for the rule:

```
Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[220,or[x1_,0]→x
```

where these rules follow from Critical Pair Lemma 16 and Substitution Lemma 12 respectively.

Critical Pair Lemma 19

The following expressions are equivalent:

```
or[x1,x2]==or[or[x1,x2],x2]
```

Proof

Note that the input for the rule:

```
or[x1_,and[x2_,x1_]]→x1
```

contains a subpattern of the form:

```
and[x2_,x1_]
```

which can be unified with the input for the rule:

```
Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[218,or[x1_,0]→x
```

where these rules follow from Critical Pair Lemma 16 and Substitution Lemma 10 respectively.

Substitution Lemma 16

It can be shown that:

```
and[x1,x2]==and[x1,and[x1,x2]]
```

Proof

We start by taking Critical Pair Lemma 17, and apply the substitution:

```
and[x1_,x2_]→and[x2,x1]
```

which follows from Axiom 3.

Critical Pair Lemma 20

The following expressions are equivalent:

```
and[x1,or[and[x1,x2],x3]]==or[and[x1,x2],and[x1,x3]]
```

Proof

Note that the input for the rule:

```
or[and[x1_,x2_],and[x1_,x3_]]→and[x1,or[x2,x3]]
```

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,and[x1_,x2_]]→and[x1,x2]`

where these rules follow from Axiom 4 and Substitution Lemma 16 respectively.

Substitution Lemma 17

It can be shown that:

`and[x1,or[and[x1,x2],x3]]==and[x1,or[x2,x3]]`

Proof

We start by taking Critical Pair Lemma 20, and apply the substitution:

`or[and[x1_,x2_],and[x1_,x3_]]→and[x1,or[x2,x3]]`

which follows from Axiom 4.

Critical Pair Lemma 21

The following expressions are equivalent:

`and[x1,or[x2,and[x1,x3]]]==or[and[x1,x2],and[x1,x3]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],and[x1_,x3_]]→and[x1,or[x2,x3]]`

contains a subpattern of the form:

`and[x1_,x3_]`

which can be unified with the input for the rule:

`and[x1_,and[x1_,x2_]]→and[x1,x2]`

where these rules follow from Axiom 4 and Substitution Lemma 16 respectively.

Substitution Lemma 18

It can be shown that:

`and[x1,or[x2,and[x1,x3]]]==and[x1,or[x2,x3]]`

Proof

We start by taking Critical Pair Lemma 21, and apply the substitution:

`or[and[x1_,x2_],and[x1_,x3_]]→and[x1,or[x2,x3]]`

which follows from Axiom 4.

Substitution Lemma 19

It can be shown that:

`or[x1,x2]==or[x1,or[x1,x2]]`

Proof

We start by taking Critical Pair Lemma 18, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 7.

Critical Pair Lemma 22

The following expressions are equivalent:

`or[x1,and[or[x1,x2],x3]]==and[or[x1,x2],or[x1,x3]]`

Proof

Note that the input for the rule:

`and[or[x1_,x2_],or[x1_,x3_]]→or[x1,and[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,or[x1_,x2_]]→or[x1,x2]`

where these rules follow from Axiom 6 and Substitution Lemma 19 respectively.

Substitution Lemma 20

It can be shown that:

`or[x1,and[or[x1,x2],x3]]==or[x1,and[x2,x3]]`

Proof

We start by taking Critical Pair Lemma 22, and apply the substitution:

`and[or[x1_,x2_],or[x1_,x3_]]→or[x1,and[x2,x3]]`

which follows from Axiom 6.

Substitution Lemma 21

It can be shown that:

`or[x1,x2]==or[x2,or[x1,x2]]`

Proof

We start by taking Critical Pair Lemma 19, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 7.

Critical Pair Lemma 23

The following expressions are equivalent:

`and[not[not[x1]],x1]==and[x1,1]`

Proof

Note that the input for the rule:

`and[x1_,or[not[x1_],x2_]]→and[x2,x1]`

contains a subpattern of the form:

`or[not[x1_],x2_]`

which can be unified with the input for the rule:

`or[x1_,not[x1_]]→1`

where these rules follow from Critical Pair Lemma 14 and Axiom 8 respectively.

Substitution Lemma 22

It can be shown that:

`and[not[not[x1]],x1]==x1`

Proof

We start by taking Critical Pair Lemma 23, and apply the substitution:

`and[x1_,1]→x1`

which follows from Substitution Lemma 2.

Substitution Lemma 23

It can be shown that:

`and[x1,not[not[x1]]]==x1`

Proof

We start by taking Substitution Lemma 22, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Axiom 3.

Critical Pair Lemma 24

The following expressions are equivalent:

`and[not[x1],and[x1,x2]]==and[not[x1],x1]`

Proof

Note that the input for the rule:

`and[not[x1_],or[x1_,x2_]]→and[not[x1],x2]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,and[x1_,x2_]]→x1`

where these rules follow from Substitution Lemma 4 and Substitution Lemma 15 respectively.

Critical Pair Lemma 25

The following expressions are equivalent:

`and[not[x1],x2]==and[not[x1],or[x2,x1]]`

Proof

Note that the input for the rule:

`and[not[x1_],or[x1_,x2_]]→and[not[x1],x2]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Substitution Lemma 4 and Axiom 7 respectively.

Substitution Lemma 24

It can be shown that:

`and[not[x1],and[x1,x2]]==and[x1,not[x1]]`

Proof

We start by taking Critical Pair Lemma 24, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Axiom 3.

Substitution Lemma 25

It can be shown that:

`and[not[x1],and[x1,x2]]==0`

Proof

We start by taking Substitution Lemma 24, and apply the substitution:

`and[x1_,not[x1_]]→0`

which follows from Axiom 5.

Critical Pair Lemma 26

The following expressions are equivalent:

`0==and[not[x1],and[x2,x1]]`

Proof

Note that the input for the rule:

`and[not[x1_],and[x1_,x2_]]→0`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,or[not[x1_],x2_]]→and[x2,x1]`

where these rules follow from Substitution Lemma 25 and Critical Pair Lemma 14 respectively.

Critical Pair Lemma 27

The following expressions are equivalent:

`and[not[not[x1]],x1]==and[not[not[x1]],1]`

Proof

Note that the input for the rule:

`and[not[x1_],or[x2_,x1_]]→and[not[x1],x2]`

contains a subpattern of the form:

`or[x2_,x1_]`

which can be unified with the input for the rule:

`or[x1_,not[x1_]]→1`

where these rules follow from Critical Pair Lemma 25 and Axiom 8 respectively.

Substitution Lemma 26

It can be shown that:

`and[not[not[x1]],x1]==not[not[x1]]`

Proof

We start by taking Critical Pair Lemma 27, and apply the substitution:

`and[x1_,1]→x1`

`and[x1_,1]→x1`

which follows from Substitution Lemma 2.

Substitution Lemma 27

It can be shown that:

`and[x1,not[not[x1]]]⩵not[not[x1]]`

Proof

We start by taking Substitution Lemma 26, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Axiom 3.

Substitution Lemma 28

It can be shown that:

`x1⩵not[not[x1]]`

Proof

We start by taking Substitution Lemma 27, and apply the substitution:

`and[x1_,not[not[x1_]]]→x1`

which follows from Substitution Lemma 23.

Substitution Lemma 29

It can be shown that:

`or[x1,and[not[x1],x2]]⩵or[x1,x2]`

Proof

Out[ ]=

We start by taking Critical Pair Lemma 3, and apply the substitution:

`and[1,x1_]→x1`

which follows from Critical Pair Lemma 6.

Critical Pair Lemma 28

The following expressions are equivalent:

`or[x1,or[not[not[x1]],x2]]⩵or[x1,and[x2,not[x1]]]`

Proof

Note that the input for the rule:

`or[x1_,and[not[x1_],x2_]]→or[x1,x2]`

contains a subpattern of the form:

`and[not[x1_],x2_]`

which can be unified with the input for the rule:

`and[x1_,or[not[x1_],x2_]]→and[x2,x1]`

where these rules follow from Substitution Lemma 29 and Critical Pair Lemma 14 respectively.

Substitution Lemma 30

It can be shown that:

`or[x1,or[x1,x2]]⩵or[x1,and[x2,not[x1]]]`

Proof

PROOF

We start by taking Critical Pair Lemma 28, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 28.

Substitution Lemma 31

It can be shown that:

`or[x1,x2]==or[x1,and[x2,not[x1]]]`

PROOF

We start by taking Substitution Lemma 30, and apply the substitution:

`or[x1_,or[x1_,x2_]]→or[x1,x2]`

which follows from Substitution Lemma 19.

Critical Pair Lemma 29

The following expressions are equivalent:

`or[x1,or[x2,not[x1]]]==or[x1,not[x1]]`

PROOF

Note that the input for the rule:

`or[x1_,and[not[x1_],x2_]]→or[x1,x2]`

contains a subpattern of the form:

`and[not[x1_],x2_]`

which can be unified with the input for the rule:

`` Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[218,or[x1_,0]→x ``

where these rules follow from Substitution Lemma 29 and Substitution Lemma 10 respectively.

Substitution Lemma 32

It can be shown that:

`or[x1,or[x2,not[x1]]]==1`

PROOF

We start by taking Critical Pair Lemma 29, and apply the substitution:

`or[x1_,not[x1_]]→1`

which follows from Axiom 8.

Critical Pair Lemma 30

The following expressions are equivalent:

`or[not[x1],x2]==or[not[x1],and[x1,x2]]`

PROOF

Note that the input for the rule:

`or[x1_,and[not[x1_],x2_]]→or[x1,x2]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Substitution Lemma 29 and Substitution Lemma 28 respectively.

Critical Pair Lemma 31

The following expressions are equivalent:

`1==or[not[x1],or[x2,x1]]`

Proof

Note that the input for the rule:

`or[x1_,or[x2_,not[x1_]]]→1`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Substitution Lemma 32 and Substitution Lemma 28 respectively.

Critical Pair Lemma 32

The following expressions are equivalent:

`1==or[not[and[x1,x2]],x1]`

Proof

Note that the input for the rule:

`or[not[x1_],or[x2_,x1_]]→1`

contains a subpattern of the form:

`or[x2_,x1_]`

which can be unified with the input for the rule:

`or[x1_,and[x1_,x2_]]→x1`

where these rules follow from Critical Pair Lemma 31 and Substitution Lemma 15 respectively.

Substitution Lemma 33

It can be shown that:

`1==or[x1,not[and[x1,x2]]]`

Proof

We start by taking Critical Pair Lemma 32, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 7.

Critical Pair Lemma 33

The following expressions are equivalent:

`and[not[and[not[x1],x2]],x1]==and[x1,1]`

Proof

Note that the input for the rule:

`and[x1_,or[not[x1_],x2_]]→and[x2,x1]`

contains a subpattern of the form:

contains a subpattern of the form:

`or[not[x1_],x2_]`

which can be unified with the input for the rule:

`or[x1_,not[and[x1_,x2_]]]→1`

where these rules follow from Critical Pair Lemma 14 and Substitution Lemma 33 respectively.

Substitution Lemma 34

It can be shown that:

`and[not[and[not[x1],x2]],x1]==x1`

Proof

We start by taking Critical Pair Lemma 33, and apply the substitution:

`and[x1_,1]→x1`

which follows from Substitution Lemma 2.

Critical Pair Lemma 34

The following expressions are equivalent:

`or[not[x1],x2]==or[not[x1],and[x2,x1]]`

Proof

Note that the input for the rule:

`or[x1_,and[x2_,not[x1_]]]→or[x1,x2]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Substitution Lemma 31 and Substitution Lemma 28 respectively.

Substitution Lemma 35

It can be shown that:

`and[x1,not[and[not[x1],x2]]]==x1`

Proof

We start by taking Substitution Lemma 34, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Axiom 3.

Critical Pair Lemma 35

The following expressions are equivalent:

`not[and[not[x1],x2]]==or[not[and[not[x1],x2]],x1]`

Proof

Note that the input for the rule:

`or[x1_,and[x2_,x1_]]→x1`

contains a subpattern of the form:

`and[x2_,x1_]`

which can be unified with the input for the rule:

`and[x1_,not[and[not[x1_],x2_]]]→x1`

where these rules follow from Critical Pair Lemma 16 and Substitution Lemma 35 respectively.

Critical Pair Lemma 36

The following expressions are equivalent:

`or[not[and[x1,x2]],not[x2]]==or[not[and[x1,x2]],0]`

Proof

Note that the input for the rule:

`or[not[x1_],and[x2_,x1_]]→or[not[x1],x2]`

contains a subpattern of the form:

`and[x2_,x1_]`

which can be unified with the input for the rule:

`and[not[x1_],and[x2_,x1_]]→0`

where these rules follow from Critical Pair Lemma 34 and Critical Pair Lemma 26 respectively.

Substitution Lemma 36

It can be shown that:

`or[not[and[x1,x2]],not[x2]]==not[and[x1,x2]]`

Proof

We start by taking Critical Pair Lemma 36, and apply the substitution:

`or[x1_,0]→x1`

which follows from Substitution Lemma 1.

Substitution Lemma 37

It can be shown that:

`or[not[x1],not[and[x2,x1]]]==not[and[x2,x1]]`

Proof

We start by taking Substitution Lemma 36, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 7.

Critical Pair Lemma 37

The following expressions are equivalent:

`and[x1,or[x2,and[x3,x2]]]==and[x1,and[x2,or[x1,x3]]]`

Proof

Note that the input for the rule:

`and[x1_,or[and[x1_,x2_],x3_]]→and[x1,or[x2,x3]]`

contains a subpattern of the form:

`or[and[x1_,x2_],x3_]`

which can be unified with the input for the rule:

`or[and[x1_,x2_],and[x3_,x2_]]→and[x2,or[x1,x3]]` or[and[x1_,x2_],and[x3_,x2_]]→and[x2,or[x1,x3]]

where these rules follow from Substitution Lemma 17 and Critical Pair Lemma 11 respectively.

Substitution Lemma 38

It can be shown that:

`and[x1,x2]==and[x1,and[x2,or[x1,x3]]]`

Proof

We start by taking Critical Pair Lemma 37, and apply the substitution:

`or[x1_,and[x2_,x1_]]→x1`

which follows from Critical Pair Lemma 16.

Critical Pair Lemma 38

The following expressions are equivalent:

`and[x1,x2]==and[x1,and[or[x1,x3],x2]]`

Proof

Note that the input for the rule:

`and[x1_,and[x2_,or[x1_,x3_]]]→and[x1,x2]`

contains a subpattern of the form:

`and[x2_,or[x1_,x3_]]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Substitution Lemma 38 and Axiom 3 respectively.

Critical Pair Lemma 39

The following expressions are equivalent:

`and[x1,x2]==and[x1,and[x2,or[x3,x1]]]`

Proof

Note that the input for the rule:

`and[x1_,and[x2_,or[x1_,x3_]]]→and[x1,x2]`

contains a subpattern of the form:

`or[x1_,x3_]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,x1_]]→or[x2,x1]`

where these rules follow from Substitution Lemma 38 and Substitution Lemma 21 respectively.

Critical Pair Lemma 40

The following expressions are equivalent:

`and[x1,x2]==and[x1,and[or[x3,x1],x2]]`

Proof

Note that the input for the rule:

`and[x1_,and[or[x1_,x2_],x3_]]→and[x1,x3]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,x1_]]→or[x2,x1]`

where these rules follow from Critical Pair Lemma 38 and Substitution Lemma 21 respectively.

Critical Pair Lemma 41

The following expressions are equivalent:

`and[and[x1,x2],x3]==and[and[x1,x2],and[x3,x1]]`

Proof

Note that the input for the rule:

`and[x1_,and[x2_,or[x3_,x1_]]]→and[x1,x2]`

contains a subpattern of the form:

`or[x3_,x1_]`

which can be unified with the input for the rule:

`or[x1_,and[x1_,x2_]]→x1`

where these rules follow from Critical Pair Lemma 39 and Substitution Lemma 15 respectively.

Critical Pair Lemma 42

The following expressions are equivalent:

`and[and[x1,x2],x3]==and[and[x1,x2],and[x2,x3]]`

Proof

Note that the input for the rule:

`and[x1_,and[or[x2_,x1_],x3_]]→and[x1,x3]`

contains a subpattern of the form:

`or[x2_,x1_]`

which can be unified with the input for the rule:

`or[x1_,and[x2_,x1_]]→x1`

where these rules follow from Critical Pair Lemma 40 and Critical Pair Lemma 16 respectively.

Critical Pair Lemma 43

The following expressions are equivalent:

`or[x1,and[x2,or[x2,x3]]]==or[x1,or[x2,and[x1,x3]]]`

Proof

Note that the input for the rule:

`or[x1_,and[or[x1_,x2_],x3_]]→or[x1,and[x2,x3]]`

contains a subpattern of the form:

`and[or[x1_,x2_],x3_]`

which can be unified with the input for the rule:

`and[or[x1_,x2_],or[x2_,x3_]]→or[x2,and[x1,x3]]`

where these rules follow from Substitution Lemma 20 and Critical Pair Lemma 2 respectively.

Substitution Lemma 39

It can be shown that:

`or[x1,x2]==or[x1,or[x2,and[x1,x3]]]`

Proof

We start by taking Critical Pair Lemma 43, and apply the substitution:

`and[x1_,or[x1_,x2_]]→x1`

which follows from Substitution Lemma 12.

Critical Pair Lemma 44

The following expressions are equivalent:

`or[x1,x2]==or[x1,or[and[x1,x3],x2]]`

Proof

Note that the input for the rule:

`or[x1_,or[x2_,and[x1_,x3_]]]→or[x1,x2]`

contains a subpattern of the form:

`or[x2_,and[x1_,x3_]]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Substitution Lemma 39 and Axiom 7 respectively.

Critical Pair Lemma 45

The following expressions are equivalent:

`or[x1,x2]==or[x1,or[x2,and[x3,x1]]]`

Proof

Note that the input for the rule:

`or[x1_,or[x2_,and[x1_,x3_]]]→or[x1,x2]`

contains a subpattern of the form:

`and[x1_,x3_]`

which can be unified with the input for the rule:

`and[x1_,or[not[x1_],x2_]]→and[x2,x1]`

where these rules follow from Substitution Lemma 39 and Critical Pair Lemma 14 respectively.

Critical Pair Lemma 46

The following expressions are equivalent:

`or[x1,x2]==or[x1,or[and[x3,x1],x2]]`

Proof

Note that the input for the rule:

`or[x1_,or[and[x1_,x2_],x3_]]→or[x1,x3]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,or[not[x1_],x2_]]→and[x2,x1]`

where these rules follow from Critical Pair Lemma 44 and Critical Pair Lemma 14 respectively.

Critical Pair Lemma 47

The following expressions are equivalent:

`or[or[x1,x2],x3]==or[or[x1,x2],or[x3,x1]]`

Proof

Note that the input for the rule:

`or[x1_,or[x2_,and[x3_,x1_]]]→or[x1,x2]`

contains a subpattern of the form:

`and[x3_,x1_]`

which can be unified with the input for the rule:

`Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[220,or[x1_,0]→x`

where these rules follow from Critical Pair Lemma 45 and Substitution Lemma 12 respectively.

Critical Pair Lemma 48

The following expressions are equivalent:

`or[or[x1,x2],x3]==or[or[x1,x2],or[x2,x3]]`

Proof

Note that the input for the rule:

`or[x1_,or[and[x2_,x1_],x3_]]→or[x1,x3]`

contains a subpattern of the form:

`and[x2_,x1_]`

which can be unified with the input for the rule:

`Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[218,or[x1_,0]→x`

where these rules follow from Critical Pair Lemma 46 and Substitution Lemma 10 respectively.

Substitution Lemma 40

It can be shown that:

`not[and[not[x1],x2]]==or[x1,not[and[not[x1],x2]]]`

Proof

We start by taking Critical Pair Lemma 35, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 7.

Critical Pair Lemma 49

The following expressions are equivalent:

`and[and[x1,x2],x3]==and[and[x3,x1],and[x1,x2]]`

Proof

Note that the input for the rule:

`and[and[x1_,x2_],and[x3_,x1_]]→and[and[x1,x2],x3]`

contains a subpattern of the form:

`and[and[x1_,x2_],and[x3_,x1_]]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Critical Pair Lemma 41 and Axiom 3 respectively.

Critical Pair Lemma 50

The following expressions are equivalent:

`or[or[x1,x2],x3]==or[or[x3,x1],or[x1,x2]]`

Proof

Note that the input for the rule:

`or[or[x1_,x2_],or[x3_,x1_]]→or[or[x1,x2],x3]`

contains a subpattern of the form:

`or[or[x1_,x2_],or[x3_,x1_]]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Critical Pair Lemma 47 and Axiom 7 respectively.

Substitution Lemma 41

It can be shown that:

`and[and[x1,x2],x3]==and[and[x3,x1],x2]`

Proof

We start by taking Critical Pair Lemma 49, and apply the substitution:

`and[and[x1_,x2_],and[x2_,x3_]]→and[and[x1,x2],x3]`

which follows from Critical Pair Lemma 42.

Critical Pair Lemma 51

The following expressions are equivalent:

`and[and[x1,x2],x3]==and[x1,and[x2,x3]]`

Proof

Note that the input for the rule:

`and[and[x1_,x2_],x3_]↔and[and[x3_,x1_],x2_]`

contains a subpattern of the form:

`and[and[x1_,x2_],x3_]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Substitution Lemma 41 and Axiom 3 respectively.

Critical Pair Lemma 52

The following expressions are equivalent:

`and[x1,and[x2,not[and[x1,x2]]]]==0`

Proof

Note that the input for the rule:

`and[and[x1_,x2_],x3_]→and[x1,and[x2,x3]]` and[and[x1_,x2_],x3_]→and[x1,and[x2,x3]]

contains a subpattern of the form:

and[and[x1_,x2_],x3_]

which can be unified with the input for the rule:

and[x1_,not[x1_]]→0

where these rules follow from Critical Pair Lemma 51 and Axiom 5 respectively.

Critical Pair Lemma 53

The following expressions are equivalent:

or[x1,and[x2,not[and[not[x1],x2]]]]==or[x1,0]

Proof

Note that the input for the rule:

or[x1_,and[not[x1_],x2_]]→or[x1,x2]

contains a subpattern of the form:

and[not[x1_],x2_]

which can be unified with the input for the rule:

and[x1_,and[x2_,not[and[x1_,x2_]]]]→0

where these rules follow from Substitution Lemma 29 and Critical Pair Lemma 52 respectively.

Substitution Lemma 42

It can be shown that:

or[x1,and[x2,not[and[not[x1],x2]]]]==x1

Proof

We start by taking Critical Pair Lemma 53, and apply the substitution:

or[x1_,0]→x1

which follows from Substitution Lemma 1.

Substitution Lemma 43

It can be shown that:

or[or[x1,x2],x3]==or[or[x3,x1],x2]

Proof

We start by taking Critical Pair Lemma 50, and apply the substitution:

or[or[x1_,x2_],or[x2_,x3_]]→or[or[x1,x2],x3]

which follows from Critical Pair Lemma 48.

Substitution Lemma 44

It can be shown that:

or[or[b,a],c]==or[a,or[b,c]]

Proof

We start by taking Hypothesis 1, and apply the substitution:

or[x1_,x2_]→or[x2,x1]

which follows from Axiom 7.

Substitution Lemma 45

It can be shown that:

`or[or[b,a],c]==or[a,or[c,b]]`

Proof

We start by taking Substitution Lemma 44, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 7.

Substitution Lemma 46

It can be shown that:

`or[or[b,a],c]==or[or[c,b],a]`

Proof

We start by taking Substitution Lemma 45, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 7.

Conclusion 2

We obtain the conclusion:

`True`

Proof

Take Substitution Lemma 46, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[or[x3,x1],x2]`

which follows from Substitution Lemma 43.

Critical Pair Lemma 54

The following expressions are equivalent:

`and[x1,or[x2,not[and[not[x2],x1]]]]==and[x1,x2]`

Proof

Note that the input for the rule:

`and[x1_,or[x2_,and[x1_,x3_]]]→and[x1,or[x2,x3]]`

contains a subpattern of the form:

`or[x2_,and[x1_,x3_]]`

which can be unified with the input for the rule:

`or[x1_,and[x2_,not[and[not[x1_],x2_]]]]→x1`

where these rules follow from Substitution Lemma 18 and Substitution Lemma 42 respectively.

Substitution Lemma 47

It can be shown that:

`and[x1,not[and[not[x2],x1]]]==and[x1,x2]`

Proof

We start by taking Critical Pair Lemma 54, and apply the substitution:

`or[x1_,not[and[not[x1_],x2_]]]→not[and[not[x1],x2]]`

which follows from Substitution Lemma 40.

Critical Pair Lemma 55

The following expressions are equivalent:

`or[not[x1],not[and[not[x2],x1]]]==or[not[x1],and[x1,x2]]`

Proof

Note that the input for the rule:

`or[not[x1_],and[x1_,x2_]]→or[not[x1],x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,not[and[not[x2_],x1_]]]→and[x1,x2]`

where these rules follow from Critical Pair Lemma 30 and Substitution Lemma 47 respectively.

Substitution Lemma 48

It can be shown that:

`not[and[not[x1],x2]]==or[not[x2],and[x2,x1]]`

Proof

We start by taking Critical Pair Lemma 55, and apply the substitution:

`or[not[x1_],not[and[x2_,x1_]]]→not[and[x2,x1]]`

which follows from Substitution Lemma 37.

Substitution Lemma 49

It can be shown that:

`not[and[not[x1],x2]]==or[not[x2],x1]`

Proof

We start by taking Substitution Lemma 48, and apply the substitution:

`or[not[x1_],and[x1_,x2_]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 30.

Critical Pair Lemma 56

The following expressions are equivalent:

`or[not[x1],not[x2]]==not[and[x2,x1]]`

Proof

Note that the input for the rule:

`not[and[not[x1_],x2_]]→or[not[x2],x1]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Substitution Lemma 49 and Substitution Lemma 28 respectively.

where these rules follow from Substitution Lemma 49 and Substitution Lemma 28 respectively.

Substitution Lemma 50

It can be shown that:

`not[or[not[or[b,a]],not[or[a,not[b]]]]]==a`

Proof

We start by taking Hypothesis 3, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 7.

Substitution Lemma 51

It can be shown that:

`not[or[not[or[a,not[b]]],not[or[b,a]]]]==a`

Proof

We start by taking Substitution Lemma 50, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 7.

Substitution Lemma 52

It can be shown that:

`not[not[and[or[b,a],or[a,not[b]]]]]==a`

Proof

We start by taking Substitution Lemma 51, and apply the substitution:

`or[not[x1_],not[x2_]]→not[and[x2,x1]]`

which follows from Critical Pair Lemma 56.

Substitution Lemma 53

It can be shown that:

`not[not[or[a,and[b,not[b]]]]]==a`

Proof

We start by taking Substitution Lemma 52, and apply the substitution:

`and[or[x1_,x2_],or[x2_,x3_]]→or[x2,and[x1,x3]]`

which follows from Critical Pair Lemma 2.

Substitution Lemma 54

It can be shown that:

`not[not[or[a,and[not[b],b]]]]==a`

Proof

We start by taking Substitution Lemma 53, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Axiom 3.

Substitution Lemma 55

It can be shown that:

`not[not[or[and[not[b],b],a]]]==a`

PROOF

We start by taking Substitution Lemma 54, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 7.

Substitution Lemma 56

It can be shown that:

`or[and[not[b],b],a]==a`

PROOF

We start by taking Substitution Lemma 55, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 28.

Substitution Lemma 57

It can be shown that:

`or[and[b,not[b]],a]==a`

PROOF

We start by taking Substitution Lemma 56, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Axiom 3.

Substitution Lemma 58

It can be shown that:

`or[0,a]==a`

PROOF

We start by taking Substitution Lemma 57, and apply the substitution:

`and[x1_,not[x1_]]→0`

which follows from Axiom 5.

Conclusion 3

We obtain the conclusion:

`True`

PROOF

Take Substitution Lemma 58, and apply the substitution:

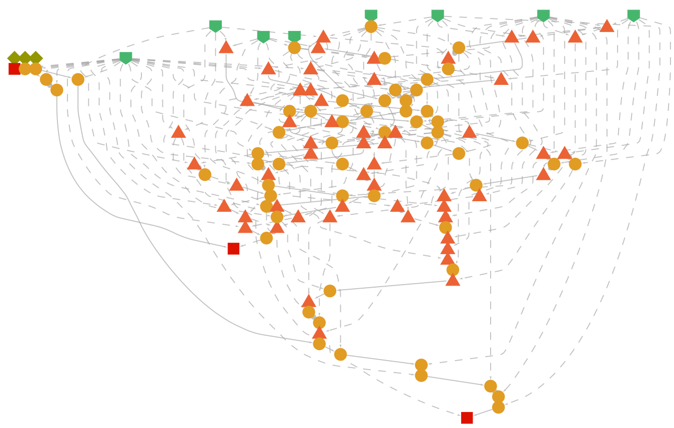
`or[0,x1_]→x1`

which follows from Critical Pair Lemma 4.

Represent the derivation of Robbins, from Boolean equational, logic as a graph. (See graph “Legend” at the end of Appendix 1 for a description of the graph symbology.)

In[∘]:=
```
proofRobbinsfromBool["ProofGraph"]
```

Out[∘]=



In[∘]:=
```
Clear[proofRobbinsfromBool]
```

Appendix 3. Derivation of AxR3 from Huntington logic

In[ ]:= proofRobbinsfromHuntington["ProofNotebook"]

Axiom 1

We are given that:

`x1==or[not[or[not[x1],x2]],not[or[not[x1],not[x2]]]]`

Axiom 2

We are given that:

`or[x1,x2]==or[x2,x1]`

Axiom 3

We are given that:

`or[x1,or[x2,x3]]==or[or[x1,x2],x3]`

Hypothesis 1

We would like to show that:

`not[or[not[or[a,b]],not[or[a,not[b]]]]]==a`

Critical Pair Lemma 1

The following expressions are equivalent:

`x1==or[not[or[x2,not[x1]]],not[or[not[x1],not[x2]]]]`

Proof

Note that the input for the rule:

`or[not[or[not[x1_],x2_]],not[or[not[x1_],not[x2_]]]]→x1`

contains a subpattern of the form:

`or[not[x1_],x2_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Axiom 1 and Axiom 2 respectively.

Critical Pair Lemma 2

The following expressions are equivalent:

`x1==or[not[or[not[x1],x2]],not[or[not[x2],not[x1]]]]`

Proof

Note that the input for the rule:

`or[not[or[not[x1_],x2_]],not[or[not[x1_],not[x2_]]]]→x1`

contains a subpattern of the form:

`or[not[x1_],not[x2_]]`

which can be unified with the input for the rule:

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Axiom 1 and Axiom 2 respectively.

Critical Pair Lemma 3

The following expressions are equivalent:

`or[x1,or[x2,x3]]==or[x3,or[x1,x2]]`

Proof

Note that the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

contains a subpattern of the form:

`or[or[x1_,x2_],x3_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Axiom 3 and Axiom 2 respectively.

Critical Pair Lemma 4

The following expressions are equivalent:

`or[not[or[not[x1],x2]],or[not[or[not[x1],not[x2]]],x3]]==or[x1,x3]`

Proof

Note that the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[not[or[not[x1_],x2_]],not[or[not[x1_],not[x2_]]]]→x1`

where these rules follow from Axiom 3 and Axiom 1 respectively.

Critical Pair Lemma 5

The following expressions are equivalent:

`or[x1,or[x2,x3]]==or[or[x2,x1],x3]`

Proof

Note that the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Axiom 3 and Axiom 2 respectively.

Substitution Lemma 1

It can be shown that:

`or[x1,or[x2,x3]]==or[x2,or[x1,x3]]`

Proof

We start by taking Critical Pair Lemma 5, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Critical Pair Lemma 6

The following expressions are equivalent:

`or[x1,or[x2,x3]]==or[x3,or[x2,x1]]`

Proof

Note that the input for the rule:

`or[x1_,or[x2_,x3_]]↔or[x3_,or[x1_,x2_]]`

contains a subpattern of the form:

`or[x2_,x3_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Critical Pair Lemma 3 and Axiom 2 respectively.

Critical Pair Lemma 7

The following expressions are equivalent:

`x1==or[not[or[x2,not[x1]]],not[or[not[x2],not[x1]]]]`

Proof

Note that the input for the rule:

`or[not[or[x1_,not[x2_]]],not[or[not[x2_],not[x1_]]]]→x2`

contains a subpattern of the form:

`or[not[x2_],not[x1_]]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Critical Pair Lemma 1 and Axiom 2 respectively.

Critical Pair Lemma 8

The following expressions are equivalent:

`or[x1,not[or[not[x1],not[not[x2]]]]]==or[not[or[not[x1],x2]],x1]`

Proof

Note that the input for the rule:

`or[not[or[not[x1_],x2_]],or[not[or[not[x1_],not[x2_]]],x3_]]→or[x1,x3]`

contains a subpattern of the form:

`or[not[or[not[x1_],not[x2_]]],x3_]`

which can be unified with the input for the rule:

`or[not[or[not[x1_],x2_]],not[or[not[x1_],not[x2_]]]]→x1`

where these rules follow from Critical Pair Lemma 4 and Axiom 1 respectively.

Substitution Lemma 2

Substitution Lemma 2

It can be shown that:

`or[x1,not[or[not[x1],not[not[x2]]]]]==or[x1,not[or[not[x1],x2]]]`

PROOF

We start by taking Critical Pair Lemma 8, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Out[ ]=

Critical Pair Lemma 9

The following expressions are equivalent:

`or[x1,or[x2,not[or[not[x1],not[not[x3]]]]]]==or[x2,or[x1,not[or[not[x1],x3]]]]`

PROOF

Note that the input for the rule:

`or[x1_,or[x2_,x3_]]↔or[x2_,or[x1_,x3_]]`

contains a subpattern of the form:

`or[x2_,x3_]`

which can be unified with the input for the rule:

`or[x1_,not[or[not[x1_],not[not[x2_]]]]]→or[x1,not[or[not[x1],x2]]]`

where these rules follow from Substitution Lemma 1 and Substitution Lemma 2 respectively.

Critical Pair Lemma 10

The following expressions are equivalent:

`or[not[or[not[not[x1]],x2]],or[x2,not[or[not[x2],x1]]]]==or[x2,not[x1]]`

PROOF

Note that the input for the rule:

`or[x1_,or[x2_,not[or[not[x1_],not[not[x3_]]]]]]→or[x2,or[x1,not[or[not[x1],x3]]]]`

contains a subpattern of the form:

`or[x2_,not[or[not[x1_],not[not[x3_]]]]]`

which can be unified with the input for the rule:

`or[not[or[not[x1_],x2_]],not[or[not[x2_],not[x1_]]]]→x1`

where these rules follow from Critical Pair Lemma 9 and Critical Pair Lemma 2 respectively.

Substitution Lemma 3

It can be shown that:

`or[not[or[not[x1],x2]],or[x1,not[or[not[not[x2]],x1]]]]==or[x1,not[x2]]`

PROOF

We start by taking Critical Pair Lemma 10, and apply the substitution:

`or[x1_,or[x2_,x3_]]→or[x3,or[x2,x1]]`

which follows from Critical Pair Lemma 6.

Substitution Lemma 4

It can be shown that:

```
or[x1,or[not[or[not[x1],x2]],not[or[not[not[x2]],x1]]]]==or[x1,not[x2]]
```

Proof

We start by taking Substitution Lemma 3, and apply the substitution:

```
or[x1_,or[x2_,x3_]]→or[x2,or[x1,x3]]
```

which follows from Substitution Lemma 1.

Critical Pair Lemma 11

The following expressions are equivalent:

```
or[not[x1],not[not[x1]]]==or[not[x1],x1]
```

Proof

Note that the input for the rule:

```
or[x1_,or[not[or[not[x1_],x2_]],not[or[not[not[x2_]],x1_]]]]→or[x1,not[x2]]
```

contains a subpattern of the form:

```
or[not[or[not[x1_],x2_]],not[or[not[not[x2_]],x1_]]]
```

which can be unified with the input for the rule:

```
or[not[or[x1_,not[x2_]]],not[or[not[x1_],not[x2_]]]]→x2
```

where these rules follow from Substitution Lemma 4 and Critical Pair Lemma 7 respectively.

Substitution Lemma 5

It can be shown that:

```
or[not[x1],not[not[x1]]]==or[x1,not[x1]]
```

Proof

We start by taking Critical Pair Lemma 11, and apply the substitution:

```
or[x1_,x2_]→or[x2,x1]
```

which follows from Axiom 2.

Critical Pair Lemma 12

The following expressions are equivalent:

```
not[x1]==or[not[or[x1,not[not[x1]]]],not[or[x1,not[x1]]]]
```

Proof

Note that the input for the rule:

```
or[not[or[x1_,not[x2_]]],not[or[not[x1_],not[x2_]]]]→x2
```

contains a subpattern of the form:

```
or[not[x1_],not[x2_]]
```

which can be unified with the input for the rule:

```
or[not[x1_],not[not[x1_]]]→or[x1,not[x1]]
```

where these rules follow from Critical Pair Lemma 7 and Substitution Lemma 5 respectively.

Substitution Lemma 6

It can be shown that:

```
not[x1]==or[not[or[x1,not[x1]]],not[or[x1,not[not[x1]]]]]
```

PROOF

We start by taking Critical Pair Lemma 12, and apply the substitution:

```
or[x1_,x2_]→or[x2,x1]
```

which follows from Axiom 2.

Critical Pair Lemma 13

The following expressions are equivalent:

```
not[not[x1]]==x1
```

PROOF

Note that the input for the rule:

```
or[not[or[x1_,not[x1_]]],not[or[x1_,not[not[x1_]]]]]→not[x1]
```

contains a subpattern of the form:

```
or[not[or[x1_,not[x1_]]],not[or[x1_,not[not[x1_]]]]]
```

which can be unified with the input for the rule:

```
or[not[or[not[x1_],x2_]],not[or[not[x1_],not[x2_]]]]→x1
```

where these rules follow from Substitution Lemma 6 and Axiom 1 respectively.

Critical Pair Lemma 14

The following expressions are equivalent:

```
not[x1]==or[not[or[x1,x2]],not[or[not[not[x1]],not[x2]]]]
```

PROOF

Note that the input for the rule:

```
or[not[or[not[x1_],x2_]],not[or[not[x1_],not[x2_]]]]→x1
```

contains a subpattern of the form:

```
not[x1_]
```

which can be unified with the input for the rule:

```
not[not[x1_]]→x1
```

where these rules follow from Axiom 1 and Critical Pair Lemma 13 respectively.

Substitution Lemma 7

It can be shown that:

```
not[x1]==or[not[or[x1,x2]],not[or[x1,not[x2]]]]
```

PROOF

We start by taking Critical Pair Lemma 14, and apply the substitution:

```
not[not[x1_]]→x1
```

which follows from Critical Pair Lemma 13.

Substitution Lemma 8

It can be shown that:

```
not[not[a]]==a
```

PROOF

We start by taking Hypothesis 1, and apply the substitution:

We start by taking Hypothesis 1, and apply the substitution:

`or[not[or[x1_,x2_]],not[or[x1_,not[x2_]]]]→not[x1]`

which follows from Substitution Lemma 7.

Conclusion 1

We obtain the conclusion:

`True`

Proof

Take Substitution Lemma 8, and apply the substitution:

`not[not[x1_]]→x1`

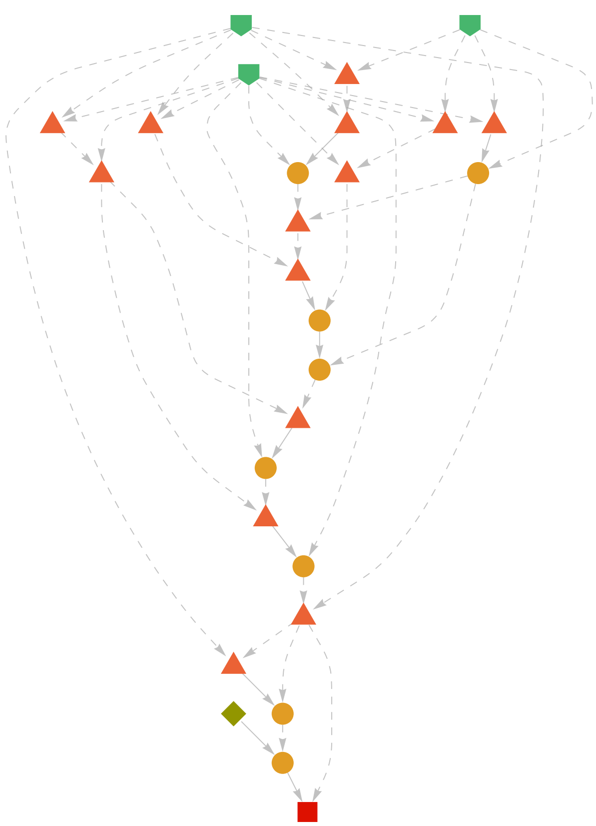
which follows from Critical Pair Lemma 13.

Represent the derivation of the third axiom of Robbins logic, from Huntington logic as a graph. (See graph “Legend” at the end of Appendix 1 for a description of the graph symbology.)

In[∘]:=

```
proofRobbinsfromHuntington["ProofGraph"]
```

Out[∘]=



In[∘]:=

```
Clear[proofRobbinsfromHuntington]
```

Appendix 4. Derivation of equational Boolean from Huntington logic

In[ ]:=
```
proofAxB1fromHunt["ProofNotebook"]
```

Out[ ]=

Axiom 1

We are given that:

`or[x1,x2]==or[x2,x1]`

Axiom 2

We are given that:

`not[or[not[x1],not[x2]]]==and[x1,x2]`

Hypothesis 1

We would like to show that:

`and[a,b]==and[b,a]`

Critical Pair Lemma 1

The following expressions are equivalent:

`and[x1,x2]==not[or[not[x2],not[x1]]]`

Proof

Note that the input for the rule:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

contains a subpattern of the form:

`or[not[x1_],not[x2_]]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Axiom 2 and Axiom 1 respectively.

Substitution Lemma 1

It can be shown that:

`and[x1,x2]==and[x2,x1]`

Proof

We start by taking Critical Pair Lemma 1, and apply the substitution:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

which follows from Axiom 2.

Conclusion 1

We obtain the conclusion:

`True`

Proof

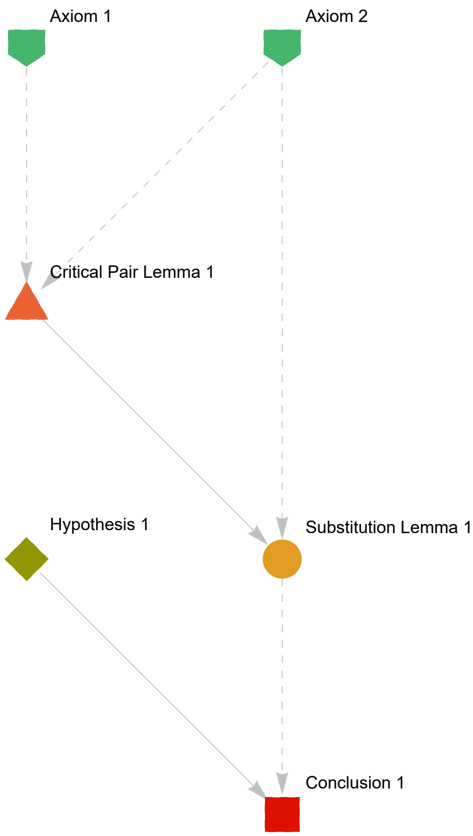
Take Hypothesis 1, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Substitution Lemma 1.

In[ ]:=
```
proofAxB1fromHunt["ProofGraph"]
```

Out[ ]=



In[ ]:=
```
Clear[proofAxB1fromHunt]
```

In[∘]:= proofAxB2fromHunt["ProofNotebook"]

Out[∘]=

Axiom 1

We are given that:

`or[x1,x2]==or[x2,x1]`

Hypothesis 1

We would like to show that:

`or[a,b]==or[b,a]`

Conclusion 1

We obtain the conclusion:

`True`

Proof

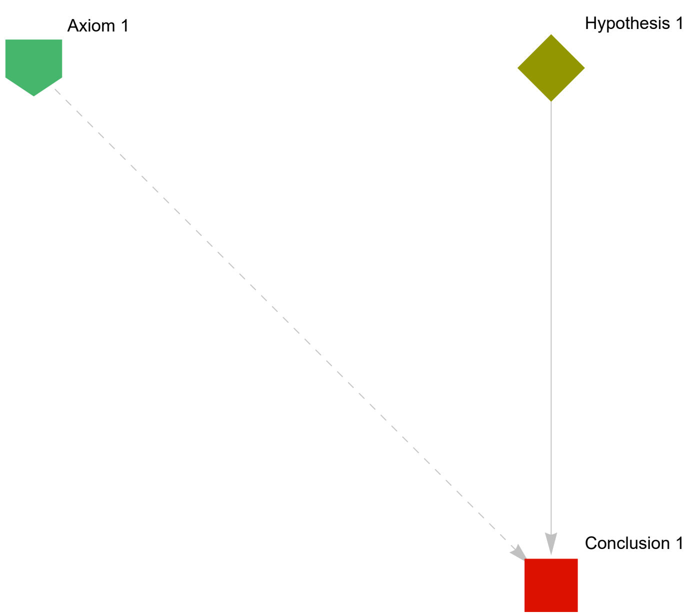
Take Hypothesis 1, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 1.

In[∘]:= proofAxB2fromHunt["ProofGraph"]

Out[∘]=



In[∘]:= Clear[proofAxB2fromHunt]

In[ ]:= `proofAxB3fromHunt["ProofNotebook"]`

Axiom 1

We are given that:

`x1==or[not[or[not[x1],x2]],not[or[not[x1],not[x2]]]]`

Axiom 2

We are given that:

`or[x1,x2]==or[x2,x1]`

Axiom 3

We are given that:

`or[x1,or[x2,x3]]==or[or[x1,x2],x3]`

Axiom 4

We are given that:

`not[or[not[x1],not[x2]]]==and[x1,x2]`

Hypothesis 1

We would like to show that:

`and[a,or[b,not[b]]]==a`

Critical Pair Lemma 1

The following expressions are equivalent:

`or[x1,or[x2,x3]]==or[x3,or[x1,x2]]`

Proof

Note that the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

contains a subpattern of the form:

`or[or[x1_,x2_],x3_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Axiom 3 and Axiom 2 respectively.

Critical Pair Lemma 2

The following expressions are equivalent:

`or[x1,or[x2,x3]]==or[or[x2,x1],x3]`

Proof

Note that the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Axiom 3 and Axiom 2 respectively.

Substitution Lemma 1

It can be shown that:

`or[x1,or[x2,x3]]==or[x2,or[x1,x3]]`

Proof

We start by taking Critical Pair Lemma 2, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Critical Pair Lemma 3

The following expressions are equivalent:

`and[x1,x2]==not[or[not[x2],not[x1]]]`

Proof

Note that the input for the rule:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

contains a subpattern of the form:

`or[not[x1_],not[x2_]]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Axiom 4 and Axiom 2 respectively.

Substitution Lemma 2

It can be shown that:

`and[x1,x2]==and[x2,x1]`

Proof

We start by taking Critical Pair Lemma 3, and apply the substitution:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

which follows from Axiom 4.

Critical Pair Lemma 4

The following expressions are equivalent:

`and[or[not[x1],not[x2]],x3]==not[or[and[x1,x2],not[x3]]]`

Proof

Note that the input for the rule:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

where these rules follow from Axiom 4 and Axiom 4 respectively.

where these rules follow from Axiom 4 and Axiom 4 respectively.

Substitution Lemma 3

It can be shown that:

`or[not[or[not[x1],x2]],and[x1,x2]]==x1`

Proof

We start by taking Axiom 1, and apply the substitution:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

which follows from Axiom 4.

Substitution Lemma 4

It can be shown that:

`or[and[x1,x2],not[or[not[x1],x2]]]==x1`

Proof

We start by taking Substitution Lemma 3, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Critical Pair Lemma 5

The following expressions are equivalent:

`x1==or[and[x2,x1],not[or[not[x1],x2]]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],not[or[not[x1_],x2_]]]→x1`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Substitution Lemma 4 and Substitution Lemma 2 respectively.

Critical Pair Lemma 6

The following expressions are equivalent:

`x1==or[and[x1,not[x2]],and[x1,x2]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],not[or[not[x1_],x2_]]]→x1`

contains a subpattern of the form:

`not[or[not[x1_],x2_]]`

which can be unified with the input for the rule:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

where these rules follow from Substitution Lemma 4 and Axiom 4 respectively.

Substitution Lemma 5

It can be shown that:

```
x1==or[and[x1,x2],and[x1,not[x2]]]
```

PROOF

We start by taking Critical Pair Lemma 6, and apply the substitution:

```
or[x1_,x2_]→or[x2,x1]
```

which follows from Axiom 2.

Critical Pair Lemma 7

The following expressions are equivalent:

```
x1==or[and[x2,x1],and[x1,not[x2]]]
```

PROOF

Note that the input for the rule:

```
or[and[x1_,x2_],and[x1_,not[x2_]]]→x1
```

contains a subpattern of the form:

```
and[x1_,x2_]
```

which can be unified with the input for the rule:

```
and[x1_,x2_]↔and[x2_,x1_]
```

where these rules follow from Substitution Lemma 5 and Substitution Lemma 2 respectively.

Critical Pair Lemma 8

The following expressions are equivalent:

```
or[and[x1,x2],or[and[x1,not[x2]],x3]]==or[x1,x3]
```

PROOF

Note that the input for the rule:

```
or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]
```

contains a subpattern of the form:

```
or[x1_,x2_]
```

which can be unified with the input for the rule:

```
or[and[x1_,x2_],and[x1_,not[x2_]]]→x1
```

where these rules follow from Axiom 3 and Substitution Lemma 5 respectively.

Critical Pair Lemma 9

The following expressions are equivalent:

```
x1==or[and[x2,x1],and[not[x2],x1]]
```

PROOF

Note that the input for the rule:

```
or[and[x1_,x2_],and[x2_,not[x1_]]]→x2
```

contains a subpattern of the form:

```
and[x2_,not[x1_]]
```

which can be unified with the input for the rule:

```
and[x1_,x2_]↔and[x2_,x1_]
```

where these rules follow from Critical Pair Lemma 7 and Substitution Lemma 2 respectively.

Critical Pair Lemma 10

The following expressions are equivalent:

`x1==or[and[x2,x1],not[or[x2,not[x1]]]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],not[or[not[x2_],x1_]]]→x2`

contains a subpattern of the form:

`or[not[x2_],x1_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Critical Pair Lemma 5 and Axiom 2 respectively.

Critical Pair Lemma 11

The following expressions are equivalent:

`and[or[not[x1],not[x2]],or[not[x2],x1]]==not[x2]`

Proof

Note that the input for the rule:

`not[or[and[x1_,x2_],not[x3_]]]→and[or[not[x1],not[x2]],x3]`

contains a subpattern of the form:

`or[and[x1_,x2_],not[x3_]]`

which can be unified with the input for the rule:

`or[and[x1_,x2_],not[or[not[x2_],x1_]]]→x2`

where these rules follow from Critical Pair Lemma 4 and Critical Pair Lemma 5 respectively.

Critical Pair Lemma 12

The following expressions are equivalent:

`or[and[x1,not[x2]],or[x3,and[x2,x1]]]==or[x3,x1]`

Proof

Note that the input for the rule:

`or[x1_,or[x2_,x3_]]↔or[x3_,or[x1_,x2_]]`

contains a subpattern of the form:

`or[x2_,x3_]`

which can be unified with the input for the rule:

`or[and[x1_,x2_],and[x2_,not[x1_]]]→x2`

where these rules follow from Critical Pair Lemma 1 and Critical Pair Lemma 7 respectively.

Critical Pair Lemma 13

The following expressions are equivalent:

`or[x1,or[x2,x3]]==or[x3,or[x2,x1]]`

Proof

PROOF

Note that the input for the rule:

`or[x1_,or[x2_,x3_]]↔or[x3_,or[x1_,x2_]]`

contains a subpattern of the form:

`or[x2_,x3_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Critical Pair Lemma 1 and Axiom 2 respectively.

Substitution Lemma 6

It can be shown that:

`and[or[not[x1],x2],or[not[x2],not[x1]]]==not[x1]`

PROOF

We start by taking Critical Pair Lemma 11, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Substitution Lemma 2.

Critical Pair Lemma 14

The following expressions are equivalent:

`or[x1,and[not[x1],not[x2]]]==or[and[x1,x2],not[x2]]`

PROOF

Note that the input for the rule:

`or[and[x1_,x2_],or[and[x1_,not[x2_]],x3_]]→or[x1,x3]`

contains a subpattern of the form:

`or[and[x1_,not[x2_]],x3_]`

which can be unified with the input for the rule:

`or[and[x1_,x2_],and[not[x1_],x2_]]→x2`

where these rules follow from Critical Pair Lemma 8 and Critical Pair Lemma 9 respectively.

Substitution Lemma 7

It can be shown that:

`or[and[x1,x2],or[x3,and[x2,not[x1]]]]==or[x3,x2]`

PROOF

We start by taking Critical Pair Lemma 12, and apply the substitution:

`or[x1_,or[x2_,x3_]]→or[x3,or[x2,x1]]`

which follows from Critical Pair Lemma 13.

Substitution Lemma 8

It can be shown that:

`or[x1,and[not[x1],not[x2]]]==or[not[x2],and[x1,x2]]`

PROOF

We start by taking Critical Pair Lemma 14, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Critical Pair Lemma 15

The following expressions are equivalent:

`or[x1,not[x1]]==or[and[x2,not[x1]],or[not[x2],and[x1,x2]]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],or[x3_,and[x2_,not[x1_]]]]→or[x3,x2]`

contains a subpattern of the form:

`or[x3_,and[x2_,not[x1_]]]`

which can be unified with the input for the rule:

`or[x1_,and[not[x1_],not[x2_]]]↔or[not[x2_],and[x1_,x2_]]`

where these rules follow from Substitution Lemma 7 and Substitution Lemma 8 respectively.

Substitution Lemma 9

It can be shown that:

`or[x1,not[x1]]==or[and[x1,x2],or[not[x2],and[x2,not[x1]]]]`

Proof

We start by taking Critical Pair Lemma 15, and apply the substitution:

`or[x1_,or[x2_,x3_]]→or[x3,or[x2,x1]]`

which follows from Critical Pair Lemma 13.

Substitution Lemma 10

It can be shown that:

`or[x1,not[x1]]==or[not[x2],x2]`

Proof

We start by taking Substitution Lemma 9, and apply the substitution:

`or[and[x1_,x2_],or[x3_,and[x2_,not[x1_]]]]→or[x3,x2]`

which follows from Substitution Lemma 7.

Substitution Lemma 11

It can be shown that:

`or[x1,not[x1]]==or[x2,not[x2]]`

Proof

We start by taking Substitution Lemma 10, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Critical Pair Lemma 16

The following expressions are equivalent:

`and[x1,not[x1]]==not[or[x2,not[x2]]]`

Proof

Note that the input for the rule:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

contains a subpattern of the form:

`or[not[x1_],not[x2_]]`

which can be unified with the input for the rule:

`or[x1_,not[x1_]]↔or[x2_,not[x2_]]`

where these rules follow from Axiom 4 and Substitution Lemma 11 respectively.

Critical Pair Lemma 17

The following expressions are equivalent:

`and[x1,not[x1]]==and[x2,not[x2]]`

Proof

Note that the input for the rule:

`and[x1_,not[x1_]]↔not[or[x2_,not[x2_]]]`

contains a subpattern of the form:

`not[or[x1_,not[x1_]]]`

which can be unified with the input for the rule:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

where these rules follow from Critical Pair Lemma 16 and Axiom 4 respectively.

Critical Pair Lemma 18

The following expressions are equivalent:

`x1==or[and[x1,not[not[x1]]],and[x2,not[x2]]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],not[or[not[x1_],x2_]]]→x1`

contains a subpattern of the form:

`not[or[not[x1_],x2_]]`

which can be unified with the input for the rule:

`and[x1_,not[x1_]]↔not[or[x2_,not[x2_]]]`

where these rules follow from Substitution Lemma 4 and Critical Pair Lemma 16 respectively.

Critical Pair Lemma 19

The following expressions are equivalent:

`x1==or[and[x1,x1],and[x2,not[x2]]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],not[or[x1_,not[x2_]]]]→x2`

contains a subpattern of the form:

`not[or[x1_,not[x2_]]]`

which can be unified with the input for the rule:

`and[x1_,not[x1_]]↔not[or[x2_,not[x2_]]]`

where these rules follow from Critical Pair Lemma 10 and Critical Pair Lemma 16 respectively.

Critical Pair Lemma 20

The following expressions are equivalent:

`or[and[x1,not[x1]],or[x2,and[x3,x3]]]==or[x2,x3]`

Proof

Note that the input for the rule:

`or[x1_,or[x2_,x3_]]↔or[x3_,or[x1_,x2_]]`

contains a subpattern of the form:

`or[x2_,x3_]`

which can be unified with the input for the rule:

`or[and[x1_,x1_],and[x2_,not[x2_]]]→x1`

where these rules follow from Critical Pair Lemma 1 and Critical Pair Lemma 19 respectively.

Critical Pair Lemma 21

The following expressions are equivalent:

`x1==not[not[x1]]`

Proof

Note that the input for the rule:

`or[and[x1_,not[not[x1_]]],and[x2_,not[x2_]]]→x1`

contains a subpattern of the form:

`or[and[x1_,not[not[x1_]]],and[x2_,not[x2_]]]`

which can be unified with the input for the rule:

`or[and[x1_,x2_],and[not[x1_],x2_]]→x2`

where these rules follow from Critical Pair Lemma 18 and Critical Pair Lemma 9 respectively.

Critical Pair Lemma 22

The following expressions are equivalent:

`or[not[x1],not[x2]]==not[and[x1,x2]]`

Proof

Note that the input for the rule:

`not[not[x1_]]→x1`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

where these rules follow from Critical Pair Lemma 21 and Axiom 4 respectively.

Critical Pair Lemma 23

The following expressions are equivalent:

`not[and[not[x1],x2]]==or[x1,not[x2]]`

Proof

Note that the input for the rule:

`or[not[x1_],not[x2_]]→not[and[x1,x2]]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Critical Pair Lemma 22 and Critical Pair Lemma 21 respectively.

Critical Pair Lemma 24

The following expressions are equivalent:

`not[and[x1,not[x2]]]==or[not[x1],x2]`

Proof

Note that the input for the rule:

`or[not[x1_],not[x2_]]→not[and[x1,x2]]`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Critical Pair Lemma 22 and Critical Pair Lemma 21 respectively.

Critical Pair Lemma 25

The following expressions are equivalent:

`or[not[x1],or[not[x2],x3]]==or[not[and[x1,x2]],x3]`

Proof

Note that the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[not[x1_],not[x2_]]→not[and[x1,x2]]`

where these rules follow from Axiom 3 and Critical Pair Lemma 22 respectively.

Critical Pair Lemma 26

The following expressions are equivalent:

`or[not[x1],or[not[x2],x3]]==or[x3,not[and[x2,x1]]]`

Proof

Note that the input for the rule:

`or[x1_,or[x2_,x3_]]↔or[x3_,or[x2_,x1_]]`

contains a subpattern of the form:

`or[x2_,x3_]`

which can be unified with the input for the rule:

`or[not[x1_],not[x2_]]→not[and[x1,x2]]`

where these rules follow from Critical Pair Lemma 13 and Critical Pair Lemma 22 respectively.

Substitution Lemma 12

It can be shown that:

`and[or[not[x1],x2],not[and[x2,x1]]]==not[x1]`

Proof

We start by taking Substitution Lemma 6, and apply the substitution:

`or[not[x1_],not[x2_]]→not[and[x1,x2]]`

which follows from Critical Pair Lemma 22.

Critical Pair Lemma 27

The following expressions are equivalent:

`and[not[x1],x2]==not[or[x1,not[x2]]]`

Proof

Note that the input for the rule:

`not[not[x1_]]→x1`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[and[not[x1_],x2_]]→or[x1,not[x2]]`

where these rules follow from Critical Pair Lemma 21 and Critical Pair Lemma 23 respectively.

Critical Pair Lemma 28

The following expressions are equivalent:

`and[x1,not[x2]]==not[or[not[x1],x2]]`

Proof

Note that the input for the rule:

`not[not[x1_]]→x1`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[and[x1_,not[x2_]]]→or[not[x1],x2]`

where these rules follow from Critical Pair Lemma 21 and Critical Pair Lemma 24 respectively.

Critical Pair Lemma 29

The following expressions are equivalent:

`or[not[and[x1,x2]],not[x3]]==or[not[x1],not[and[x2,x3]]]`

Proof

Note that the input for the rule:

`or[not[x1_],or[not[x2_],x3_]]→or[not[and[x1,x2]],x3]`

contains a subpattern of the form:

`or[not[x2_],x3_]`

which can be unified with the input for the rule:

`or[not[x1_],not[x2_]]→not[and[x1,x2]]`

where these rules follow from Critical Pair Lemma 25 and Critical Pair Lemma 22 respectively.

Substitution Lemma 13

It can be shown that:

`not[and[and[x1,x2],x3]]==or[not[x1],not[and[x2,x3]]]`

Proof

We start by taking Critical Pair Lemma 29, and apply the substitution:

`or[not[x1_],not[x2_]]→not[and[x1,x2]]`

which follows from Critical Pair Lemma 22.

Substitution Lemma 14

It can be shown that:

`not[and[and[x1,x2],x3]]==not[and[x1,and[x2,x3]]]`

Proof

We start by taking Substitution Lemma 13, and apply the substitution:

`or[not[x1_],not[x2_]]→not[and[x1,x2]]`

which follows from Critical Pair Lemma 22.

Critical Pair Lemma 30

The following expressions are equivalent:

`and[and[x1,x2],x3]==not[not[and[x1,and[x2,x3]]]]`

Proof

Note that the input for the rule:

`not[not[x1_]]→x1`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[and[and[x1_,x2_],x3_]]→not[and[x1,and[x2,x3]]]`

where these rules follow from Critical Pair Lemma 21 and Substitution Lemma 14 respectively.

Substitution Lemma 15

It can be shown that:

`and[and[x1,x2],x3]==and[x1,and[x2,x3]]`

Proof

We start by taking Critical Pair Lemma 30, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 21.

Critical Pair Lemma 31

The following expressions are equivalent:

`and[x1,and[x2,x3]]==and[x3,and[x1,x2]]`

Proof

Note that the input for the rule:

`and[and[x1_,x2_],x3_]→and[x1,and[x2,x3]]`

contains a subpattern of the form:

`and[and[x1_,x2_],x3_]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Substitution Lemma 15 and Substitution Lemma 2 respectively.

Critical Pair Lemma 32

The following expressions are equivalent:

`and[x1,and[x2,x3]]==and[and[x2,x1],x3]`

Proof

Note that the input for the rule:

`and[and[x1_,x2_],x3_]→and[x1,and[x2,x3]]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Substitution Lemma 15 and Substitution Lemma 2 respectively.

Substitution Lemma 16

It can be shown that:

`and[x1,and[x2,x3]]==and[x2,and[x1,x3]]`

Proof

We start by taking Critical Pair Lemma 32, and apply the substitution:

`and[and[x1_,x2_],x3_]→and[x1,and[x2,x3]]`

which follows from Substitution Lemma 15.

Substitution Lemma 17

It can be shown that:

`or[not[and[x1,x2]],x3]==or[x3,not[and[x2,x1]]]`

Proof

We start by taking Critical Pair Lemma 26, and apply the substitution:

`or[not[x1_],or[not[x2_],x3_]]→or[not[and[x1,x2]],x3]`

which follows from Critical Pair Lemma 25.

Critical Pair Lemma 33

Critical Pair Lemma 33

The following expressions are equivalent:

`and[not[not[and[x1,x2]]],x3]==not[or[not[x3],not[and[x2,x1]]]]`

PROOF

Note that the input for the rule:

`not[or[x1_,not[x2_]]]→and[not[x1],x2]`

contains a subpattern of the form:

`or[x1_,not[x2_]]`

which can be unified with the input for the rule:

`or[not[and[x1_,x2_]],x3_]↔or[x3_,not[and[x2_,x1_]]]`

where these rules follow from Critical Pair Lemma 27 and Substitution Lemma 17 respectively.

Substitution Lemma 18

It can be shown that:

`and[and[x1,x2],x3]==not[or[not[x3],not[and[x2,x1]]]]`

PROOF

We start by taking Critical Pair Lemma 33, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 21.

Substitution Lemma 19

It can be shown that:

`and[x1,and[x2,x3]]==not[or[not[x3],not[and[x2,x1]]]]`

PROOF

We start by taking Substitution Lemma 18, and apply the substitution:

`and[and[x1_,x2_],x3_]→and[x1,and[x2,x3]]`

which follows from Substitution Lemma 15.

Substitution Lemma 20

It can be shown that:

`and[x1,and[x2,x3]]==and[x3,not[not[and[x2,x1]]]]`

PROOF

We start by taking Substitution Lemma 19, and apply the substitution:

`not[or[not[x1_],x2_]]→and[x1,not[x2]]`

which follows from Critical Pair Lemma 28.

Substitution Lemma 21

It can be shown that:

`and[x1,and[x2,x3]]==and[x3,and[x2,x1]]`

PROOF

We start by taking Substitution Lemma 20, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 21.

Critical Pair Lemma 34

The following expressions are equivalent:

`not[not[x1]]⩵and[or[x1,x2],not[and[x2,not[x1]]]]`

Proof

Note that the input for the rule:

`and[or[not[x1_],x2_],not[and[x2_,x1_]]]→not[x1]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Substitution Lemma 12 and Critical Pair Lemma 21 respectively.

Substitution Lemma 22

It can be shown that:

`x1⩵and[or[x1,x2],not[and[x2,not[x1]]]]`

Proof

We start by taking Critical Pair Lemma 34, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 21.

Substitution Lemma 23

It can be shown that:

`x1⩵and[or[x1,x2],or[not[x2],x1]]`

Proof

We start by taking Substitution Lemma 22, and apply the substitution:

Out[•]=

`not[and[x1_,not[x2_]]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 24.

Critical Pair Lemma 35

The following expressions are equivalent:

`not[not[x1]]⩵and[or[not[not[x1]],x1],not[and[x2,not[x2]]]]`

Proof

Note that the input for the rule:

`and[or[not[x1_],x2_],not[and[x2_,x1_]]]→not[x1]`

contains a subpattern of the form:

`and[x2_,x1_]`

which can be unified with the input for the rule:

`and[x1_,not[x1_]]↔and[x2_,not[x2_]]`

where these rules follow from Substitution Lemma 12 and Critical Pair Lemma 17 respectively.

Substitution Lemma 24

It can be shown that:

`x1==and[or[not[not[x1]],x1],not[and[x2,not[x2]]]]`

Proof

We start by taking Critical Pair Lemma 35, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 21.

Substitution Lemma 25

It can be shown that:

`x1==and[or[x1,x1],not[and[x2,not[x2]]]]`

Proof

We start by taking Substitution Lemma 24, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 21.

Substitution Lemma 26

It can be shown that:

`x1==and[or[x1,x1],or[not[x2],x2]]`

Proof

We start by taking Substitution Lemma 25, and apply the substitution:

`not[and[x1_,not[x2_]]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 24.

Critical Pair Lemma 36

The following expressions are equivalent:

`and[x1,x1]==and[x1,or[not[and[x2,not[x2]]],and[x1,x1]]]`

Proof

Note that the input for the rule:

`and[or[x1_,x2_],or[not[x2_],x1_]]→x1`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[and[x1_,x1_],and[x2_,not[x2_]]]→x1`

where these rules follow from Substitution Lemma 23 and Critical Pair Lemma 19 respectively.

Substitution Lemma 27

It can be shown that:

`and[x1,x1]==and[x1,or[or[not[x2],x2],and[x1,x1]]]`

Proof

We start by taking Critical Pair Lemma 36, and apply the substitution:

`not[and[x1_,not[x2_]]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 24.

Substitution Lemma 28

It can be shown that:

`and[x1,x1]==and[x1,or[not[x2],or[x2,and[x1,x1]]]]`

Proof

We start by taking Substitution Lemma 27, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Critical Pair Lemma 37

The following expressions are equivalent:

`x1==and[or[x2,not[x2]],or[not[not[x1]],x1]]`

Proof

Note that the input for the rule:

`and[or[x1_,x2_],or[not[x2_],x1_]]→x1`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,not[x1_]]↔or[x2_,not[x2_]]`

where these rules follow from Substitution Lemma 23 and Substitution Lemma 11 respectively.

Substitution Lemma 29

It can be shown that:

`x1==and[or[x2,not[x2]],or[x1,x1]]`

Proof

We start by taking Critical Pair Lemma 37, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 21.

Critical Pair Lemma 38

The following expressions are equivalent:

`x1==and[or[x1,x2],or[x1,not[x2]]]`

Proof

Note that the input for the rule:

`and[or[x1_,x2_],or[not[x2_],x1_]]→x1`

contains a subpattern of the form:

`or[not[x2_],x1_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Substitution Lemma 23 and Axiom 2 respectively.

where these rules follow from Substitution Lemma 23 and Axiom 2 respectively.

Substitution Lemma 30

It can be shown that:

`x1==and[or[x1,x1],or[x2,not[x2]]]`

Proof

We start by taking Substitution Lemma 26, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Substitution Lemma 31

It can be shown that:

`and[x1,x1]==and[x1,or[x2,or[not[x2],and[x1,x1]]]]`

Proof

We start by taking Substitution Lemma 28, and apply the substitution:

`or[x1_,or[x2_,x3_]]→or[x2,or[x1,x3]]`

which follows from Substitution Lemma 1.

Critical Pair Lemma 39

The following expressions are equivalent:

`and[x1,x1]==and[x1,or[not[and[x2,not[x2]]],x1]]`

Proof

Note that the input for the rule:

`and[x1_,or[x2_,or[not[x2_],and[x1_,x1_]]]]→and[x1,x1]`

contains a subpattern of the form:

`or[x2_,or[not[x2_],and[x1_,x1_]]]`

which can be unified with the input for the rule:

`or[and[x1_,not[x1_]],or[x2_,and[x3_,x3_]]]→or[x2,x3]`

where these rules follow from Substitution Lemma 31 and Critical Pair Lemma 20 respectively.

Substitution Lemma 32

It can be shown that:

`and[x1,x1]==and[x1,or[or[not[x2],x2],x1]]`

Proof

We start by taking Critical Pair Lemma 39, and apply the substitution:

`not[and[x1_,not[x2_]]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 24.

Substitution Lemma 33

It can be shown that:

`and[x1,x1]==and[x1,or[not[x2],or[x2,x1]]]`

Proof

We start by taking Substitution Lemma 32, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Substitution Lemma 34

It can be shown that:

`and[x1,x1]==and[x1,or[x2,or[not[x2],x1]]]`

Proof

We start by taking Substitution Lemma 33, and apply the substitution:

`or[x1_,or[x2_,x3_]]→or[x2,or[x1,x3]]`

which follows from Substitution Lemma 1.

Critical Pair Lemma 40

The following expressions are equivalent:

`and[x1,x1]==and[x1,or[x1,or[not[x2],x2]]]`

Proof

Note that the input for the rule:

`and[x1_,or[x2_,or[not[x2_],x1_]]]→and[x1,x1]`

contains a subpattern of the form:

`or[x2_,or[not[x2_],x1_]]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,x3_]]↔or[x3_,or[x2_,x1_]]`

where these rules follow from Substitution Lemma 34 and Critical Pair Lemma 13 respectively.

Substitution Lemma 35

It can be shown that:

`and[x1,x1]==and[x1,or[x1,or[x2,not[x2]]]]`

Proof

We start by taking Critical Pair Lemma 40, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Critical Pair Lemma 41

The following expressions are equivalent:

`and[and[x1,x2],and[x1,x2]]==and[and[x1,x2],or[x1,not[and[x1,not[x2]]]]]`

Proof

Note that the input for the rule:

`and[x1_,or[x1_,or[x2_,not[x2_]]]]→and[x1,x1]`

contains a subpattern of the form:

`or[x1_,or[x2_,not[x2_]]]`

which can be unified with the input for the rule:

`or[and[x1_,x2_],or[and[x1_,not[x2_]],x3_]]→or[x1,x3]`

where these rules follow from Substitution Lemma 35 and Critical Pair Lemma 8 respectively.

Substitution Lemma 36

It can be shown that:

```
and[x1,and[x2,and[x1,x2]]]==and[and[x1,x2],or[x1,not[and[x1,not[x2]]]]]
```

Proof

We start by taking Critical Pair Lemma 41, and apply the substitution:

```
and[and[x1_,x2_],x3_]→and[x1,and[x2,x3]]
```

which follows from Substitution Lemma 15.

Substitution Lemma 37

It can be shown that:

```
and[x1,and[x2,and[x1,x2]]]==and[x1,and[x2,or[x1,not[and[x1,not[x2]]]]]]
```

Proof

We start by taking Substitution Lemma 36, and apply the substitution:

```
and[and[x1_,x2_],x3_]→and[x1,and[x2,x3]]
```

which follows from Substitution Lemma 15.

Substitution Lemma 38

It can be shown that:

```
and[x1,and[x2,and[x1,x2]]]==and[x1,and[x2,or[x1,or[not[x1],x2]]]]
```

Proof

We start by taking Substitution Lemma 37, and apply the substitution:

```
not[and[x1_,not[x2_]]]→or[not[x1],x2]
```

which follows from Critical Pair Lemma 24.

Substitution Lemma 39

It can be shown that:

```
and[x1,and[x2,and[x1,x2]]]==and[x1,and[x2,x2]]
```

Proof

We start by taking Substitution Lemma 38, and apply the substitution:

```
and[x1_,or[x2_,or[not[x2_],x1_]]]→and[x1,x1]
```

which follows from Substitution Lemma 34.

Critical Pair Lemma 42

The following expressions are equivalent:

```
and[x1,and[x2,x2]]==and[x2,and[x1,and[x1,x2]]]
```

Proof

Note that the input for the rule:

```
and[x1_,and[x2_,and[x1_,x2_]]]→and[x1,and[x2,x2]]
```

contains a subpattern of the form:

```
and[x1_,and[x2_,and[x1_,x2_]]]
```

and[x1_,and[x2_,and[x1_,x2_]]]

which can be unified with the input for the rule:

and[x1_,and[x2_,x3_]]↔and[x2_,and[x1_,x3_]]

where these rules follow from Substitution Lemma 39 and Substitution Lemma 16 respectively.

Critical Pair Lemma 43

The following expressions are equivalent:

and[x1,and[x2,x2]]==and[x1,and[x1,and[x2,x2]]]

Proof

Note that the input for the rule:

and[x1_,and[x2_,and[x1_,x2_]]]→and[x1,and[x2,x2]]

contains a subpattern of the form:

and[x2_,and[x1_,x2_]]

which can be unified with the input for the rule:

and[x1_,and[x2_,x3_]]↔and[x2_,and[x1_,x3_]]

where these rules follow from Substitution Lemma 39 and Substitution Lemma 16 respectively.

Critical Pair Lemma 44

The following expressions are equivalent:

and[x1,and[x2,x2]]==and[x1,and[x2,and[x2,x1]]]

Proof

Note that the input for the rule:

and[x1_,and[x2_,and[x1_,x2_]]]→and[x1,and[x2,x2]]

contains a subpattern of the form:

and[x2_,and[x1_,x2_]]

which can be unified with the input for the rule:

and[x1_,and[x2_,x3_]]↔and[x3_,and[x1_,x2_]]

where these rules follow from Substitution Lemma 39 and Critical Pair Lemma 31 respectively.

Substitution Lemma 40

It can be shown that:

and[x1,and[x2,x2]]==and[x2,and[x1,x1]]

Proof

We start by taking Critical Pair Lemma 44, and apply the substitution:

and[x1_,and[x2_,and[x2_,x1_]]]→and[x2,and[x1,x1]]

which follows from Critical Pair Lemma 42.

Critical Pair Lemma 45

The following expressions are equivalent:

and[x1,and[x2,x2]]==and[x1,and[x1,x2]]

Proof

Note that the input for the rule:

`and[x1_,and[x2_,x2_]]↔and[x2_,and[x1_,x1_]]`

contains a subpattern of the form:

`and[x1_,and[x2_,x2_]]`

which can be unified with the input for the rule:

`and[x1_,and[x2_,x3_]]↔and[x3_,and[x2_,x1_]]`

where these rules follow from Substitution Lemma 40 and Substitution Lemma 21 respectively.

Critical Pair Lemma 46

The following expressions are equivalent:

`and[x1,and[x1,x2]]==and[x2,and[x1,x2]]`

Proof

Note that the input for the rule:

`and[x1_,and[x2_,x2_]]↔and[x1_,and[x1_,x2_]]`

contains a subpattern of the form:

`and[x1_,and[x2_,x2_]]`

which can be unified with the input for the rule:

`and[x1_,and[x2_,x3_]]↔and[x2_,and[x1_,x3_]]`

where these rules follow from Critical Pair Lemma 45 and Substitution Lemma 16 respectively.

Critical Pair Lemma 47

The following expressions are equivalent:

`and[x1,and[and[x2,x2],and[x2,x2]]]==and[x1,and[x2,x2]]`

Proof

Note that the input for the rule:

`and[x1_,and[x2_,x2_]]↔and[x1_,and[x1_,x2_]]`

contains a subpattern of the form:

`and[x1_,and[x1_,x2_]]`

which can be unified with the input for the rule:

`and[x1_,and[x1_,and[x2_,x2_]]]→and[x1,and[x2,x2]]`

where these rules follow from Critical Pair Lemma 45 and Critical Pair Lemma 43 respectively.

Substitution Lemma 41

It can be shown that:

`and[x1,and[x2,and[x2,and[x2,x2]]]]==and[x1,and[x2,x2]]`

Proof

We start by taking Critical Pair Lemma 47, and apply the substitution:

`and[and[x1_,x2_],x3_]→and[x1,and[x2,x3]]`

which follows from Substitution Lemma 15.

Substitution Lemma 42

It can be shown that:

`and[x1,and[x2,and[x2,x2]]]==and[x1,and[x2,x2]]`

Proof

We start by taking Substitution Lemma 41, and apply the substitution:

`and[x1_,and[x2_,and[x2_,x1_]]]→and[x2,and[x1,x1]]`

which follows from Critical Pair Lemma 42.

Critical Pair Lemma 48

The following expressions are equivalent:

`and[x1,and[x1,x1]]==or[and[x2,and[x1,x1]],and[not[x2],and[x1,and[x1,x1]]]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],and[not[x1_],x2_]]→x2`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,and[x2_,and[x2_,x2_]]]→and[x1,and[x2,x2]]`

where these rules follow from Critical Pair Lemma 9 and Substitution Lemma 42 respectively.

Substitution Lemma 43

It can be shown that:

`and[x1,and[x1,x1]]==or[and[x2,and[x1,x1]],and[not[x2],and[x1,x1]]]`

Proof

We start by taking Critical Pair Lemma 48, and apply the substitution:

`and[x1_,and[x2_,and[x2_,x2_]]]→and[x1,and[x2,x2]]`

which follows from Substitution Lemma 42.

Substitution Lemma 44

It can be shown that:

`and[x1,and[x1,x1]]==and[x1,x1]`

Proof

We start by taking Substitution Lemma 43, and apply the substitution:

`or[and[x1_,x2_],and[not[x1_],x2_]]→x2`

which follows from Critical Pair Lemma 9.

Critical Pair Lemma 49

The following expressions are equivalent:

`or[x1,not[and[not[x1],not[x1]]]]==not[and[not[x1],not[x1]]]`

Proof

Note that the input for the rule:

`not[and[not[x1_],x2_]]→or[x1,not[x2]]`

contains a subpattern of the form:

`and[not[x1_],x2_]`

which can be unified with the input for the rule:

`and[x1_,and[x1_,x1_]]→and[x1,x1]`

where these rules follow from Critical Pair Lemma 23 and Substitution Lemma 44 respectively.

Substitution Lemma 45

It can be shown that:

`or[x1,or[x1,not[not[x1]]]]==not[and[not[x1],not[x1]]]`

Proof

We start by taking Critical Pair Lemma 49, and apply the substitution:

`not[and[not[x1_],x2_]]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 23.

Substitution Lemma 46

It can be shown that:

`or[x1,or[x1,x1]]==not[and[not[x1],not[x1]]]`

Proof

We start by taking Substitution Lemma 45, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 21.

Substitution Lemma 47

It can be shown that:

`or[x1,or[x1,x1]]==or[x1,not[not[x1]]]`

Proof

We start by taking Substitution Lemma 46, and apply the substitution:

`not[and[not[x1_],x2_]]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 23.

Substitution Lemma 48

It can be shown that:

`or[x1,or[x1,x1]]==or[x1,x1]`

Proof

We start by taking Substitution Lemma 47, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 21.

Critical Pair Lemma 50

The following expressions are equivalent:

`or[x1,or[x2,or[x1,x1]]]==or[x2,or[x1,x1]]`

Proof

Note that the input for the rule:

`or[x1_,or[x2_,x3_]]↔or[x2_,or[x1_,x3_]]`

contains a subpattern of the form:

`or[x2_,x3_]`

which can be unified with the input for the rule:

`or[x1_,or[x1_,x1_]]→or[x1,x1]`

where these rules follow from Substitution Lemma 1 and Substitution Lemma 48 respectively.

Critical Pair Lemma 51

The following expressions are equivalent:

`and[or[x1,x2],and[or[x1,x2],or[x1,not[x2]]]]==and[or[x1,not[x2]],x1]`

Proof

Note that the input for the rule:

`and[x1_,and[x1_,x2_]]↔and[x2_,and[x1_,x2_]]`

contains a subpattern of the form:

`and[x2_,x1_]`

which can be unified with the input for the rule:

`and[or[x1_,x2_],or[x1_,not[x2_]]]→x1`

where these rules follow from Critical Pair Lemma 46 and Critical Pair Lemma 38 respectively.

Substitution Lemma 49

It can be shown that:

`and[or[x1,x2],x1]==and[or[x1,not[x2]],x1]`

Proof

We start by taking Critical Pair Lemma 51, and apply the substitution:

`and[or[x1_,x2_],or[x1_,not[x2_]]]→x1`

which follows from Critical Pair Lemma 38.

Substitution Lemma 50

It can be shown that:

`and[x1,or[x1,x2]]==and[or[x1,not[x2]],x1]`

Proof

We start by taking Substitution Lemma 49, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Substitution Lemma 2.

Substitution Lemma 51

It can be shown that:

`and[x1,or[x1,x2]]==and[x1,or[x1,not[x2]]]`

Proof

We start by taking Substitution Lemma 50, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Substitution Lemma 2.

Critical Pair Lemma 52

Critical Pair Lemma 52

The following expressions are equivalent:

`and[or[x1,x1],or[or[x1,x1],or[x1,x1]]]==x1`

Proof

Note that the input for the rule:

`and[x1_,or[x1_,not[x2_]]]→and[x1,or[x1,x2]]`

contains a subpattern of the form:

`and[x1_,or[x1_,not[x2_]]]`

which can be unified with the input for the rule:

`and[or[x1_,x1_],or[x2_,not[x2_]]]→x1`

where these rules follow from Substitution Lemma 51 and Substitution Lemma 30 respectively.

Substitution Lemma 52

It can be shown that:

`and[or[x1,x1],or[x1,or[x1,or[x1,x1]]]]==x1`

Proof

We start by taking Critical Pair Lemma 52, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Substitution Lemma 53

It can be shown that:

`and[or[x1,x1],or[x1,or[x1,x1]]]==x1`

Proof

We start by taking Substitution Lemma 52, and apply the substitution:

`or[x1_,or[x2_,or[x1_,x1_]]]→or[x2,or[x1,x1]]`

which follows from Critical Pair Lemma 50.

Substitution Lemma 54

It can be shown that:

`and[or[x1,x1],or[x1,x1]]==x1`

Proof

We start by taking Substitution Lemma 53, and apply the substitution:

`or[x1_,or[x1_,x1_]]→or[x1,x1]`

which follows from Substitution Lemma 48.

Critical Pair Lemma 53

The following expressions are equivalent:

`and[or[x1,x1],or[x1,x1]]==and[or[x1,x1],x1]`

Proof

Note that the input for the rule:

`and[x1_,and[x1_,x1_]]→and[x1,x1]`

contains a subpattern of the form:

`and[x1_,x1_]`

which can be unified with the input for the rule:

`and[or[x1_,x1_],or[x1_,x1_]]→x1`

where these rules follow from Substitution Lemma 44 and Substitution Lemma 54 respectively.

Substitution Lemma 55

It can be shown that:

`x1==and[or[x1,x1],x1]`

Proof

We start by taking Critical Pair Lemma 53, and apply the substitution:

`and[or[x1_,x1_],or[x1_,x1_]]→x1`

which follows from Substitution Lemma 54.

Substitution Lemma 56

It can be shown that:

`x1==and[x1,or[x1,x1]]`

Proof

We start by taking Substitution Lemma 55, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Substitution Lemma 2.

Critical Pair Lemma 54

The following expressions are equivalent:

`or[x1,not[or[not[x1],not[x1]]]]==not[not[x1]]`

Proof

Note that the input for the rule:

`not[and[not[x1_],x2_]]→or[x1,not[x2]]`

contains a subpattern of the form:

`and[not[x1_],x2_]`

which can be unified with the input for the rule:

`and[x1_,or[x1_,x1_]]→x1`

where these rules follow from Critical Pair Lemma 23 and Substitution Lemma 56 respectively.

Substitution Lemma 57

It can be shown that:

`or[x1,and[x1,not[not[x1]]]]==not[not[x1]]`

Proof

We start by taking Critical Pair Lemma 54, and apply the substitution:

`not[or[not[x1_],x2_]]→and[x1,not[x2]]`

which follows from Critical Pair Lemma 28.

Substitution Lemma 58

It can be shown that:

`or[x1,and[x1,x1]]==not[not[x1]]`

Proof

We start by taking Substitution Lemma 57, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 21.

Substitution Lemma 59

It can be shown that:

`or[x1,and[x1,x1]]==x1`

Proof

We start by taking Substitution Lemma 58, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 21.

Critical Pair Lemma 55

The following expressions are equivalent:

`x1==and[x1,or[x1,not[and[x1,x1]]]]`

Proof

Note that the input for the rule:

`and[or[x1_,x2_],or[x1_,not[x2_]]]→x1`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,and[x1_,x1_]]→x1`

where these rules follow from Critical Pair Lemma 38 and Substitution Lemma 59 respectively.

Substitution Lemma 60

It can be shown that:

`x1==and[x1,or[x1,and[x1,x1]]]`

Proof

We start by taking Critical Pair Lemma 55, and apply the substitution:

`and[x1_,or[x1_,not[x2_]]]→and[x1,or[x1,x2]]`

which follows from Substitution Lemma 51.

Substitution Lemma 61

It can be shown that:

`x1==and[x1,x1]`

Proof

We start by taking Substitution Lemma 60, and apply the substitution:

We start by taking Substitution Lemma [illegible], and apply the substitution:

`or[x1_,and[x1_,x1_]]→x1`

which follows from Substitution Lemma 59.

Critical Pair Lemma 56

The following expressions are equivalent:

`or[not[not[x1]],x1]==not[not[x1]]`

Proof

Note that the input for the rule:

`not[and[x1_,not[x2_]]]→or[not[x1],x2]`

contains a subpattern of the form:

`and[x1_,not[x2_]]`

which can be unified with the input for the rule:

`and[x1_,x1_]→x1`

where these rules follow from Critical Pair Lemma 24 and Substitution Lemma 61 respectively.

Substitution Lemma 62

It can be shown that:

`or[x1,x1]==not[not[x1]]`

Proof

We start by taking Critical Pair Lemma 56, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 21.

Substitution Lemma 63

It can be shown that:

`or[x1,x1]==x1`

Proof

We start by taking Substitution Lemma 62, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 21.

Substitution Lemma 64

It can be shown that:

`and[or[x1,not[x1]],x2]==x2`

Proof

We start by taking Substitution Lemma 29, and apply the substitution:

`or[x1_,x1_]→x1`

which follows from Substitution Lemma 63.

Substitution Lemma 65

It can be shown that:

`and[or[b,not[b]],a]==a`

Proof

We start by taking Hypothesis 1, and apply the substitution:

```
and[x1_,x2_]→and[x2,x1]
```

which follows from Substitution Lemma 2.

Conclusion 1

We obtain the conclusion:

```
True
```

Proof

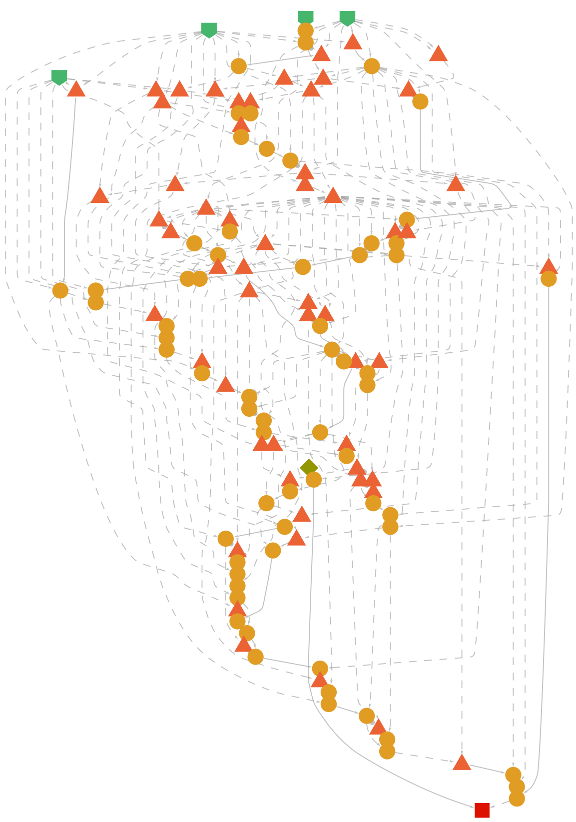
Take Substitution Lemma 65, and apply the substitution:

```
and[or[x1_,not[x1_]],x2_]→x2
```

which follows from Substitution Lemma 64.

In[∘]:=
```
proofAxB3fromHunt["ProofGraph"]
```

Out[∘]=



In[∘]:=
```
Clear[proofAxB3fromHunt]
```

In[ ]:= proofAxB4fromHunt["ProofNotebook"]

Axiom 1

We are given that:

`x1==or[not[or[not[x1],x2]],not[or[not[x1],not[x2]]]]`

Axiom 2

We are given that:

`or[x1,x2]==or[x2,x1]`

Axiom 3

We are given that:

`or[x1,or[x2,x3]]==or[or[x1,x2],x3]`

Axiom 4

We are given that:

`not[or[not[x1],not[x2]]]==and[x1,x2]`

Hypothesis 1

We would like to show that:

`or[a,and[b,not[b]]]==a`

Critical Pair Lemma 1

The following expressions are equivalent:

`or[x1,or[x2,x3]]==or[x3,or[x1,x2]]`

Proof

Note that the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

contains a subpattern of the form:

`or[or[x1_,x2_],x3_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Axiom 3 and Axiom 2 respectively.

Critical Pair Lemma 2

The following expressions are equivalent:

`or[x1,or[x2,x3]]==or[or[x2,x1],x3]`

Proof

Note that the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Axiom 3 and Axiom 2 respectively.

Substitution Lemma 1

It can be shown that:

`or[x1,or[x2,x3]]==or[x2,or[x1,x3]]`

Proof

We start by taking Critical Pair Lemma 2, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Critical Pair Lemma 3

The following expressions are equivalent:

`and[x1,x2]==not[or[not[x2],not[x1]]]`

Proof

Note that the input for the rule:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

contains a subpattern of the form:

`or[not[x1_],not[x2_]]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Axiom 4 and Axiom 2 respectively.

Substitution Lemma 2

It can be shown that:

`and[x1,x2]==and[x2,x1]`

Proof

We start by taking Critical Pair Lemma 3, and apply the substitution:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

which follows from Axiom 4.

Critical Pair Lemma 4

The following expressions are equivalent:

`and[or[not[x1],not[x2]],x3]==not[or[and[x1,x2],not[x3]]]`

Proof

Note that the input for the rule:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

where these rules follow from Axiom 4 and Axiom 4 respectively.

where these rules follow from Axiom 4 and Axiom 4 respectively.

Substitution Lemma 3

It can be shown that:

`or[not[or[not[x1],x2]],and[x1,x2]]==x1`

Proof

We start by taking Axiom 1, and apply the substitution:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

which follows from Axiom 4.

Substitution Lemma 4

It can be shown that:

`or[and[x1,x2],not[or[not[x1],x2]]]==x1`

Proof

We start by taking Substitution Lemma 3, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Critical Pair Lemma 5

The following expressions are equivalent:

`x1==or[and[x2,x1],not[or[not[x1],x2]]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],not[or[not[x1_],x2_]]]→x1`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Substitution Lemma 4 and Substitution Lemma 2 respectively.

Critical Pair Lemma 6

The following expressions are equivalent:

`x1==or[and[x1,not[x2]],and[x1,x2]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],not[or[not[x1_],x2_]]]→x1`

contains a subpattern of the form:

`not[or[not[x1_],x2_]]`

which can be unified with the input for the rule:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

where these rules follow from Substitution Lemma 4 and Axiom 4 respectively.

Substitution Lemma 5

Substitution Lemma 6

It can be shown that:

`x1==or[and[x1,x2],and[x1,not[x2]]]`

Proof

We start by taking Critical Pair Lemma 6, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Critical Pair Lemma 7

The following expressions are equivalent:

`x1==or[and[x2,x1],and[x1,not[x2]]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],and[x1_,not[x2_]]]→x1`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Substitution Lemma 5 and Substitution Lemma 2 respectively.

Critical Pair Lemma 8

The following expressions are equivalent:

`or[and[x1,x2],or[and[x1,not[x2]],x3]]==or[x1,x3]`

Proof

Note that the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[and[x1_,x2_],and[x1_,not[x2_]]]→x1`

where these rules follow from Axiom 3 and Substitution Lemma 5 respectively.

Critical Pair Lemma 9

The following expressions are equivalent:

`x1==or[and[x2,x1],and[not[x2],x1]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],and[x2_,not[x1_]]]→x2`

contains a subpattern of the form:

`and[x2_,not[x1_]]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

and[x1_,x2_]↔and[x2_,x1_]

where these rules follow from Critical Pair Lemma 7 and Substitution Lemma 2 respectively.

Critical Pair Lemma 10

The following expressions are equivalent:

`x1⩵or[and[x2,x1],not[or[x2,not[x1]]]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],not[or[not[x2_],x1_]]]→x2`

contains a subpattern of the form:

`or[not[x2_],x1_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Critical Pair Lemma 5 and Axiom 2 respectively.

Critical Pair Lemma 11

The following expressions are equivalent:

`and[or[not[x1],not[x2]],or[not[x2],x1]]⩵not[x2]`

Proof

Note that the input for the rule:

`not[or[and[x1_,x2_],not[x3_]]]→and[or[not[x1],not[x2]],x3]`

contains a subpattern of the form:

`or[and[x1_,x2_],not[x3_]]`

which can be unified with the input for the rule:

`or[and[x1_,x2_],not[or[not[x2_],x1_]]]→x2`

where these rules follow from Critical Pair Lemma 4 and Critical Pair Lemma 5 respectively.

Critical Pair Lemma 12

The following expressions are equivalent:

`or[and[x1,not[x2]],or[x3,and[x2,x1]]]⩵or[x3,x1]`

Proof

Note that the input for the rule:

`or[x1_,or[x2_,x3_]]↔or[x3_,or[x1_,x2_]]`

contains a subpattern of the form:

`or[x2_,x3_]`

which can be unified with the input for the rule:

`or[and[x1_,x2_],and[x2_,not[x1_]]]→x2`

where these rules follow from Critical Pair Lemma 1 and Critical Pair Lemma 7 respectively.

Critical Pair Lemma 13

The following expressions are equivalent:

`or[x1,or[x2,x3]]⩵or[x3,or[x2,x1]]`

Proof

PROOF

Note that the input for the rule:

`or[x1_,or[x2_,x3_]]↔or[x3_,or[x1_,x2_]]`

contains a subpattern of the form:

`or[x2_,x3_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Critical Pair Lemma 1 and Axiom 2 respectively.

Substitution Lemma 6

It can be shown that:

`and[or[not[x1],x2],or[not[x2],not[x1]]]==not[x1]`

PROOF

We start by taking Critical Pair Lemma 11, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Substitution Lemma 2.

Critical Pair Lemma 14

The following expressions are equivalent:

`or[x1,and[not[x1],not[x2]]]==or[and[x1,x2],not[x2]]`

PROOF

Note that the input for the rule:

`or[and[x1_,x2_],or[and[x1_,not[x2_]],x3_]]→or[x1,x3]`

contains a subpattern of the form:

`or[and[x1_,not[x2_]],x3_]`

which can be unified with the input for the rule:

`or[and[x1_,x2_],and[not[x1_],x2_]]→x2`

where these rules follow from Critical Pair Lemma 8 and Critical Pair Lemma 9 respectively.

Substitution Lemma 7

It can be shown that:

`or[and[x1,x2],or[x3,and[x2,not[x1]]]]==or[x3,x2]`

PROOF

We start by taking Critical Pair Lemma 12, and apply the substitution:

`or[x1_,or[x2_,x3_]]→or[x3,or[x2,x1]]`

which follows from Critical Pair Lemma 13.

Substitution Lemma 8

It can be shown that:

`or[x1,and[not[x1],not[x2]]]==or[not[x2],and[x1,x2]]`

PROOF

We start by taking Critical Pair Lemma 14, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Critical Pair Lemma 15

The following expressions are equivalent:

`or[x1,not[x1]]==or[and[x2,not[x1]],or[not[x2],and[x1,x2]]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],or[x3_,and[x2_,not[x1_]]]]→or[x3,x2]`

contains a subpattern of the form:

`or[x3_,and[x2_,not[x1_]]]`

which can be unified with the input for the rule:

`or[x1_,and[not[x1_],not[x2_]]]↔or[not[x2_],and[x1_,x2_]]`

where these rules follow from Substitution Lemma 7 and Substitution Lemma 8 respectively.

Substitution Lemma 9

It can be shown that:

`or[x1,not[x1]]==or[and[x1,x2],or[not[x2],and[x2,not[x1]]]]`

Proof

We start by taking Critical Pair Lemma 15, and apply the substitution:

`or[x1_,or[x2_,x3_]]→or[x3,or[x2,x1]]`

which follows from Critical Pair Lemma 13.

Substitution Lemma 10

It can be shown that:

`or[x1,not[x1]]==or[not[x2],x2]`

Proof

We start by taking Substitution Lemma 9, and apply the substitution:

`or[and[x1_,x2_],or[x3_,and[x2_,not[x1_]]]]→or[x3,x2]`

which follows from Substitution Lemma 7.

Substitution Lemma 11

It can be shown that:

`or[x1,not[x1]]==or[x2,not[x2]]`

Proof

We start by taking Substitution Lemma 10, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Critical Pair Lemma 16

The following expressions are equivalent:

`and[x1,not[x1]]==not[or[x2,not[x2]]]`

Proof

Note that the input for the rule:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

contains a subpattern of the form:

`or[not[x1_],not[x2_]]`

which can be unified with the input for the rule:

`or[x1_,not[x1_]]↔or[x2_,not[x2_]]`

where these rules follow from Axiom 4 and Substitution Lemma 11 respectively.

Critical Pair Lemma 17

The following expressions are equivalent:

`and[x1,not[x1]]⩵and[x2,not[x2]]`

Proof

Note that the input for the rule:

`and[x1_,not[x1_]]↔not[or[x2_,not[x2_]]]`

contains a subpattern of the form:

`not[or[x1_,not[x1_]]]`

which can be unified with the input for the rule:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

where these rules follow from Critical Pair Lemma 16 and Axiom 4 respectively.

Critical Pair Lemma 18

The following expressions are equivalent:

`x1⩵or[and[x1,not[not[x1]]],and[x2,not[x2]]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],not[or[not[x1_],x2_]]]→x1`

contains a subpattern of the form:

`not[or[not[x1_],x2_]]`

which can be unified with the input for the rule:

`and[x1_,not[x1_]]↔not[or[x2_,not[x2_]]]`

where these rules follow from Substitution Lemma 4 and Critical Pair Lemma 16 respectively.

Critical Pair Lemma 19

The following expressions are equivalent:

`x1⩵or[and[x1,x1],and[x2,not[x2]]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],not[or[x1_,not[x2_]]]]→x2`

contains a subpattern of the form:

`not[or[x1_,not[x2_]]]`

which can be unified with the input for the rule:

`and[x1_,not[x1_]]↔not[or[x2_,not[x2_]]]`

where these rules follow from Critical Pair Lemma 10 and Critical Pair Lemma 16 respectively.

Critical Pair Lemma 20

The following expressions are equivalent:

`x1==or[and[x2,not[x2]],not[or[not[x1],not[x1]]]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],not[or[not[x1_],x2_]]]→x1`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,not[x1_]]↔and[x2_,not[x2_]]`

where these rules follow from Substitution Lemma 4 and Critical Pair Lemma 17 respectively.

Substitution Lemma 12

It can be shown that:

`x1==or[and[x2,not[x2]],and[x1,x1]]`

Proof

We start by taking Critical Pair Lemma 20, and apply the substitution:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

which follows from Axiom 4.

Critical Pair Lemma 21

The following expressions are equivalent:

`or[and[x1,not[x1]],or[x2,and[x3,x3]]]==or[x2,x3]`

Proof

Note that the input for the rule:

`or[x1_,or[x2_,x3_]]↔or[x3_,or[x1_,x2_]]`

contains a subpattern of the form:

`or[x2_,x3_]`

which can be unified with the input for the rule:

`or[and[x1_,x1_],and[x2_,not[x2_]]]→x1`

where these rules follow from Critical Pair Lemma 1 and Critical Pair Lemma 19 respectively.

Critical Pair Lemma 22

The following expressions are equivalent:

`x1==not[not[x1]]`

Proof

Note that the input for the rule:

`or[and[x1_,not[not[x1_]]],and[x2_,not[x2_]]]→x1` `or[and[x1_,not[not[x1_]]],and[x1_,not[x1_]]]→x1`

contains a subpattern of the form:

`or[and[x1_,not[not[x1_]]],and[x2_,not[x2_]]]`

which can be unified with the input for the rule:

`or[and[x1_,x2_],and[not[x1_],x2_]]→x2`

where these rules follow from Critical Pair Lemma 18 and Critical Pair Lemma 9 respectively.

Critical Pair Lemma 23

The following expressions are equivalent:

`or[not[x1],not[x2]]==not[and[x1,x2]]`

Proof

Note that the input for the rule:

`not[not[x1_]]→x1`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

where these rules follow from Critical Pair Lemma 22 and Axiom 4 respectively.

Critical Pair Lemma 24

The following expressions are equivalent:

`not[and[not[x1],x2]]==or[x1,not[x2]]`

Proof

Note that the input for the rule:

`or[not[x1_],not[x2_]]→not[and[x1,x2]]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Critical Pair Lemma 23 and Critical Pair Lemma 22 respectively.

Critical Pair Lemma 25

The following expressions are equivalent:

`not[and[x1,not[x2]]]==or[not[x1],x2]`

Proof

Note that the input for the rule:

`or[not[x1_],not[x2_]]→not[and[x1,x2]]`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Critical Pair Lemma 23 and Critical Pair Lemma 22 respectively.

Critical Pair Lemma 26

The following expressions are equivalent:

`or[not[x1],or[not[x2],x3]]==or[not[and[x1,x2]],x3]`

Proof

Note that the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[not[x1_],not[x2_]]→not[and[x1,x2]]`

where these rules follow from Axiom 3 and Critical Pair Lemma 23 respectively.

Critical Pair Lemma 27

The following expressions are equivalent:

`or[not[x1],or[not[x2],x3]]==or[x3,not[and[x2,x1]]]`

Proof

Note that the input for the rule:

`or[x1_,or[x2_,x3_]]↔or[x3_,or[x2_,x1_]]`

contains a subpattern of the form:

`or[x2_,x3_]`

which can be unified with the input for the rule:

`or[not[x1_],not[x2_]]→not[and[x1,x2]]`

where these rules follow from Critical Pair Lemma 13 and Critical Pair Lemma 23 respectively.

Substitution Lemma 13

It can be shown that:

`and[or[not[x1],x2],not[and[x2,x1]]]==not[x1]`

Proof

We start by taking Substitution Lemma 6, and apply the substitution:

`or[not[x1_],not[x2_]]→not[and[x1,x2]]`

which follows from Critical Pair Lemma 23.

Critical Pair Lemma 28

The following expressions are equivalent:

`and[not[x1],x2]==not[or[x1,not[x2]]]`

Proof

Note that the input for the rule:

`not[not[x1_]]→x1`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

which can be unified with the input for the rule:

`not[and[not[x1_],x2_]]→or[x1,not[x2]]`

where these rules follow from Critical Pair Lemma 22 and Critical Pair Lemma 24 respectively.

Critical Pair Lemma 29

The following expressions are equivalent:

`and[x1,not[x2]]==not[or[not[x1],x2]]`

Proof

Note that the input for the rule:

`not[not[x1_]]→x1`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[and[x1_,not[x2_]]]→or[not[x1],x2]`

where these rules follow from Critical Pair Lemma 22 and Critical Pair Lemma 25 respectively.

Critical Pair Lemma 30

The following expressions are equivalent:

`or[not[and[x1,x2]],not[x3]]==or[not[x1],not[and[x2,x3]]]`

Proof

Note that the input for the rule:

`or[not[x1_],or[not[x2_],x3_]]→or[not[and[x1,x2]],x3]`

contains a subpattern of the form:

`or[not[x2_],x3_]`

which can be unified with the input for the rule:

`or[not[x1_],not[x2_]]→not[and[x1,x2]]`

where these rules follow from Critical Pair Lemma 26 and Critical Pair Lemma 23 respectively.

Substitution Lemma 14

It can be shown that:

`not[and[and[x1,x2],x3]]==or[not[x1],not[and[x2,x3]]]`

Proof

We start by taking Critical Pair Lemma 30, and apply the substitution:

`or[not[x1_],not[x2_]]→not[and[x1,x2]]`

which follows from Critical Pair Lemma 23.

Substitution Lemma 15

It can be shown that:

`not[and[and[x1,x2],x3]]==not[and[x1,and[x2,x3]]]`

Proof

We start by taking Substitution Lemma 14, and apply the substitution:

`or[not[x1_],not[x2_]]→not[and[x1,x2]]`

which follows from Critical Pair Lemma 23.

which follows from Critical Pair Lemma 23.

Critical Pair Lemma 31

The following expressions are equivalent:

`and[and[x1,x2],x3]==not[not[and[x1,and[x2,x3]]]]`

Proof

Note that the input for the rule:

`not[not[x1_]]→x1`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[and[and[x1_,x2_],x3_]]→not[and[x1,and[x2,x3]]]`

where these rules follow from Critical Pair Lemma 22 and Substitution Lemma 15 respectively.

Substitution Lemma 16

It can be shown that:

`and[and[x1,x2],x3]==and[x1,and[x2,x3]]`

Proof

We start by taking Critical Pair Lemma 31, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 22.

Critical Pair Lemma 32

The following expressions are equivalent:

`and[x1,and[x2,x3]]==and[x3,and[x1,x2]]`

Proof

Note that the input for the rule:

`and[and[x1_,x2_],x3_]→and[x1,and[x2,x3]]`

contains a subpattern of the form:

`and[and[x1_,x2_],x3_]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Substitution Lemma 16 and Substitution Lemma 2 respectively.

Critical Pair Lemma 33

The following expressions are equivalent:

`and[x1,and[x2,x3]]==and[and[x2,x1],x3]`

Proof

Note that the input for the rule:

`and[and[x1_,x2_],x3_]→and[x1,and[x2,x3]]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Substitution Lemma 16 and Substitution Lemma 2 respectively.

Substitution Lemma 17

It can be shown that:

`and[x1,and[x2,x3]]==and[x2,and[x1,x3]]`

Proof

We start by taking Critical Pair Lemma 33, and apply the substitution:

`and[and[x1_,x2_],x3_]→and[x1,and[x2,x3]]`

which follows from Substitution Lemma 16.

Substitution Lemma 18

It can be shown that:

`or[not[and[x1,x2]],x3]==or[x3,not[and[x2,x1]]]`

Proof

We start by taking Critical Pair Lemma 27, and apply the substitution:

`or[not[x1_],or[not[x2_],x3_]]→or[not[and[x1,x2]],x3]`

which follows from Critical Pair Lemma 26.

Critical Pair Lemma 34

The following expressions are equivalent:

`and[not[not[and[x1,x2]]],x3]==not[or[not[x3],not[and[x2,x1]]]]`

Proof

Note that the input for the rule:

`not[or[x1_,not[x2_]]]→and[not[x1],x2]`

contains a subpattern of the form:

`or[x1_,not[x2_]]`

which can be unified with the input for the rule:

`or[not[and[x1_,x2_]],x3_]↔or[x3_,not[and[x2_,x1_]]]`

where these rules follow from Critical Pair Lemma 28 and Substitution Lemma 18 respectively.

Substitution Lemma 19

It can be shown that:

`and[and[x1,x2],x3]==not[or[not[x3],not[and[x2,x1]]]]`

Proof

We start by taking Critical Pair Lemma 34, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 22.

Substitution Lemma 20

It can be shown that:

`and[x1,and[x2,x3]]==not[or[not[x3],not[and[x2,x1]]]]`

PROOF

We start by taking Substitution Lemma 19, and apply the substitution:

`and[and[x1_,x2_],x3_]→and[x1,and[x2,x3]]`

which follows from Substitution Lemma 16.

Substitution Lemma 21

It can be shown that:

`and[x1,and[x2,x3]]==and[x3,not[not[and[x2,x1]]]]`

PROOF

We start by taking Substitution Lemma 20, and apply the substitution:

`not[or[not[x1_],x2_]]→and[x1,not[x2]]`

which follows from Critical Pair Lemma 29.

Out[•]=

Substitution Lemma 22

It can be shown that:

`and[x1,and[x2,x3]]==and[x3,and[x2,x1]]`

PROOF

We start by taking Substitution Lemma 21, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 22.

Critical Pair Lemma 35

The following expressions are equivalent:

`not[not[x1]]==and[or[x1,x2],not[and[x2,not[x1]]]]`

PROOF

Note that the input for the rule:

`and[or[not[x1_],x2_],not[and[x2_,x1_]]]→not[x1]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Substitution Lemma 13 and Critical Pair Lemma 22 respectively.

Substitution Lemma 23

It can be shown that:

`x1==and[or[x1,x2],not[and[x2,not[x1]]]]`

PROOF

We start by taking Critical Pair Lemma 35, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 22.

Substitution Lemma 24

It can be shown that:

`x1==and[or[x1,x2],or[not[x2],x1]]`

Proof

We start by taking Substitution Lemma 23, and apply the substitution:

`not[and[x1_,not[x2_]]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 25.

Critical Pair Lemma 36

The following expressions are equivalent:

`not[not[x1]]==and[or[not[not[x1]],x1],not[and[x2,not[x2]]]]`

Proof

Note that the input for the rule:

`and[or[not[x1_],x2_],not[and[x2_,x1_]]]→not[x1]`

contains a subpattern of the form:

`and[x2_,x1_]`

which can be unified with the input for the rule:

`and[x1_,not[x1_]]↔and[x2_,not[x2_]]`

where these rules follow from Substitution Lemma 13 and Critical Pair Lemma 17 respectively.

Substitution Lemma 25

It can be shown that:

`x1==and[or[not[not[x1]],x1],not[and[x2,not[x2]]]]`

Proof

We start by taking Critical Pair Lemma 36, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 22.

Substitution Lemma 26

It can be shown that:

`x1==and[or[x1,x1],not[and[x2,not[x2]]]]`

Proof

We start by taking Substitution Lemma 25, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 22.

Substitution Lemma 27

It can be shown that:

`x1==and[or[x1,x1],or[not[x2],x2]]`

Proof

We start by taking Substitution Lemma 26, and apply the substitution:

`not[and[x1_,not[x2_]]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 25.

Critical Pair Lemma 37

The following expressions are equivalent:

`and[x1,x1]==and[x1,or[not[and[x2,not[x2]]],and[x1,x1]]]`

Proof

Note that the input for the rule:

`and[or[x1_,x2_],or[not[x2_],x1_]]→x1`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[and[x1_,x1_],and[x2_,not[x2_]]]→x1`

where these rules follow from Substitution Lemma 24 and Critical Pair Lemma 19 respectively.

Substitution Lemma 28

It can be shown that:

`and[x1,x1]==and[x1,or[or[not[x2],x2],and[x1,x1]]]`

Proof

We start by taking Critical Pair Lemma 37, and apply the substitution:

`not[and[x1_,not[x2_]]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 25.

Substitution Lemma 29

It can be shown that:

`and[x1,x1]==and[x1,or[not[x2],or[x2,and[x1,x1]]]]`

Proof

We start by taking Substitution Lemma 28, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Critical Pair Lemma 38

The following expressions are equivalent:

`x1==and[or[x1,x2],or[x1,not[x2]]]`

Proof

Note that the input for the rule:

`and[or[x1_,x2_],or[not[x2_],x1_]]→x1`

contains a subpattern of the form:

`or[not[x2_],x1_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Substitution Lemma 24 and Axiom 2 respectively.

where these rules follow from Substitution Lemma 24 and Axiom 2 respectively.

Substitution Lemma 30

It can be shown that:

`x1==and[or[x1,x1],or[x2,not[x2]]]`

Proof

We start by taking Substitution Lemma 27, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Substitution Lemma 31

It can be shown that:

`and[x1,x1]==and[x1,or[x2,or[not[x2],and[x1,x1]]]]`

Proof

We start by taking Substitution Lemma 29, and apply the substitution:

`or[x1_,or[x2_,x3_]]→or[x2,or[x1,x3]]`

which follows from Substitution Lemma 1.

Critical Pair Lemma 39

The following expressions are equivalent:

`and[x1,x1]==and[x1,or[not[and[x2,not[x2]]],x1]]`

Proof

Note that the input for the rule:

`and[x1_,or[x2_,or[not[x2_],and[x1_,x1_]]]]→and[x1,x1]`

contains a subpattern of the form:

`or[x2_,or[not[x2_],and[x1_,x1_]]]`

which can be unified with the input for the rule:

`or[and[x1_,not[x1_]],or[x2_,and[x3_,x3_]]]→or[x2,x3]`

where these rules follow from Substitution Lemma 31 and Critical Pair Lemma 21 respectively.

Substitution Lemma 32

It can be shown that:

`and[x1,x1]==and[x1,or[or[not[x2],x2],x1]]`

Proof

We start by taking Critical Pair Lemma 39, and apply the substitution:

`not[and[x1_,not[x2_]]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 25.

Substitution Lemma 33

It can be shown that:

`and[x1,x1]==and[x1,or[not[x2],or[x2,x1]]]`

Proof

We start by taking Substitution Lemma 32, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Substitution Lemma 34

It can be shown that:

`and[x1,x1]==and[x1,or[x2,or[not[x2],x1]]]`

Proof

We start by taking Substitution Lemma 33, and apply the substitution:

`or[x1_,or[x2_,x3_]]→or[x2,or[x1,x3]]`

which follows from Substitution Lemma 1.

Critical Pair Lemma 40

The following expressions are equivalent:

`and[x1,x1]==and[x1,or[x1,or[not[x2],x2]]]`

Proof

Note that the input for the rule:

`and[x1_,or[x2_,or[not[x2_],x1_]]]→and[x1,x1]`

contains a subpattern of the form:

`or[x2_,or[not[x2_],x1_]]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,x3_]]↔or[x3_,or[x2_,x1_]]`

where these rules follow from Substitution Lemma 34 and Critical Pair Lemma 13 respectively.

Substitution Lemma 35

It can be shown that:

`and[x1,x1]==and[x1,or[x1,or[x2,not[x2]]]]`

Proof

We start by taking Critical Pair Lemma 40, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Critical Pair Lemma 41

The following expressions are equivalent:

`and[and[x1,x2],and[x1,x2]]==and[and[x1,x2],or[x1,not[and[x1,not[x2]]]]]`

Proof

Note that the input for the rule:

`and[x1_,or[x1_,or[x2_,not[x2_]]]]→and[x1,x1]`

contains a subpattern of the form:

`or[x1_,or[x2_,not[x2_]]]`

which can be unified with the input for the rule:

`or[and[x1_,x2_],or[and[x1_,not[x2_]],x3_]]→or[x1,x3]`

where these rules follow from Substitution Lemma 35 and Critical Pair Lemma 8 respectively.

Substitution Lemma 36

It can be shown that:

`and[x1,and[x2,and[x1,x2]]]==and[and[x1,x2],or[x1,not[and[x1,not[x2]]]]]`

Proof

We start by taking Critical Pair Lemma 41, and apply the substitution:

`and[and[x1_,x2_],x3_]→and[x1,and[x2,x3]]`

which follows from Substitution Lemma 16.

Substitution Lemma 37

It can be shown that:

`and[x1,and[x2,and[x1,x2]]]==and[x1,and[x2,or[x1,not[and[x1,not[x2]]]]]]`

Proof

We start by taking Substitution Lemma 36, and apply the substitution:

`and[and[x1_,x2_],x3_]→and[x1,and[x2,x3]]`

which follows from Substitution Lemma 16.

Substitution Lemma 38

It can be shown that:

`and[x1,and[x2,and[x1,x2]]]==and[x1,and[x2,or[x1,or[not[x1],x2]]]]`

Proof

We start by taking Substitution Lemma 37, and apply the substitution:

`not[and[x1_,not[x2_]]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 25.

Substitution Lemma 39

It can be shown that:

`and[x1,and[x2,and[x1,x2]]]==and[x1,and[x2,x2]]`

Proof

We start by taking Substitution Lemma 38, and apply the substitution:

`and[x1_,or[x2_,or[not[x2_],x1_]]]→and[x1,x1]`

which follows from Substitution Lemma 34.

Critical Pair Lemma 42

The following expressions are equivalent:

`and[x1,and[x2,x2]]==and[x2,and[x1,and[x1,x2]]]`

Proof

Note that the input for the rule:

`and[x1_,and[x2_,and[x1_,x2_]]]→and[x1,and[x2,x2]]`

contains a subpattern of the form:

`and[x1_,and[x2_,and[x1_,x2_]]]`

and[x1_,and[x2_,and[x1_,x2_]]]

which can be unified with the input for the rule:

and[x1_,and[x2_,x3_]]↔and[x2_,and[x1_,x3_]]

where these rules follow from Substitution Lemma 39 and Substitution Lemma 17 respectively.

Critical Pair Lemma 43

The following expressions are equivalent:

and[x1,and[x2,x2]]==and[x1,and[x1,and[x2,x2]]]

Proof

Note that the input for the rule:

and[x1_,and[x2_,and[x1_,x2_]]]→and[x1,and[x2,x2]]

contains a subpattern of the form:

and[x2_,and[x1_,x2_]]

which can be unified with the input for the rule:

and[x1_,and[x2_,x3_]]↔and[x2_,and[x1_,x3_]]

where these rules follow from Substitution Lemma 39 and Substitution Lemma 17 respectively.

Critical Pair Lemma 44

The following expressions are equivalent:

and[x1,and[x2,x2]]==and[x1,and[x2,and[x2,x1]]]

Proof

Note that the input for the rule:

and[x1_,and[x2_,and[x1_,x2_]]]→and[x1,and[x2,x2]]

contains a subpattern of the form:

and[x2_,and[x1_,x2_]]

which can be unified with the input for the rule:

and[x1_,and[x2_,x3_]]↔and[x3_,and[x1_,x2_]]

where these rules follow from Substitution Lemma 39 and Critical Pair Lemma 32 respectively.

Substitution Lemma 40

It can be shown that:

and[x1,and[x2,x2]]==and[x2,and[x1,x1]]

Proof

We start by taking Critical Pair Lemma 44, and apply the substitution:

and[x1_,and[x2_,and[x2_,x1_]]]→and[x2,and[x1,x1]]

which follows from Critical Pair Lemma 42.

Critical Pair Lemma 45

The following expressions are equivalent:

and[x1,and[x2,x2]]==and[x1,and[x1,x2]]

Proof

Note that the input for the rule:

```
and[x1_,and[x2_,x2_]]↔and[x2_,and[x1_,x1_]]
```
contains a subpattern of the form:
```
and[x1_,and[x2_,x2_]]
```
which can be unified with the input for the rule:
```
and[x1_,and[x2_,x3_]]↔and[x3_,and[x2_,x1_]]
```
where these rules follow from Substitution Lemma 40 and Substitution Lemma 22 respectively.

Critical Pair Lemma 46

The following expressions are equivalent:
```
and[x1,and[x1,x2]]==and[x2,and[x1,x2]]
```
Proof

Note that the input for the rule:
```
and[x1_,and[x2_,x2_]]↔and[x1_,and[x1_,x2_]]
```
contains a subpattern of the form:
```
and[x1_,and[x2_,x2_]]
```
which can be unified with the input for the rule:
```
and[x1_,and[x2_,x3_]]↔and[x2_,and[x1_,x3_]]
```
where these rules follow from Critical Pair Lemma 45 and Substitution Lemma 17 respectively.

Critical Pair Lemma 47

The following expressions are equivalent:
```
and[x1,and[and[x2,x2],and[x2,x2]]]==and[x1,and[x2,x2]]
```
Proof

Note that the input for the rule:
```
and[x1_,and[x2_,x2_]]↔and[x1_,and[x1_,x2_]]
```
contains a subpattern of the form:
```
and[x1_,and[x1_,x2_]]
```
which can be unified with the input for the rule:
```
and[x1_,and[x1_,and[x2_,x2_]]]→and[x1,and[x2,x2]]
```
where these rules follow from Critical Pair Lemma 45 and Critical Pair Lemma 43 respectively.

Substitution Lemma 41

It can be shown that:
```
and[x1,and[x2,and[x2,and[x2,x2]]]]==and[x1,and[x2,x2]]
```
Proof

We start by taking Critical Pair Lemma 47, and apply the substitution:
```
and[and[x1_,x2_],x3_]→and[x1,and[x2,x3]]
```
which follows from Substitution Lemma 16.

Substitution Lemma 42

It can be shown that:
```
and[x1,and[x2,and[x2,x2]]]==and[x1,and[x2,x2]]
```

Proof

We start by taking Substitution Lemma 41, and apply the substitution:

```
and[x1_,and[x2_,and[x2_,x1_]]]→and[x2,and[x1,x1]]
```

which follows from Critical Pair Lemma 42.

Critical Pair Lemma 48

The following expressions are equivalent:

```
and[x1,and[x1,x1]]==or[and[x2,and[x1,x1]],and[not[x2],and[x1,and[x1,x1]]]]
```

Proof

Note that the input for the rule:

```
or[and[x1_,x2_],and[not[x1_],x2_]]→x2
```

contains a subpattern of the form:

```
and[x1_,x2_]
```

which can be unified with the input for the rule:

```
and[x1_,and[x2_,and[x2_,x2_]]]→and[x1,and[x2,x2]]
```

where these rules follow from Critical Pair Lemma 9 and Substitution Lemma 42 respectively.

Substitution Lemma 43

It can be shown that:

```
and[x1,and[x1,x1]]==or[and[x2,and[x1,x1]],and[not[x2],and[x1,x1]]]
```

Proof

We start by taking Critical Pair Lemma 48, and apply the substitution:

```
and[x1_,and[x2_,and[x2_,x2_]]]→and[x1,and[x2,x2]]
```

which follows from Substitution Lemma 42.

Substitution Lemma 44

It can be shown that:

```
and[x1,and[x1,x1]]==and[x1,x1]
```

Proof

We start by taking Substitution Lemma 43, and apply the substitution:

```
or[and[x1_,x2_],and[not[x1_],x2_]]→x2
```

which follows from Critical Pair Lemma 9.

Critical Pair Lemma 49

The following expressions are equivalent:

```
or[x1,not[and[not[x1],not[x1]]]]==not[and[not[x1],not[x1]]]
```

Proof

Note that the input for the rule:

```
not[and[not[x1_],x2_]]→or[x1,not[x2]]
```

contains a subpattern of the form:

```
and[not[x1_],x2_]
```

which can be unified with the input for the rule:

`and[x1_,and[x1_,x1_]]→and[x1,x1]`

where these rules follow from Critical Pair Lemma 24 and Substitution Lemma 44 respectively.

Substitution Lemma 45

It can be shown that:

`or[x1,or[x1,not[not[x1]]]]==not[and[not[x1],not[x1]]]`

Proof

We start by taking Critical Pair Lemma 49, and apply the substitution:

`not[and[not[x1_],x2_]]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 24.

Substitution Lemma 46

It can be shown that:

`or[x1,or[x1,x1]]==not[and[not[x1],not[x1]]]`

Proof

We start by taking Substitution Lemma 45, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 22.

Substitution Lemma 47

It can be shown that:

`or[x1,or[x1,x1]]==or[x1,not[not[x1]]]`

Proof

We start by taking Substitution Lemma 46, and apply the substitution:

`not[and[not[x1_],x2_]]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 24.

Substitution Lemma 48

It can be shown that:

`or[x1,or[x1,x1]]==or[x1,x1]`

Proof

We start by taking Substitution Lemma 47, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 22.

Critical Pair Lemma 50

The following expressions are equivalent:

`or[x1,or[x2,or[x1,x1]]]==or[x2,or[x1,x1]]`

Proof

Note that the input for the rule:

`or[x1_,or[x2_,x3_]]↔or[x2_,or[x1_,x3_]]`

contains a subpattern of the form:

`or[x2_,x3_]`

which can be unified with the input for the rule:

`or[x1_,or[x1_,x1_]]→or[x1,x1]`

where these rules follow from Substitution Lemma 1 and Substitution Lemma 48 respectively.

Critical Pair Lemma 51

The following expressions are equivalent:

`and[or[x1,x2],and[or[x1,x2],or[x1,not[x2]]]]==and[or[x1,not[x2]],x1]`

Proof

Note that the input for the rule:

`and[x1_,and[x1_,x2_]]↔and[x2_,and[x1_,x2_]]`

contains a subpattern of the form:

`and[x2_,x1_]`

which can be unified with the input for the rule:

`and[or[x1_,x2_],or[x1_,not[x2_]]]→x1`

where these rules follow from Critical Pair Lemma 46 and Critical Pair Lemma 38 respectively.

Substitution Lemma 49

It can be shown that:

`and[or[x1,x2],x1]==and[or[x1,not[x2]],x1]`

Proof

We start by taking Critical Pair Lemma 51, and apply the substitution:

`and[or[x1_,x2_],or[x1_,not[x2_]]]→x1`

which follows from Critical Pair Lemma 38.

Substitution Lemma 50

It can be shown that:

`and[x1,or[x1,x2]]==and[or[x1,not[x2]],x1]`

Proof

We start by taking Substitution Lemma 49, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Substitution Lemma 2.

Substitution Lemma 51

It can be shown that:

`and[x1,or[x1,x2]]==and[x1,or[x1,not[x2]]]`

Proof

We start by taking Substitution Lemma 50, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Substitution Lemma 2.

Critical Pair Lemma 52

Critical Pair Lemma 52

The following expressions are equivalent:

`and[or[x1,x1],or[or[x1,x1],or[x1,x1]]]==x1`

Proof

Note that the input for the rule:

`and[x1_,or[x1_,not[x2_]]]→and[x1,or[x1,x2]]`

contains a subpattern of the form:

`and[x1_,or[x1_,not[x2_]]]`

which can be unified with the input for the rule:

`and[or[x1_,x1_],or[x2_,not[x2_]]]→x1`

where these rules follow from Substitution Lemma 51 and Substitution Lemma 30 respectively.

Substitution Lemma 52

It can be shown that:

`and[or[x1,x1],or[x1,or[x1,or[x1,x1]]]]==x1`

Proof

We start by taking Critical Pair Lemma 52, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Substitution Lemma 53

It can be shown that:

`and[or[x1,x1],or[x1,or[x1,x1]]]==x1`

Proof

We start by taking Substitution Lemma 52, and apply the substitution:

`or[x1_,or[x2_,or[x1_,x1_]]]→or[x2,or[x1,x1]]`

which follows from Critical Pair Lemma 50.

Substitution Lemma 54

It can be shown that:

`and[or[x1,x1],or[x1,x1]]==x1`

Proof

We start by taking Substitution Lemma 53, and apply the substitution:

`or[x1_,or[x1_,x1_]]→or[x1,x1]`

which follows from Substitution Lemma 48.

Critical Pair Lemma 53

The following expressions are equivalent:

`and[or[x1,x1],or[x1,x1]]==and[or[x1,x1],x1]`

Proof

Note that the input for the rule:

`and[x1_,and[x1_,x1_]]→and[x1,x1]`

`and[x1_,and[x1_,x1_]]→and[x1,x1]`

contains a subpattern of the form:

`and[x1_,x1_]`

which can be unified with the input for the rule:

`and[or[x1_,x1_],or[x1_,x1_]]→x1`

where these rules follow from Substitution Lemma 44 and Substitution Lemma 54 respectively.

Substitution Lemma 55

It can be shown that:

`x1==and[or[x1,x1],x1]`

Proof

We start by taking Critical Pair Lemma 53, and apply the substitution:

`and[or[x1_,x1_],or[x1_,x1_]]→x1`

which follows from Substitution Lemma 54.

Substitution Lemma 56

It can be shown that:

`x1==and[x1,or[x1,x1]]`

Proof

We start by taking Substitution Lemma 55, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Substitution Lemma 2.

Critical Pair Lemma 54

The following expressions are equivalent:

`or[x1,not[or[not[x1],not[x1]]]]==not[not[x1]]`

Proof

Note that the input for the rule:

`not[and[not[x1_],x2_]]→or[x1,not[x2]]`

contains a subpattern of the form:

`and[not[x1_],x2_]`

which can be unified with the input for the rule:

`and[x1_,or[x1_,x1_]]→x1`

where these rules follow from Critical Pair Lemma 24 and Substitution Lemma 56 respectively.

Substitution Lemma 57

It can be shown that:

`or[x1,and[x1,not[not[x1]]]]==not[not[x1]]`

Proof

We start by taking Critical Pair Lemma 54, and apply the substitution:

`not[or[not[x1_],x2_]]→and[x1,not[x2]]`

which follows from Critical Pair Lemma 29.

Substitution Lemma 58

It can be shown that:

`or[x1,and[x1,x1]]==not[not[x1]]`

Proof

We start by taking Substitution Lemma 57, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 22.

Substitution Lemma 59

It can be shown that:

`or[x1,and[x1,x1]]==x1`

Proof

We start by taking Substitution Lemma 58, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 22.

Critical Pair Lemma 55

The following expressions are equivalent:

`x1==and[x1,or[x1,not[and[x1,x1]]]]`

Proof

Note that the input for the rule:

`and[or[x1_,x2_],or[x1_,not[x2_]]]→x1`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,and[x1_,x1_]]→x1`

where these rules follow from Critical Pair Lemma 38 and Substitution Lemma 59 respectively.

Substitution Lemma 60

It can be shown that:

`x1==and[x1,or[x1,and[x1,x1]]]`

Proof

We start by taking Critical Pair Lemma 55, and apply the substitution:

`and[x1_,or[x1_,not[x2_]]]→and[x1,or[x1,x2]]`

which follows from Substitution Lemma 51.

Substitution Lemma 61

It can be shown that:

`x1==and[x1,x1]`

Proof

We start by taking Substitution Lemma 60, and apply the substitution:

`or[x1_,and[x1_,x1_]]→x1`

which follows from Substitution Lemma 59.

Substitution Lemma 62

It can be shown that:

`or[and[x1,not[x1]],x2]==x2`

Proof

We start by taking Substitution Lemma 12, and apply the substitution:

`and[x1_,x1_]→x1`

which follows from Substitution Lemma 61.

Substitution Lemma 63

It can be shown that:

`or[and[b,not[b]],a]==a`

Proof

We start by taking Hypothesis 1, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Conclusion 1

We obtain the conclusion:

`True`

Proof

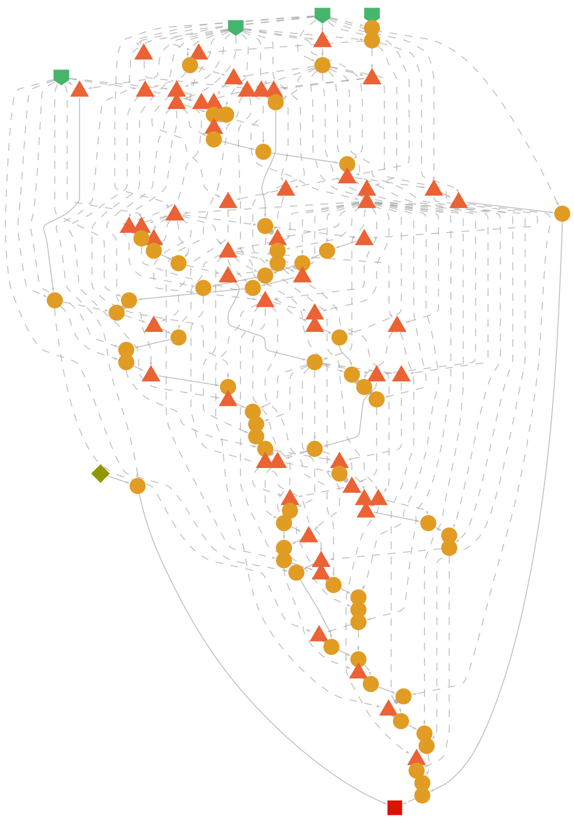
Take Substitution Lemma 63, and apply the substitution:

`or[and[x1_,not[x1_]],x2_]→x2`

which follows from Substitution Lemma 62.

In[ ]:=
```
proofAxB4fromHunt["ProofGraph"]
```

Out[ ]=



In[ ]:= Clear[proofAxB4fromHunt]

In[ ]:= proofAxB5fromHunt["ProofNotebook"]

Axiom 1

We are given that:

`x1==or[not[or[not[x1],x2]],not[or[not[x1],not[x2]]]]`

Axiom 2

We are given that:

`or[x1,x2]==or[x2,x1]`

Axiom 3

We are given that:

`or[x1,or[x2,x3]]==or[or[x1,x2],x3]`

Axiom 4

We are given that:

`not[or[not[x1],not[x2]]]==and[x1,x2]`

Hypothesis 1

We would like to show that:

`or[and[a,b],and[a,c]]==and[a,or[b,c]]`

Critical Pair Lemma 1

The following expressions are equivalent:

`or[x1,or[x2,x3]]==or[x3,or[x1,x2]]`

Proof

Note that the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

contains a subpattern of the form:

`or[or[x1_,x2_],x3_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Axiom 3 and Axiom 2 respectively.

Critical Pair Lemma 2

The following expressions are equivalent:

`or[x1,or[x2,x3]]==or[or[x2,x1],x3]`

Proof

Note that the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Axiom 3 and Axiom 2 respectively.

Substitution Lemma 1

It can be shown that:

`or[x1,or[x2,x3]]==or[x2,or[x1,x3]]`

Proof

We start by taking Critical Pair Lemma 2, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Critical Pair Lemma 3

The following expressions are equivalent:

`and[x1,x2]==not[or[not[x2],not[x1]]]`

Proof

Note that the input for the rule:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

contains a subpattern of the form:

`or[not[x1_],not[x2_]]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Axiom 4 and Axiom 2 respectively.

Substitution Lemma 2

It can be shown that:

`and[x1,x2]==and[x2,x1]`

Proof

We start by taking Critical Pair Lemma 3, and apply the substitution:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

which follows from Axiom 4.

Critical Pair Lemma 4

The following expressions are equivalent:

`and[or[not[x1],not[x2]],x3]==not[or[and[x1,x2],not[x3]]]`

Proof

Note that the input for the rule:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

where these rules follow from Axiom 4 and Axiom 4 respectively.

Substitution Lemma 3

It can be shown that:

`or[not[or[not[x1],x2]],and[x1,x2]]==x1`

Proof

We start by taking Axiom 1, and apply the substitution:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

which follows from Axiom 4.

Substitution Lemma 4

It can be shown that:

`or[and[x1,x2],not[or[not[x1],x2]]]==x1`

Proof

We start by taking Substitution Lemma 3, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Critical Pair Lemma 5

The following expressions are equivalent:

`x1==or[and[x2,x1],not[or[not[x1],x2]]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],not[or[not[x1_],x2_]]]→x1`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Substitution Lemma 4 and Substitution Lemma 2 respectively.

Critical Pair Lemma 6

The following expressions are equivalent:

`x1==or[and[x1,not[x2]],and[x1,x2]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],not[or[not[x1_],x2_]]]→x1`

contains a subpattern of the form:

`not[or[not[x1_],x2_]]`

which can be unified with the input for the rule:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

where these rules follow from Substitution Lemma 4 and Axiom 4 respectively.

Substitution Lemma 5

It can be shown that:

`x1==or[and[x1,x2],and[x1,not[x2]]]`

Proof

We start by taking Critical Pair Lemma 6, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Critical Pair Lemma 7

The following expressions are equivalent:

`x1==or[and[x2,x1],and[x1,not[x2]]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],and[x1_,not[x2_]]]→x1`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Substitution Lemma 5 and Substitution Lemma 2 respectively.

Critical Pair Lemma 8

The following expressions are equivalent:

`x1==or[and[x1,x2],and[not[x2],x1]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],and[x1_,not[x2_]]]→x1`

contains a subpattern of the form:

`and[x1_,not[x2_]]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Substitution Lemma 5 and Substitution Lemma 2 respectively.

Critical Pair Lemma 9

The following expressions are equivalent:

`or[and[x1,x2],or[and[x1,not[x2]],x3]]==or[x1,x3]`

Proof

Note that the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

```
or[and[x1_,x2_],and[x1_,not[x2_]]]→x1
```

where these rules follow from Axiom 3 and Substitution Lemma 5 respectively.

Critical Pair Lemma 10

The following expressions are equivalent:

```
x1==or[and[x2,x1],and[not[x2],x1]]
```

Proof

Note that the input for the rule:

```
or[and[x1_,x2_],and[x2_,not[x1_]]]→x2
```

contains a subpattern of the form:

```
and[x2_,not[x1_]]
```

which can be unified with the input for the rule:

```
and[x1_,x2_]↔and[x2_,x1_]
```

where these rules follow from Critical Pair Lemma 7 and Substitution Lemma 2 respectively.

Critical Pair Lemma 11

The following expressions are equivalent:

```
or[and[x1,x2],or[and[not[x2],x1],x3]]==or[x1,x3]
```

Proof

Note that the input for the rule:

```
or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]
```

contains a subpattern of the form:

```
or[x1_,x2_]
```

which can be unified with the input for the rule:

```
or[and[x1_,x2_],and[not[x2_],x1_]]→x1
```

where these rules follow from Axiom 3 and Critical Pair Lemma 8 respectively.

Critical Pair Lemma 12

The following expressions are equivalent:

```
x1==or[and[x2,x1],not[or[x2,not[x1]]]]
```

Proof

Note that the input for the rule:

```
or[and[x1_,x2_],not[or[not[x2_],x1_]]]→x2
```

contains a subpattern of the form:

```
or[not[x2_],x1_]
```

which can be unified with the input for the rule:

```
or[x1_,x2_]↔or[x2_,x1_]
```

where these rules follow from Critical Pair Lemma 5 and Axiom 2 respectively.

Critical Pair Lemma 13

The following expressions are equivalent:

```
and[or[not[x1],not[x2]],or[not[x2],x1]]==not[x2]
```

Proof

Note that the input for the rule:

`not[or[and[x1_,x2_],not[x3_]]]→and[or[not[x1],not[x2]],x3]`

contains a subpattern of the form:

`or[and[x1_,x2_],not[x3_]]`

which can be unified with the input for the rule:

`or[and[x1_,x2_],not[or[not[x2_],x1_]]]→x2`

where these rules follow from Critical Pair Lemma 4 and Critical Pair Lemma 5 respectively.

Critical Pair Lemma 14

The following expressions are equivalent:

`or[and[not[x1],x2],or[x3,and[x1,x2]]]==or[x3,x2]`

Proof

Note that the input for the rule:

`or[x1_,or[x2_,x3_]]↔or[x3_,or[x1_,x2_]]`

contains a subpattern of the form:

`or[x2_,x3_]`

which can be unified with the input for the rule:

`or[and[x1_,x2_],and[not[x1_],x2_]]→x2`

where these rules follow from Critical Pair Lemma 1 and Critical Pair Lemma 10 respectively.

Critical Pair Lemma 15

The following expressions are equivalent:

`or[and[x1,not[x2]],or[x3,and[x1,x2]]]==or[x3,x1]`

Proof

Note that the input for the rule:

`or[x1_,or[x2_,x3_]]↔or[x3_,or[x1_,x2_]]`

contains a subpattern of the form:

`or[x2_,x3_]`

which can be unified with the input for the rule:

`or[and[x1_,x2_],and[x1_,not[x2_]]]→x1`

where these rules follow from Critical Pair Lemma 1 and Substitution Lemma 5 respectively.

Critical Pair Lemma 16

The following expressions are equivalent:

`or[and[x1,not[x2]],or[x3,and[x2,x1]]]==or[x3,x1]`

Proof

Note that the input for the rule:

`or[x1_,or[x2_,x3_]]↔or[x3_,or[x1_,x2_]]`

contains a subpattern of the form:

`or[x2_,x3_]`

which can be unified with the input for the rule:

`or[and[x1_,x2_],and[x2_,not[x1_]]]→x2`

where these rules follow from Critical Pair Lemma 1 and Critical Pair Lemma 7 respectively.

Critical Pair Lemma 17

The following expressions are equivalent:

`or[x1,or[x2,x3]]==or[x3,or[x2,x1]]`

Proof

Note that the input for the rule:

`or[x1_,or[x2_,x3_]]↔or[x3_,or[x1_,x2_]]`

contains a subpattern of the form:

`or[x2_,x3_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Critical Pair Lemma 1 and Axiom 2 respectively.

Substitution Lemma 6

It can be shown that:

`and[or[not[x1],x2],or[not[x2],not[x1]]]==not[x1]`

Proof

We start by taking Critical Pair Lemma 13, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Substitution Lemma 2.

Critical Pair Lemma 18

The following expressions are equivalent:

`or[x1,and[not[x1],not[x2]]]==or[and[x1,x2],not[x2]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],or[and[x1_,not[x2_]],x3_]]→or[x1,x3]`

contains a subpattern of the form:

`or[and[x1_,not[x2_]],x3_]`

which can be unified with the input for the rule:

`or[and[x1_,x2_],and[not[x1_],x2_]]→x2`

where these rules follow from Critical Pair Lemma 9 and Critical Pair Lemma 10 respectively.

Critical Pair Lemma 19

The following expressions are equivalent:

`or[x1,and[not[x2],not[x1]]]==or[and[x1,x2],not[x2]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],or[and[x1_,not[x2_]],x3_]]→or[x1,x3]`

contains a subpattern of the form:

`or[and[x1_,not[x2_]],x3_]`

which can be unified with the input for the rule:

`or[and[x1_,x2_],and[x2_,not[x1_]]]→x2`

where these rules follow from Critical Pair Lemma 9 and Critical Pair Lemma 7 respectively.

Substitution Lemma 7

It can be shown that:

`or[and[x1,x2],or[x3,and[not[x1],x2]]]==or[x3,x2]`

Proof

We start by taking Critical Pair Lemma 14, and apply the substitution:

`or[x1_,or[x2_,x3_]]→or[x3,or[x2,x1]]`

which follows from Critical Pair Lemma 17.

Substitution Lemma 8

It can be shown that:

`or[and[x1,x2],or[x3,and[x1,not[x2]]]]==or[x3,x1]`

Proof

We start by taking Critical Pair Lemma 15, and apply the substitution:

`or[x1_,or[x2_,x3_]]→or[x3,or[x2,x1]]`

which follows from Critical Pair Lemma 17.

Substitution Lemma 9

It can be shown that:

`or[and[x1,x2],or[x3,and[x2,not[x1]]]]==or[x3,x2]`

Proof

We start by taking Critical Pair Lemma 16, and apply the substitution:

`or[x1_,or[x2_,x3_]]→or[x3,or[x2,x1]]`

which follows from Critical Pair Lemma 17.

Substitution Lemma 10

It can be shown that:

`or[x1,and[not[x1],not[x2]]]==or[not[x2],and[x1,x2]]`

Proof

We start by taking Critical Pair Lemma 18, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Critical Pair Lemma 20

The following expressions are equivalent:

`or[x1,not[x1]]==or[and[x2,not[x1]],or[not[x2],and[x1,x2]]]`

Proof

PROOF

Note that the input for the rule:

`or[and[x1_,x2_],or[x3_,and[x2_,not[x1_]]]]→or[x3,x2]`

contains a subpattern of the form:

`or[x3_,and[x2_,not[x1_]]]`

which can be unified with the input for the rule:

`or[x1_,and[not[x1_],not[x2_]]]↔or[not[x2_],and[x1_,x2_]]`

where these rules follow from Substitution Lemma 9 and Substitution Lemma 10 respectively.

Substitution Lemma 11

It can be shown that:

`or[x1,not[x1]]==or[and[x1,x2],or[not[x2],and[x2,not[x1]]]]`

PROOF

We start by taking Critical Pair Lemma 20, and apply the substitution:

`or[x1_,or[x2_,x3_]]→or[x3,or[x2,x1]]`

which follows from Critical Pair Lemma 17.

Substitution Lemma 12

It can be shown that:

`or[x1,not[x1]]==or[not[x2],x2]`

PROOF

We start by taking Substitution Lemma 11, and apply the substitution:

`or[and[x1_,x2_],or[x3_,and[x2_,not[x1_]]]]→or[x3,x2]`

which follows from Substitution Lemma 9.

Substitution Lemma 13

It can be shown that:

`or[x1,not[x1]]==or[x2,not[x2]]`

PROOF

We start by taking Substitution Lemma 12, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Critical Pair Lemma 21

The following expressions are equivalent:

`and[x1,not[x1]]==not[or[x2,not[x2]]]`

PROOF

Note that the input for the rule:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

contains a subpattern of the form:

`or[not[x1_],not[x2_]]`

which can be unified with the input for the rule:

`or[x1_,not[x1_]]↔or[x2_,not[x2_]]`

where these rules follow from Axiom 4 and Substitution Lemma 13 respectively.

Critical Pair Lemma 22

The following expressions are equivalent:

`and[x1,not[x1]]==and[x2,not[x2]]`

Proof

Note that the input for the rule:

`and[x1_,not[x1_]]↔not[or[x2_,not[x2_]]]`

contains a subpattern of the form:

`not[or[x1_,not[x1_]]]`

which can be unified with the input for the rule:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

where these rules follow from Critical Pair Lemma 21 and Axiom 4 respectively.

Critical Pair Lemma 23

The following expressions are equivalent:

`x1==or[and[x1,not[not[x1]]],and[x2,not[x2]]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],not[or[not[x1_],x2_]]]→x1`

contains a subpattern of the form:

`not[or[not[x1_],x2_]]`

which can be unified with the input for the rule:

`and[x1_,not[x1_]]↔not[or[x2_,not[x2_]]]`

where these rules follow from Substitution Lemma 4 and Critical Pair Lemma 21 respectively.

Critical Pair Lemma 24

The following expressions are equivalent:

`x1==or[and[x1,x1],and[x2,not[x2]]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],not[or[x1_,not[x2_]]]]→x2`

contains a subpattern of the form:

`not[or[x1_,not[x2_]]]`

which can be unified with the input for the rule:

`and[x1_,not[x1_]]↔not[or[x2_,not[x2_]]]`

where these rules follow from Critical Pair Lemma 12 and Critical Pair Lemma 21 respectively.

Critical Pair Lemma 25

The following expressions are equivalent:

`or[and[x1,not[x1]],or[x2,and[x3,x3]]]==or[x2,x3]`

Proof

Note that the input for the rule:

`or[x1_,or[x2_,x3_]]↔or[x3_,or[x1_,x2_]]`

contains a subpattern of the form:

`or[x2_,x3_]`

which can be unified with the input for the rule:

`or[and[x1_,x1_],and[x2_,not[x2_]]]→x1`

where these rules follow from Critical Pair Lemma 1 and Critical Pair Lemma 24 respectively.

Critical Pair Lemma 26

The following expressions are equivalent:

`x1==not[not[x1]]`

Proof

Note that the input for the rule:

`or[and[x1_,not[not[x1_]]],and[x2_,not[x2_]]]→x1`

contains a subpattern of the form:

`or[and[x1_,not[not[x1_]]],and[x2_,not[x2_]]]`

which can be unified with the input for the rule:

`or[and[x1_,x2_],and[not[x1_],x2_]]→x2`

where these rules follow from Critical Pair Lemma 23 and Critical Pair Lemma 10 respectively.

Critical Pair Lemma 27

The following expressions are equivalent:

`or[not[x1],not[x2]]==not[and[x1,x2]]`

Proof

Note that the input for the rule:

`not[not[x1_]]→x1`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

where these rules follow from Critical Pair Lemma 26 and Axiom 4 respectively.

Critical Pair Lemma 28

The following expressions are equivalent:

`not[and[not[x1],x2]]==or[x1,not[x2]]`

Proof

Note that the input for the rule:

`or[not[x1_],not[x2_]]→not[and[x1,x2]]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Critical Pair Lemma 27 and Critical Pair Lemma 26 respectively.

Critical Pair Lemma 29

The following expressions are equivalent:

`not[and[x1,not[x2]]]==or[not[x1],x2]`

Proof

Note that the input for the rule:

`or[not[x1_],not[x2_]]→not[and[x1,x2]]`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Critical Pair Lemma 27 and Critical Pair Lemma 26 respectively.

Critical Pair Lemma 30

The following expressions are equivalent:

`or[not[x1],or[not[x2],x3]]==or[not[and[x1,x2]],x3]`

Proof

Note that the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[not[x1_],not[x2_]]→not[and[x1,x2]]`

where these rules follow from Axiom 3 and Critical Pair Lemma 27 respectively.

Critical Pair Lemma 31

The following expressions are equivalent:

`or[not[x1],or[not[x2],x3]]==or[x3,not[and[x2,x1]]]`

Proof

Note that the input for the rule:

`or[x1_,or[x2_,x3_]]↔or[x3_,or[x2_,x1_]]`

contains a subpattern of the form:

`or[x2_,x3_]`

which can be unified with the input for the rule:

`or[not[x1_],not[x2_]]→not[and[x1,x2]]`

where these rules follow from Critical Pair Lemma 17 and Critical Pair Lemma 27 respectively.

Critical Pair Lemma 32

The following expressions are equivalent:

`or[not[x1],or[x2,not[x3]]]==or[x2,not[and[x1,x3]]]`

Proof

Note that the input for the rule:

`or[x1_,or[x2_,x3_]]↔or[x2_,or[x1_,x3_]]`

contains a subpattern of the form:

`or[x2_,x3_]`

which can be unified with the input for the rule:

`or[not[x1_],not[x2_]]→not[and[x1,x2]]`

where these rules follow from Substitution Lemma 1 and Critical Pair Lemma 27 respectively.

Substitution Lemma 14

It can be shown that:

`and[or[not[x1],x2],not[and[x2,x1]]]==not[x1]`

Proof

We start by taking Substitution Lemma 6, and apply the substitution:

`or[not[x1_],not[x2_]]→not[and[x1,x2]]`

which follows from Critical Pair Lemma 27.

Critical Pair Lemma 33

The following expressions are equivalent:

`and[not[x1],x2]==not[or[x1,not[x2]]]`

Proof

Note that the input for the rule:

`not[not[x1_]]→x1`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[and[not[x1_],x2_]]→or[x1,not[x2]]`

where these rules follow from Critical Pair Lemma 26 and Critical Pair Lemma 28 respectively.

Critical Pair Lemma 34

The following expressions are equivalent:

`and[x1,not[x2]]==not[or[not[x1],x2]]`

Proof

Note that the input for the rule:

`not[not[x1_]]→x1`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[and[x1_,not[x2_]]]→or[not[x1],x2]`

where these rules follow from Critical Pair Lemma 26 and Critical Pair Lemma 29 respectively.

Critical Pair Lemma 35

The following expressions are equivalent:

`and[not[x1],not[x2]]==not[or[x1,x2]]`

Proof

Note that the input for the rule:

`not[or[x1_,not[x2_]]]→and[not[x1],x2]`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Critical Pair Lemma 33 and Critical Pair Lemma 26 respectively.

Critical Pair Lemma 36

The following expressions are equivalent:

`or[not[and[x1,x2]],not[x3]]==or[not[x1],not[and[x2,x3]]]`

Proof

Note that the input for the rule:

`or[not[x1_],or[not[x2_],x3_]]→or[not[and[x1,x2]],x3]`

contains a subpattern of the form:

`or[not[x2_],x3_]`

which can be unified with the input for the rule:

`or[not[x1_],not[x2_]]→not[and[x1,x2]]`

where these rules follow from Critical Pair Lemma 30 and Critical Pair Lemma 27 respectively.

Substitution Lemma 15

It can be shown that:

`not[and[and[x1,x2],x3]]==or[not[x1],not[and[x2,x3]]]`

Proof

We start by taking Critical Pair Lemma 36, and apply the substitution:

`or[not[x1_],not[x2_]]→not[and[x1,x2]]`

which follows from Critical Pair Lemma 27.

Substitution Lemma 16

It can be shown that:

`not[and[and[x1,x2],x3]]==not[and[x1,and[x2,x3]]]`

Proof

We start by taking Substitution Lemma 15, and apply the substitution:

`or[not[x1_],not[x2_]]→not[and[x1,x2]]`

which follows from Critical Pair Lemma 27.

Critical Pair Lemma 37

The following expressions are equivalent:

`and[and[x1,x2],x3]==not[not[and[x1,and[x2,x3]]]]`

Proof

Note that the input for the rule:

`not[not[x1_]]→x1`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[and[and[x1_,x2_],x3_]]→not[and[x1,and[x2,x3]]]`

where these rules follow from Critical Pair Lemma 26 and Substitution Lemma 16 respectively.

Substitution Lemma 17

It can be shown that:

`and[and[x1,x2],x3]==and[x1,and[x2,x3]]`

Proof

We start by taking Critical Pair Lemma 37, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 26.

Critical Pair Lemma 38

The following expressions are equivalent:

`and[x1,and[x2,x3]]==and[x3,and[x1,x2]]`

Proof

Note that the input for the rule:

`and[and[x1_,x2_],x3_]→and[x1,and[x2,x3]]`

contains a subpattern of the form:

`and[and[x1_,x2_],x3_]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Substitution Lemma 17 and Substitution Lemma 2 respectively.

Critical Pair Lemma 39

The following expressions are equivalent:

`and[x1,and[x2,x3]]==and[and[x2,x1],x3]`

Proof

Note that the input for the rule:

`and[and[x1_,x2_],x3_]→and[x1,and[x2,x3]]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Substitution Lemma 17 and Substitution Lemma 2 respectively.

Substitution Lemma 18

It can be shown that:

`and[x1,and[x2,x3]]==and[x2,and[x1,x3]]`

Proof

We start by taking Critical Pair Lemma 39, and apply the substitution:

`and[and[x1_,x2_],x3_]→and[x1,and[x2,x3]]`

which follows from Substitution Lemma 17.

Substitution Lemma 19

It can be shown that:

`or[not[and[x1,x2]],x3]==or[x3,not[and[x2,x1]]]`

Proof

We start by taking Critical Pair Lemma 31, and apply the substitution:

`or[not[x1_],or[not[x2_],x3_]]→or[not[and[x1,x2]],x3]`

which follows from Critical Pair Lemma 30.

Critical Pair Lemma 40

The following expressions are equivalent:

`and[not[not[and[x1,x2]]],x3]==not[or[not[x3],not[and[x2,x1]]]]`

Proof

Note that the input for the rule:

`not[or[x1_,not[x2_]]]→and[not[x1],x2]`

contains a subpattern of the form:

`or[x1_,not[x2_]]`

which can be unified with the input for the rule:

`or[not[and[x1_,x2_]],x3_]↔or[x3_,not[and[x2_,x1_]]]`

where these rules follow from Critical Pair Lemma 33 and Substitution Lemma 19 respectively.

Substitution Lemma 20

It can be shown that:

`and[and[x1,x2],x3]==not[or[not[x3],not[and[x2,x1]]]]`

Proof

We start by taking Critical Pair Lemma 40, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 26.

Substitution Lemma 21

It can be shown that:

`and[x1,and[x2,x3]]==not[or[not[x3],not[and[x2,x1]]]]`

`and[x1,and[x2,x3]]==not[or[not[x3],not[and[x2,x1]]]]`

PROOF

We start by taking Substitution Lemma 20, and apply the substitution:

`and[and[x1_,x2_],x3_]→and[x1,and[x2,x3]]`

which follows from Substitution Lemma 17.

Substitution Lemma 22

It can be shown that:

`and[x1,and[x2,x3]]==and[x3,not[not[and[x2,x1]]]]`

PROOF

We start by taking Substitution Lemma 21, and apply the substitution:

`not[or[not[x1_],x2_]]→and[x1,not[x2]]`

which follows from Critical Pair Lemma 34.

Substitution Lemma 23

It can be shown that:

`and[x1,and[x2,x3]]==and[x3,and[x2,x1]]`

PROOF

We start by taking Substitution Lemma 22, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 26.

Critical Pair Lemma 41

The following expressions are equivalent:

`not[not[x1]]==and[or[x1,x2],not[and[x2,not[x1]]]]`

PROOF

Note that the input for the rule:

`and[or[not[x1_],x2_],not[and[x2_,x1_]]]→not[x1]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Substitution Lemma 14 and Critical Pair Lemma 26 respectively.

Substitution Lemma 24

It can be shown that:

`x1==and[or[x1,x2],not[and[x2,not[x1]]]]`

PROOF

We start by taking Critical Pair Lemma 41, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 26.

which follows from Critical Pair Lemma 26.

Substitution Lemma 25

It can be shown that:

`x1==and[or[x1,x2],or[not[x2],x1]]`

Proof

We start by taking Substitution Lemma 24, and apply the substitution:

`not[and[x1_,not[x2_]]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 29.

Critical Pair Lemma 42

The following expressions are equivalent:

`not[not[x1]]==and[or[not[not[x1]],x1],not[and[x2,not[x2]]]]`

Proof

Note that the input for the rule:

`and[or[not[x1_],x2_],not[and[x2_,x1_]]]→not[x1]`

contains a subpattern of the form:

`and[x2_,x1_]`

which can be unified with the input for the rule:

`and[x1_,not[x1_]]↔and[x2_,not[x2_]]`

where these rules follow from Substitution Lemma 14 and Critical Pair Lemma 22 respectively.

Substitution Lemma 26

It can be shown that:

`x1==and[or[not[not[x1]],x1],not[and[x2,not[x2]]]]`

Proof

We start by taking Critical Pair Lemma 42, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 26.

Substitution Lemma 27

It can be shown that:

`x1==and[or[x1,x1],not[and[x2,not[x2]]]]`

Proof

We start by taking Substitution Lemma 26, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 26.

Substitution Lemma 28

It can be shown that:

`x1==and[or[x1,x1],or[not[x2],x2]]`

Proof

Proof

We start by taking Substitution Lemma 27, and apply the substitution:

`not[and[x1_,not[x2_]]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 29.

Critical Pair Lemma 43

The following expressions are equivalent:

`and[x1,x1]==and[x1,or[not[and[x2,not[x2]]],and[x1,x1]]]`

Proof

Note that the input for the rule:

`and[or[x1_,x2_],or[not[x2_],x1_]]→x1`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[and[x1_,x1_],and[x2_,not[x2_]]]→x1`

where these rules follow from Substitution Lemma 25 and Critical Pair Lemma 24 respectively.

Substitution Lemma 29

It can be shown that:

`and[x1,x1]==and[x1,or[or[not[x2],x2],and[x1,x1]]]`

Proof

We start by taking Critical Pair Lemma 43, and apply the substitution:

`not[and[x1_,not[x2_]]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 29.

Substitution Lemma 30

It can be shown that:

`and[x1,x1]==and[x1,or[not[x2],or[x2,and[x1,x1]]]]`

Proof

We start by taking Substitution Lemma 29, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Critical Pair Lemma 44

The following expressions are equivalent:

`x1==and[or[x2,x1],or[not[x2],x1]]`

Proof

Note that the input for the rule:

`and[or[x1_,x2_],or[not[x2_],x1_]]→x1`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Substitution Lemma 25 and Axiom 2 respectively.

Critical Pair Lemma 45

The following expressions are equivalent:

`x1==and[or[x1,x2],or[x1,not[x2]]]`

Proof

Note that the input for the rule:

`and[or[x1_,x2_],or[not[x2_],x1_]]→x1`

contains a subpattern of the form:

`or[not[x2_],x1_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Substitution Lemma 25 and Axiom 2 respectively.

Substitution Lemma 31

It can be shown that:

`x1==and[or[x1,x1],or[x2,not[x2]]]`

Proof

We start by taking Substitution Lemma 28, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Substitution Lemma 32

It can be shown that:

`or[x1,not[or[x2,x1]]]==or[and[x1,x2],not[x2]]`

Proof

We start by taking Critical Pair Lemma 19, and apply the substitution:

`and[not[x1_],not[x2_]]→not[or[x1,x2]]`

which follows from Critical Pair Lemma 35.

Substitution Lemma 33

It can be shown that:

`or[x1,not[or[x2,x1]]]==or[not[x2],and[x1,x2]]`

Proof

We start by taking Substitution Lemma 32, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Substitution Lemma 34

It can be shown that:

`and[x1,x1]==and[x1,or[x2,or[not[x2],and[x1,x1]]]]`

PROOF

We start by taking Substitution Lemma 30, and apply the substitution:

`or[x1_,or[x2_,x3_]]→or[x2,or[x1,x3]]`

which follows from Substitution Lemma 1.

Critical Pair Lemma 46

The following expressions are equivalent:

`and[x1,x1]==and[x1,or[not[and[x2,not[x2]]],x1]]`

PROOF

Note that the input for the rule:

`and[x1_,or[x2_,or[not[x2_],and[x1_,x1_]]]]→and[x1,x1]`

contains a subpattern of the form:

`or[x2_,or[not[x2_],and[x1_,x1_]]]`

which can be unified with the input for the rule:

`or[and[x1_,not[x1_]],or[x2_,and[x3_,x3_]]]→or[x2,x3]`

where these rules follow from Substitution Lemma 34 and Critical Pair Lemma 25 respectively.

Substitution Lemma 35

It can be shown that:

`and[x1,x1]==and[x1,or[or[not[x2],x2],x1]]`

PROOF

We start by taking Critical Pair Lemma 46, and apply the substitution:

`not[and[x1_,not[x2_]]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 29.

Substitution Lemma 36

It can be shown that:

`and[x1,x1]==and[x1,or[not[x2],or[x2,x1]]]`

PROOF

We start by taking Substitution Lemma 35, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Substitution Lemma 37

It can be shown that:

`and[x1,x1]==and[x1,or[x2,or[not[x2],x1]]]`

PROOF

We start by taking Substitution Lemma 36, and apply the substitution:

`or[x1_,or[x2_,x3_]]→or[x2,or[x1,x3]]`

which follows from Substitution Lemma 1

which follows from Substitution Lemma 1.

Critical Pair Lemma 47

The following expressions are equivalent:

`and[x1,x1]==and[x1,or[x1,or[not[x2],x2]]]`

Proof

Note that the input for the rule:

`and[x1_,or[x2_,or[not[x2_],x1_]]]→and[x1,x1]`

contains a subpattern of the form:

`or[x2_,or[not[x2_],x1_]]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,x3_]]↔or[x3_,or[x2_,x1_]]`

where these rules follow from Substitution Lemma 37 and Critical Pair Lemma 17 respectively.

Substitution Lemma 38

It can be shown that:

`and[x1,x1]==and[x1,or[x1,or[x2,not[x2]]]]`

Proof

We start by taking Critical Pair Lemma 47, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Critical Pair Lemma 48

The following expressions are equivalent:

`and[and[x1,x2],and[x1,x2]]==and[and[x1,x2],or[x1,not[and[x1,not[x2]]]]]`

Proof

Note that the input for the rule:

`and[x1_,or[x1_,or[x2_,not[x2_]]]]→and[x1,x1]`

contains a subpattern of the form:

`or[x1_,or[x2_,not[x2_]]]`

which can be unified with the input for the rule:

`or[and[x1_,x2_],or[and[x1_,not[x2_]],x3_]]→or[x1,x3]`

where these rules follow from Substitution Lemma 38 and Critical Pair Lemma 9 respectively.

Substitution Lemma 39

It can be shown that:

`and[x1,and[x2,and[x1,x2]]]==and[and[x1,x2],or[x1,not[and[x1,not[x2]]]]]`

Proof

We start by taking Critical Pair Lemma 48, and apply the substitution:

`and[and[x1_,x2_],x3_]→and[x1,and[x2,x3]]`

which follows from Substitution Lemma 17.

Substitution Lemma 40

It can be shown that:

`and[x1,and[x2,and[x1,x2]]]==and[x1,and[x2,or[x1,not[and[x1,not[x2]]]]]]`

Proof

We start by taking Substitution Lemma 39, and apply the substitution:

`and[and[x1_,x2_],x3_]→and[x1,and[x2,x3]]`

which follows from Substitution Lemma 17.

Substitution Lemma 41

It can be shown that:

`and[x1,and[x2,and[x1,x2]]]==and[x1,and[x2,or[x1,or[not[x1],x2]]]]`

Proof

We start by taking Substitution Lemma 40, and apply the substitution:

`not[and[x1_,not[x2_]]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 29.

Substitution Lemma 42

It can be shown that:

`and[x1,and[x2,and[x1,x2]]]==and[x1,and[x2,x2]]`

Proof

We start by taking Substitution Lemma 41, and apply the substitution:

`and[x1_,or[x2_,or[not[x2_],x1_]]]→and[x1,x1]`

which follows from Substitution Lemma 37.

Critical Pair Lemma 49

The following expressions are equivalent:

`and[x1,and[x2,x2]]==and[x2,and[x1,and[x1,x2]]]`

Proof

Note that the input for the rule:

`and[x1_,and[x2_,and[x1_,x2_]]]→and[x1,and[x2,x2]]`

contains a subpattern of the form:

`and[x1_,and[x2_,and[x1_,x2_]]]`

which can be unified with the input for the rule:

`and[x1_,and[x2_,x3_]]↔and[x2_,and[x1_,x3_]]`

where these rules follow from Substitution Lemma 42 and Substitution Lemma 18 respectively.

Critical Pair Lemma 50

The following expressions are equivalent:

`and[x1,and[x2,x2]]==and[x1,and[x1,and[x2,x2]]]`

Proof

Note that the input for the rule:

`and[x1_,and[x2_,and[x1_,x2_]]]→and[x1,and[x2,x2]]`

contains a subpattern of the form:

`and[x2_,and[x1_,x2_]]`

which can be unified with the input for the rule:

`and[x1_,and[x2_,x3_]]↔and[x2_,and[x1_,x3_]]`

where these rules follow from Substitution Lemma 42 and Substitution Lemma 18 respectively.

Critical Pair Lemma 51

The following expressions are equivalent:

`and[x1,and[x2,x2]]==and[x1,and[x2,and[x2,x1]]]`

Proof

Out[ ]=

Note that the input for the rule:

`and[x1_,and[x2_,and[x1_,x2_]]]→and[x1,and[x2,x2]]`

contains a subpattern of the form:

`and[x2_,and[x1_,x2_]]`

which can be unified with the input for the rule:

`and[x1_,and[x2_,x3_]]↔and[x3_,and[x1_,x2_]]`

where these rules follow from Substitution Lemma 42 and Critical Pair Lemma 38 respectively.

Substitution Lemma 43

It can be shown that:

`and[x1,and[x2,x2]]==and[x2,and[x1,x1]]`

Proof

We start by taking Critical Pair Lemma 51, and apply the substitution:

`and[x1_,and[x2_,and[x2_,x1_]]]→and[x2,and[x1,x1]]`

which follows from Critical Pair Lemma 49.

Critical Pair Lemma 52

The following expressions are equivalent:

`and[x1,and[x2,x2]]==and[x1,and[x1,x2]]`

Proof

Note that the input for the rule:

`and[x1_,and[x2_,x2_]]↔and[x2_,and[x1_,x1_]]`

contains a subpattern of the form:

`and[x1_,and[x2_,x2_]]`

which can be unified with the input for the rule:

`and[x1_,and[x2_,x3_]]↔and[x3_,and[x2_,x1_]]`

where these rules follow from Substitution Lemma 43 and Substitution Lemma 23 respectively.

respectively.

Critical Pair Lemma 53

The following expressions are equivalent:

`and[x1,and[x1,x2]]==and[x2,and[x1,x2]]`

Proof

Note that the input for the rule:

`and[x1_,and[x2_,x2_]]↔and[x1_,and[x1_,x2_]]`

contains a subpattern of the form:

`and[x1_,and[x2_,x2_]]`

which can be unified with the input for the rule:

`and[x1_,and[x2_,x3_]]↔and[x2_,and[x1_,x3_]]`

where these rules follow from Critical Pair Lemma 52 and Substitution Lemma 18 respectively.

Critical Pair Lemma 54

The following expressions are equivalent:

`and[x1,and[and[x2,x2],and[x2,x2]]]==and[x1,and[x2,x2]]`

Proof

Note that the input for the rule:

`and[x1_,and[x2_,x2_]]↔and[x1_,and[x1_,x2_]]`

contains a subpattern of the form:

`and[x1_,and[x1_,x2_]]`

which can be unified with the input for the rule:

`and[x1_,and[x1_,and[x2_,x2_]]]→and[x1,and[x2,x2]]`

where these rules follow from Critical Pair Lemma 52 and Critical Pair Lemma 50 respectively.

Substitution Lemma 44

It can be shown that:

`and[x1,and[x2,and[x2,and[x2,x2]]]]==and[x1,and[x2,x2]]`

Proof

We start by taking Critical Pair Lemma 54, and apply the substitution:

`and[and[x1_,x2_],x3_]→and[x1,and[x2,x3]]`

which follows from Substitution Lemma 17.

Substitution Lemma 45

It can be shown that:

`and[x1,and[x2,and[x2,x2]]]==and[x1,and[x2,x2]]`

Proof

We start by taking Substitution Lemma 44, and apply the substitution:

`and[x1_,and[x2_,and[x2_,x1_]]]→and[x2,and[x1,x1]]`

which follows from Critical Pair Lemma 49.

Critical Pair Lemma 55

The following expressions are equivalent:

`and[x1,and[x1,x1]]==or[and[x2,and[x1,x1]],and[not[x2],and[x1,and[x1,x1]]]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],and[not[x1_],x2_]]→x2`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,and[x2_,and[x2_,x2_]]]→and[x1,and[x2,x2]]`

where these rules follow from Critical Pair Lemma 10 and Substitution Lemma 45 respectively.

Substitution Lemma 46

It can be shown that:

`and[x1,and[x1,x1]]==or[and[x2,and[x1,x1]],and[not[x2],and[x1,x1]]]`

Proof

We start by taking Critical Pair Lemma 55, and apply the substitution:

`and[x1_,and[x2_,and[x2_,x2_]]]→and[x1,and[x2,x2]]`

which follows from Substitution Lemma 45.

Substitution Lemma 47

It can be shown that:

`and[x1,and[x1,x1]]==and[x1,x1]`

Proof

We start by taking Substitution Lemma 46, and apply the substitution:

`or[and[x1_,x2_],and[not[x1_],x2_]]→x2`

which follows from Critical Pair Lemma 10.

Critical Pair Lemma 56

The following expressions are equivalent:

`or[x1,not[and[not[x1],not[x1]]]]==not[and[not[x1],not[x1]]]`

Proof

Note that the input for the rule:

`not[and[not[x1_],x2_]]→or[x1,not[x2]]`

contains a subpattern of the form:

`and[not[x1_],x2_]`

which can be unified with the input for the rule:

`and[x1_,and[x1_,x1_]]→and[x1,x1]`

where these rules follow from Critical Pair Lemma 28 and Substitution Lemma 47 respectively.

Substitution Lemma 48

It can be shown that:

`or[x1,or[x1,not[not[x1]]]]==not[and[not[x1],not[x1]]]`

Proof

We start by taking Critical Pair Lemma 56, and apply the substitution:

`not[and[not[x1_],x2_]]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 28.

Substitution Lemma 49

It can be shown that:

`or[x1,or[x1,x1]]==not[and[not[x1],not[x1]]]`

Proof

We start by taking Substitution Lemma 48, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 26.

Substitution Lemma 50

It can be shown that:

`or[x1,or[x1,x1]]==or[x1,not[not[x1]]]`

Proof

We start by taking Substitution Lemma 49, and apply the substitution:

`not[and[not[x1_],x2_]]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 28.

Substitution Lemma 51

It can be shown that:

`or[x1,or[x1,x1]]==or[x1,x1]`

Proof

We start by taking Substitution Lemma 50, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 26.

Critical Pair Lemma 57

The following expressions are equivalent:

`or[x1,or[x2,or[x1,x1]]]==or[x2,or[x1,x1]]`

Proof

Note that the input for the rule:

`or[x1_,or[x2_,x3_]]↔or[x2_,or[x1_,x3_]]`

contains a subpattern of the form:

`or[x2_,x3_]`

which can be unified with the input for the rule:

which can be unified with the input for the rule:

`or[x1_,or[x1_,x1_]]→or[x1,x1]`

where these rules follow from Substitution Lemma 1 and Substitution Lemma 51 respectively.

Critical Pair Lemma 58

The following expressions are equivalent:

`and[or[x1,x2],and[or[x1,x2],or[x1,not[x2]]]]==and[or[x1,not[x2]],x1]`

Proof

Note that the input for the rule:

`and[x1_,and[x1_,x2_]]↔and[x2_,and[x1_,x2_]]`

contains a subpattern of the form:

`and[x2_,x1_]`

which can be unified with the input for the rule:

`and[or[x1_,x2_],or[x1_,not[x2_]]]→x1`

where these rules follow from Critical Pair Lemma 53 and Critical Pair Lemma 45 respectively.

Substitution Lemma 52

It can be shown that:

`and[or[x1,x2],x1]==and[or[x1,not[x2]],x1]`

Proof

We start by taking Critical Pair Lemma 58, and apply the substitution:

`and[or[x1_,x2_],or[x1_,not[x2_]]]→x1`

which follows from Critical Pair Lemma 45.

Substitution Lemma 53

It can be shown that:

`and[x1,or[x1,x2]]==and[or[x1,not[x2]],x1]`

Proof

We start by taking Substitution Lemma 52, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Substitution Lemma 2.

Substitution Lemma 54

It can be shown that:

`and[x1,or[x1,x2]]==and[x1,or[x1,not[x2]]]`

Proof

We start by taking Substitution Lemma 53, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Substitution Lemma 2.

Critical Pair Lemma 59

The following expressions are equivalent:

`and[or[x1,x1],or[or[x1,x1],or[x1,x1]]]==x1` `and[or[x1,x1],or[or[x1,x1],or[x1,x1]]]==x1`

PROOF

Note that the input for the rule:

`and[x1_,or[x1_,not[x2_]]]→and[x1,or[x1,x2]]`

contains a subpattern of the form:

`and[x1_,or[x1_,not[x2_]]]`

which can be unified with the input for the rule:

`and[or[x1_,x1_],or[x2_,not[x2_]]]→x1`

where these rules follow from Substitution Lemma 54 and Substitution Lemma 31 respectively.

Substitution Lemma 55

It can be shown that:

`and[or[x1,x1],or[x1,or[x1,or[x1,x1]]]]==x1`

PROOF

We start by taking Critical Pair Lemma 59, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Substitution Lemma 56

It can be shown that:

`and[or[x1,x1],or[x1,or[x1,x1]]]==x1`

PROOF

We start by taking Substitution Lemma 55, and apply the substitution:

`or[x1_,or[x2_,or[x1_,x1_]]]→or[x2,or[x1,x1]]`

which follows from Critical Pair Lemma 57.

Substitution Lemma 57

It can be shown that:

`and[or[x1,x1],or[x1,x1]]==x1`

PROOF

We start by taking Substitution Lemma 56, and apply the substitution:

`or[x1_,or[x1_,x1_]]→or[x1,x1]`

which follows from Substitution Lemma 51.

Critical Pair Lemma 60

The following expressions are equivalent:

`and[or[x1,x1],or[x1,x1]]==and[or[x1,x1],x1]`

PROOF

Note that the input for the rule:

`and[x1_,and[x1_,x1_]]→and[x1,x1]`

contains a subpattern of the form:

`and[x1_,x1_]`

which can be unified with the input for the rule:

`and[or[x1_,x1_],or[x1_,x1_]]→x1`

where these rules follow from Substitution Lemma 47 and Substitution Lemma 57 respectively.

Substitution Lemma 58

It can be shown that:

`x1==and[or[x1,x1],x1]`

Proof

We start by taking Critical Pair Lemma 60, and apply the substitution:

`and[or[x1_,x1_],or[x1_,x1_]]→x1`

which follows from Substitution Lemma 57.

Substitution Lemma 59

It can be shown that:

`x1==and[x1,or[x1,x1]]`

Proof

We start by taking Substitution Lemma 58, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Substitution Lemma 2.

Critical Pair Lemma 61

The following expressions are equivalent:

`or[x1,not[or[not[x1],not[x1]]]]==not[not[x1]]`

Proof

Note that the input for the rule:

`not[and[not[x1_],x2_]]→or[x1,not[x2]]`

contains a subpattern of the form:

`and[not[x1_],x2_]`

which can be unified with the input for the rule:

`and[x1_,or[x1_,x1_]]→x1`

where these rules follow from Critical Pair Lemma 28 and Substitution Lemma 59 respectively.

Substitution Lemma 60

It can be shown that:

`or[x1,and[x1,not[not[x1]]]]==not[not[x1]]`

Proof

We start by taking Critical Pair Lemma 61, and apply the substitution:

`not[or[not[x1_],x2_]]→and[x1,not[x2]]`

which follows from Critical Pair Lemma 34.

which follows from Critical Pair Lemma 54.

Substitution Lemma 61

It can be shown that:

`or[x1,and[x1,x1]]==not[not[x1]]`

Proof

We start by taking Substitution Lemma 60, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 26.

Substitution Lemma 62

It can be shown that:

`or[x1,and[x1,x1]]==x1`

Proof

We start by taking Substitution Lemma 61, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 26.

Critical Pair Lemma 62

The following expressions are equivalent:

`x1==and[x1,or[x1,not[and[x1,x1]]]]`

Proof

Note that the input for the rule:

`and[or[x1_,x2_],or[x1_,not[x2_]]]→x1`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,and[x1_,x1_]]→x1`

where these rules follow from Critical Pair Lemma 45 and Substitution Lemma 62 respectively.

Substitution Lemma 63

It can be shown that:

`x1==and[x1,or[x1,and[x1,x1]]]`

Proof

We start by taking Critical Pair Lemma 62, and apply the substitution:

`and[x1_,or[x1_,not[x2_]]]→and[x1,or[x1,x2]]`

which follows from Substitution Lemma 54.

Substitution Lemma 64

It can be shown that:

`x1==and[x1,x1]`

Proof

PROOF

We start by taking Substitution Lemma 63, and apply the substitution:

`or[x1_,and[x1_,x1_]]→x1`

which follows from Substitution Lemma 62.

Critical Pair Lemma 63

The following expressions are equivalent:

`and[x1,x2]⩵and[x1,and[and[x1,x2],x2]]`

Proof

Note that the input for the rule:

`and[x1_,x1_]→x1`

contains a subpattern of the form:

`and[x1_,x1_]`

which can be unified with the input for the rule:

`and[x1_,and[x2_,x3_]]↔and[x2_,and[x1_,x3_]]`

where these rules follow from Substitution Lemma 64 and Substitution Lemma 18 respectively.

Substitution Lemma 65

It can be shown that:

`and[x1,x2]⩵and[x1,and[x1,and[x2,x2]]]`

Proof

We start by taking Critical Pair Lemma 63, and apply the substitution:

`and[and[x1_,x2_],x3_]→and[x1,and[x2,x3]]`

which follows from Substitution Lemma 17.

Substitution Lemma 66

It can be shown that:

`and[x1,x2]⩵and[x1,and[x1,x2]]`

Proof

We start by taking Substitution Lemma 65, and apply the substitution:

`and[x1_,x1_]→x1`

which follows from Substitution Lemma 64.

Critical Pair Lemma 64

The following expressions are equivalent:

`or[not[not[x1]],x1]⩵not[not[x1]]`

Proof

Note that the input for the rule:

`not[and[x1_,not[x2_]]]→or[not[x1],x2]`

contains a subpattern of the form:

`and[x1_,not[x2_]]`

which can be unified with the input for the rule:

`and[x1_,x1_]→x1`

where these rules follow from Critical Pair Lemma 29 and Substitution Lemma 64 respectively.

Substitution Lemma 67

It can be shown that:

`or[x1,x1]==not[not[x1]]`

Proof

We start by taking Critical Pair Lemma 64, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 26.

Substitution Lemma 68

It can be shown that:

`or[x1,x1]==x1`

Proof

We start by taking Substitution Lemma 67, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 26.

Critical Pair Lemma 65

The following expressions are equivalent:

`or[x1,x2]==or[x1,or[or[x1,x2],x2]]`

Proof

Note that the input for the rule:

`or[x1_,x1_]→x1`

contains a subpattern of the form:

`or[x1_,x1_]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,x3_]]↔or[x2_,or[x1_,x3_]]`

where these rules follow from Substitution Lemma 68 and Substitution Lemma 1 respectively.

Substitution Lemma 69

It can be shown that:

`or[x1,x2]==or[x1,or[x1,or[x2,x2]]]`

Proof

We start by taking Critical Pair Lemma 65, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Substitution Lemma 70

It can be shown that:

`or[x1,x2]==or[x1,or[x1,x2]]`

PROOF

We start by taking Substitution Lemma 69, and apply the substitution:

`or[x1_,x1_]→x1`

which follows from Substitution Lemma 68.

Critical Pair Lemma 66

The following expressions are equivalent:

`and[or[x1,x2],or[x1,not[x2]]]==and[or[x1,x2],x1]`

PROOF

Note that the input for the rule:

`and[x1_,and[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[or[x1_,x2_],or[x1_,not[x2_]]]→x1`

where these rules follow from Substitution Lemma 66 and Critical Pair Lemma 45 respectively.

Substitution Lemma 71

It can be shown that:

`x1==and[or[x1,x2],x1]`

PROOF

We start by taking Critical Pair Lemma 66, and apply the substitution:

`and[or[x1_,x2_],or[x1_,not[x2_]]]→x1`

which follows from Critical Pair Lemma 45.

Critical Pair Lemma 67

The following expressions are equivalent:

`and[or[x1,x2],or[not[x1],x2]]==and[or[x1,x2],x2]`

PROOF

Note that the input for the rule:

`and[x1_,and[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[or[x1_,x2_],or[not[x1_],x2_]]→x2`

where these rules follow from Substitution Lemma 66 and Critical Pair Lemma 44 respectively.

Substitution Lemma 72

It can be shown that:

`x1==and[or[x2,x1],x1]`

PROOF

We start by taking Critical Pair Lemma 67, and apply the substitution:

`and[or[x1_,x2_],or[not[x1_],x2_]]→x2`

which follows from Critical Pair Lemma 44.

Substitution Lemma 73

It can be shown that:

`x1==and[x1,or[x1,x2]]`

PROOF

We start by taking Substitution Lemma 71, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Substitution Lemma 2.

Substitution Lemma 74

It can be shown that:

`x1==and[x1,or[x2,x1]]`

PROOF

We start by taking Substitution Lemma 72, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Substitution Lemma 2.

Critical Pair Lemma 68

The following expressions are equivalent:

`or[and[x1,x2],and[x2,not[x1]]]==or[and[x1,x2],x2]`

PROOF

Note that the input for the rule:

`or[x1_,or[x1_,x2_]]→or[x1,x2]`

contains a subpattern of the form:

`or[x1_,or[x1_,x2_]]`

which can be unified with the input for the rule:

`or[and[x1_,x2_],or[x3_,and[x2_,not[x1_]]]]→or[x3,x2]`

where these rules follow from Substitution Lemma 70 and Substitution Lemma 9 respectively.

Substitution Lemma 75

It can be shown that:

`x1==or[and[x2,x1],x1]`

PROOF

We start by taking Critical Pair Lemma 68, and apply the substitution:

`or[and[x1_,x2_],and[x2_,not[x1_]]]→x2`

which follows from Critical Pair Lemma 7.

Critical Pair Lemma 69

The following expressions are equivalent:

`or[and[x1,x2],and[x1,not[x2]]]==or[and[x1,x2],x1]`

Proof

Note that the input for the rule:

`or[x1_,or[x1_,x2_]]→or[x1,x2]`

contains a subpattern of the form:

`or[x1_,or[x1_,x2_]]`

which can be unified with the input for the rule:

`or[and[x1_,x2_],or[x3_,and[x1_,not[x2_]]]]→or[x3,x1]`

where these rules follow from Substitution Lemma 70 and Substitution Lemma 8 respectively.

Substitution Lemma 76

It can be shown that:

`x1==or[and[x1,x2],x1]`

Proof

We start by taking Critical Pair Lemma 69, and apply the substitution:

`or[and[x1_,x2_],and[x1_,not[x2_]]]→x1`

which follows from Substitution Lemma 5.

Critical Pair Lemma 70

The following expressions are equivalent:

`or[and[x1,x2],or[x3,and[x2,not[x1]]]]==or[and[x1,x2],or[x3,x2]]`

Proof

Note that the input for the rule:

`or[x1_,or[x1_,x2_]]→or[x1,x2]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[and[x1_,x2_],or[x3_,and[x2_,not[x1_]]]]→or[x3,x2]`

where these rules follow from Substitution Lemma 70 and Substitution Lemma 9 respectively.

Substitution Lemma 77

It can be shown that:

`or[x1,x2]==or[and[x3,x2],or[x1,x2]]`

Proof

We start by taking Critical Pair Lemma 70, and apply the substitution:

`or[and[x1_,x2_],or[x3_,and[x2_,not[x1_]]]]→or[x3,x2]`

which follows from Substitution Lemma 9.

Critical Pair Lemma 71

Critical Pair Lemma 71

The following expressions are equivalent:

`or[and[x1,x2],or[and[not[x2],x1],x3]]==or[and[x1,x2],or[x1,x3]]`

Proof

Note that the input for the rule:

`or[x1_,or[x1_,x2_]]→or[x1,x2]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[and[x1_,x2_],or[and[not[x2_],x1_],x3_]]→or[x1,x3]`

where these rules follow from Substitution Lemma 70 and Critical Pair Lemma 11 respectively.

Substitution Lemma 78

It can be shown that:

`or[x1,x2]==or[and[x1,x3],or[x1,x2]]`

Proof

We start by taking Critical Pair Lemma 71, and apply the substitution:

`or[and[x1_,x2_],or[and[not[x2_],x1_],x3_]]→or[x1,x3]`

which follows from Critical Pair Lemma 11.

Critical Pair Lemma 72

The following expressions are equivalent:

`or[not[x1],and[x2,x1]]==or[not[x1],or[x2,not[or[x1,x2]]]]`

Proof

Note that the input for the rule:

`or[x1_,or[x1_,x2_]]→or[x1,x2]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,not[or[x2_,x1_]]]↔or[not[x2_],and[x1_,x2_]]`

where these rules follow from Substitution Lemma 70 and Substitution Lemma 33 respectively.

Substitution Lemma 79

It can be shown that:

`or[not[x1],and[x2,x1]]==or[x2,not[and[x1,or[x1,x2]]]]`

Proof

We start by taking Critical Pair Lemma 72, and apply the substitution:

`or[not[x1_],or[x2_,not[x3_]]]→or[x2,not[and[x1,x3]]]`

which follows from Critical Pair Lemma 32.

Substitution Lemma 80

It can be shown that:

`or[not[x1],and[x2,x1]]==or[x2,not[x1]]`

Proof

We start by taking Substitution Lemma 79, and apply the substitution:

`and[x1_,or[x1_,x2_]]→x1`

which follows from Substitution Lemma 73.

Substitution Lemma 81

It can be shown that:

`x1==or[x1,and[x2,x1]]`

Proof

We start by taking Substitution Lemma 75, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Substitution Lemma 82

It can be shown that:

`x1==or[x1,and[x1,x2]]`

Proof

We start by taking Substitution Lemma 76, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Critical Pair Lemma 73

The following expressions are equivalent:

`or[x1,not[not[x2]]]==or[x2,and[x1,not[x2]]]`

Proof

Note that the input for the rule:

`or[not[x1_],and[x2_,x1_]]→or[x2,not[x1]]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Substitution Lemma 80 and Critical Pair Lemma 26 respectively.

Substitution Lemma 83

It can be shown that:

`or[x1,x2]==or[x2,and[x1,not[x2]]]`

Proof

We start by taking Critical Pair Lemma 73, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 26.

Critical Pair Lemma 74

The following expressions are equivalent:

`or[or[x1,x2],not[or[x1,not[x2]]]]==or[not[or[x1,not[x2]]],x1]`

Proof

Note that the input for the rule:

`or[not[x1_],and[x2_,x1_]]→or[x2,not[x1]]`

contains a subpattern of the form:

`and[x2_,x1_]`

which can be unified with the input for the rule:

`and[or[x1_,x2_],or[x1_,not[x2_]]]→x1`

where these rules follow from Substitution Lemma 80 and Critical Pair Lemma 45 respectively.

Substitution Lemma 84

It can be shown that:

`or[x1,or[x2,not[or[x1,not[x2]]]]]==or[not[or[x1,not[x2]]],x1]`

Proof

We start by taking Critical Pair Lemma 74, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Substitution Lemma 85

It can be shown that:

`or[x1,or[x2,and[not[x1],x2]]]==or[not[or[x1,not[x2]]],x1]`

Proof

We start by taking Substitution Lemma 84, and apply the substitution:

`not[or[x1_,not[x2_]]]→and[not[x1],x2]`

which follows from Critical Pair Lemma 33.

Substitution Lemma 86

It can be shown that:

`or[x1,x2]==or[not[or[x1,not[x2]]],x1]`

Proof

We start by taking Substitution Lemma 85, and apply the substitution:

`or[x1_,and[x2_,x1_]]→x1`

which follows from Substitution Lemma 81.

Substitution Lemma 87

It can be shown that:

`or[x1,x2]==or[and[not[x1],x2],x1]`

PROOF

We start by taking Substitution Lemma 86, and apply the substitution:

`not[or[x1_,not[x2_]]]→and[not[x1],x2]`

which follows from Critical Pair Lemma 33.

Critical Pair Lemma 75

The following expressions are equivalent:

`or[and[x1,x2],x3]==or[x3,and[x1,and[x2,not[x3]]]]`

PROOF

Note that the input for the rule:

`or[x1_,and[x2_,not[x1_]]]→or[x2,x1]`

contains a subpattern of the form:

`and[x2_,not[x1_]]`

which can be unified with the input for the rule:

`and[and[x1_,x2_],x3_]→and[x1,and[x2,x3]]`

where these rules follow from Substitution Lemma 83 and Substitution Lemma 17 respectively.

Critical Pair Lemma 76

The following expressions are equivalent:

`and[x1,not[x2]]==and[or[x1,x2],or[not[x2],and[x1,not[x2]]]]`

PROOF

Note that the input for the rule:

`and[or[x1_,x2_],or[not[x1_],x2_]]→x2`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,and[x2_,not[x1_]]]→or[x2,x1]`

where these rules follow from Critical Pair Lemma 44 and Substitution Lemma 83 respectively.

Substitution Lemma 88

It can be shown that:

`and[x1,not[x2]]==and[or[x1,x2],not[x2]]`

PROOF

We start by taking Critical Pair Lemma 76, and apply the substitution:

`or[x1_,and[x2_,x1_]]→x1`

which follows from Substitution Lemma 81.

Substitution Lemma 89

It can be shown that:

`or[x1,x2]==or[x1,and[not[x1],x2]]`

PROOF

We start by taking Substitution Lemma 87, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Critical Pair Lemma 77

The following expressions are equivalent:

`and[not[x1],x2]==and[or[x1,x2],or[not[x1],and[not[x1],x2]]]`

PROOF

Note that the input for the rule:

`and[or[x1_,x2_],or[not[x1_],x2_]]→x2`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,and[not[x1_],x2_]]→or[x1,x2]`

where these rules follow from Critical Pair Lemma 44 and Substitution Lemma 89 respectively.

Substitution Lemma 90

It can be shown that:

`and[not[x1],x2]==and[or[x1,x2],not[x1]]`

PROOF

We start by taking Critical Pair Lemma 77, and apply the substitution:

`or[x1_,and[x1_,x2_]]→x1`

which follows from Substitution Lemma 82.

Critical Pair Lemma 78

The following expressions are equivalent:

`and[not[not[x1]],x2]==and[or[x1,and[not[not[x1]],x2]],or[not[x1],x2]]`

PROOF

Note that the input for the rule:

`and[or[x1_,x2_],or[not[x1_],x2_]]→x2`

contains a subpattern of the form:

`or[not[x1_],x2_]`

which can be unified with the input for the rule:

`or[x1_,and[not[x1_],x2_]]→or[x1,x2]`

where these rules follow from Critical Pair Lemma 44 and Substitution Lemma 89 respectively.

Substitution Lemma 91

It can be shown that:

`and[x1,x2]==and[or[x1,and[not[not[x1]],x2]],or[not[x1],x2]]`

PROOF

We start by taking Critical Pair Lemma 78, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 26.

Substitution Lemma 92

It can be shown that:

`and[x1,x2]==and[or[x1,and[x1,x2]],or[not[x1],x2]]`

PROOF

We start by taking Substitution Lemma 91, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 26.

Substitution Lemma 93

It can be shown that:

`and[x1,x2]==and[x1,or[not[x1],x2]]`

PROOF

We start by taking Substitution Lemma 92, and apply the substitution:

`or[x1_,and[x1_,x2_]]→x1`

which follows from Substitution Lemma 82.

Substitution Lemma 94

It can be shown that:

`and[x1,not[x2]]==and[not[x2],or[x1,x2]]`

PROOF

We start by taking Substitution Lemma 88, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Substitution Lemma 2.

Substitution Lemma 95

It can be shown that:

`and[not[x1],x2]==and[not[x1],or[x1,x2]]`

PROOF

We start by taking Substitution Lemma 90, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Substitution Lemma 2.

Substitution Lemma 96

It can be shown that:

`or[x1,x2]==or[x2,or[x1,and[x3,x2]]]`

Proof

We start by taking Substitution Lemma 77, and apply the substitution:

`or[x1_,or[x2_,x3_]]→or[x3,or[x2,x1]]`

which follows from Critical Pair Lemma 17.

Critical Pair Lemma 79

The following expressions are equivalent:

`and[not[x1],or[x2,and[x3,x1]]]==and[not[x1],or[x2,x1]]`

Proof

Note that the input for the rule:

`and[not[x1_],or[x1_,x2_]]→and[not[x1],x2]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,and[x3_,x1_]]]→or[x2,x1]`

where these rules follow from Substitution Lemma 95 and Substitution Lemma 96 respectively.

Substitution Lemma 97

It can be shown that:

`and[not[x1],or[x2,and[x3,x1]]]==and[x2,not[x1]]`

Proof

We start by taking Critical Pair Lemma 79, and apply the substitution:

`and[not[x1_],or[x2_,x1_]]→and[x2,not[x1]]`

which follows from Substitution Lemma 94.

Substitution Lemma 98

It can be shown that:

`or[x1,x2]==or[x1,or[and[x1,x3],x2]]`

Proof

We start by taking Substitution Lemma 78, and apply the substitution:

`or[x1_,or[x2_,x3_]]→or[x2,or[x1,x3]]`

which follows from Substitution Lemma 1.

Critical Pair Lemma 80

The following expressions are equivalent:

`and[x1,or[and[not[x1],x2],x3]]==and[x1,or[not[x1],x3]]`

Proof

Note that the input for the rule:

`and[x1_,or[not[x1_],x2_]]→and[x1,x2]`

contains a subpattern of the form:

contains a subpattern of the form:

`or[not[x1_],x2_]`

which can be unified with the input for the rule:

`or[x1_,or[and[x1_,x2_],x3_]]→or[x1,x3]`

where these rules follow from Substitution Lemma 93 and Substitution Lemma 98 respectively.

Substitution Lemma 99

It can be shown that:

`and[x1,or[and[not[x1],x2],x3]]==and[x1,x3]`

Proof

We start by taking Critical Pair Lemma 80, and apply the substitution:

`and[x1_,or[not[x1_],x2_]]→and[x1,x2]`

which follows from Substitution Lemma 93.

Critical Pair Lemma 81

The following expressions are equivalent:

`and[x1,or[x2,and[not[not[x1]],x3]]]==and[x1,or[x2,x3]]`

Proof

Note that the input for the rule:

`and[x1_,or[and[not[x1_],x2_],x3_]]→and[x1,x3]`

contains a subpattern of the form:

`or[and[not[x1_],x2_],x3_]`

which can be unified with the input for the rule:

`or[and[x1_,x2_],or[x3_,and[not[x1_],x2_]]]→or[x3,x2]`

where these rules follow from Substitution Lemma 99 and Substitution Lemma 7 respectively.

Substitution Lemma 100

It can be shown that:

`and[x1,or[x2,and[x1,x3]]]==and[x1,or[x2,x3]]`

Proof

We start by taking Critical Pair Lemma 81, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 26.

Critical Pair Lemma 82

The following expressions are equivalent:

`and[x1,not[and[x2,not[x1]]]]==and[not[and[x2,not[x1]]],or[and[x3,x2],x1]]`

Proof

Note that the input for the rule:

`and[not[x1_],or[x2_,and[x3_,x1_]]]→and[x2,not[x1]]`

contains a subpattern of the form:

`or[x2_,and[x3_,x1_]]`

which can be unified with the input for the rule:

`or[x1_,and[x2_,and[x3_,not[x1_]]]]→or[and[x2,x3],x1]`

where these rules follow from Substitution Lemma 97 and Critical Pair Lemma 75 respectively.

Substitution Lemma 101

It can be shown that:

`and[x1,or[not[x2],x1]]==and[not[and[x2,not[x1]]],or[and[x3,x2],x1]]`

Proof

We start by taking Critical Pair Lemma 82, and apply the substitution:

`not[and[x1_,not[x2_]]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 29.

Substitution Lemma 102

It can be shown that:

`x1==and[not[and[x2,not[x1]]],or[and[x3,x2],x1]]`

Proof

We start by taking Substitution Lemma 101, and apply the substitution:

`and[x1_,or[x2_,x1_]]→x1`

which follows from Substitution Lemma 74.

Substitution Lemma 103

It can be shown that:

`x1==and[or[not[x2],x1],or[and[x3,x2],x1]]`

Proof

We start by taking Substitution Lemma 102, and apply the substitution:

`not[and[x1_,not[x2_]]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 29.

Critical Pair Lemma 83

The following expressions are equivalent:

`x1==and[or[x2,x1],or[and[x3,not[x2]],x1]]`

Proof

Note that the input for the rule:

`and[or[not[x1_],x2_],or[and[x3_,x1_],x2_]]→x2`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Substitution Lemma 103 and Critical Pair Lemma 26 respectively.

Critical Pair Lemma 84

The following expressions are equivalent:

`or[and[not[not[x1]],x2],x3]==and[or[x1,or[and[not[not[x1]],x2],x3]],or[x2,x3]]`

Proof

Note that the input for the rule:

`and[or[x1_,x2_],or[and[x3_,not[x1_]],x2_]]→x2`

contains a subpattern of the form:

`or[and[x3_,not[x1_]],x2_]`

which can be unified with the input for the rule:

`or[and[x1_,x2_],or[and[not[x2_],x1_],x3_]]→or[x1,x3]`

where these rules follow from Critical Pair Lemma 83 and Critical Pair Lemma 11 respectively.

Substitution Lemma 104

It can be shown that:

`or[and[x1,x2],x3]==and[or[x1,or[and[not[not[x1]],x2],x3]],or[x2,x3]]`

Proof

We start by taking Critical Pair Lemma 84, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 26.

Substitution Lemma 105

It can be shown that:

`or[and[x1,x2],x3]==and[or[x1,or[and[x1,x2],x3]],or[x2,x3]]`

Proof

We start by taking Substitution Lemma 104, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Critical Pair Lemma 26.

Substitution Lemma 106

It can be shown that:

`or[and[x1,x2],x3]==and[or[x1,x3],or[x2,x3]]`

Proof

We start by taking Substitution Lemma 105, and apply the substitution:

`or[x1_,or[and[x1_,x2_],x3_]]→or[x1,x3]`

which follows from Substitution Lemma 98.

Critical Pair Lemma 85

The following expressions are equivalent:

`or[and[x1,x2],and[x1,x3]]==and[x1,or[x2,and[x1,x3]]]`

Proof

Note that the input for the rule:

Note that the input for the rule:

`and[or[x1_,x2_],or[x3_,x2_]]→or[and[x1,x3],x2]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,and[x1_,x2_]]→x1`

where these rules follow from Substitution Lemma 106 and Substitution Lemma 82 respectively.

Substitution Lemma 107

It can be shown that:

`or[and[x1,x2],and[x1,x3]]==and[x1,or[x2,x3]]`

Proof

We start by taking Critical Pair Lemma 85, and apply the substitution:

`and[x1_,or[x2_,and[x1_,x3_]]]→and[x1,or[x2,x3]]`

which follows from Substitution Lemma 100.

Conclusion 1

We obtain the conclusion:

`True`

Proof

Take Hypothesis 1, and apply the substitution:

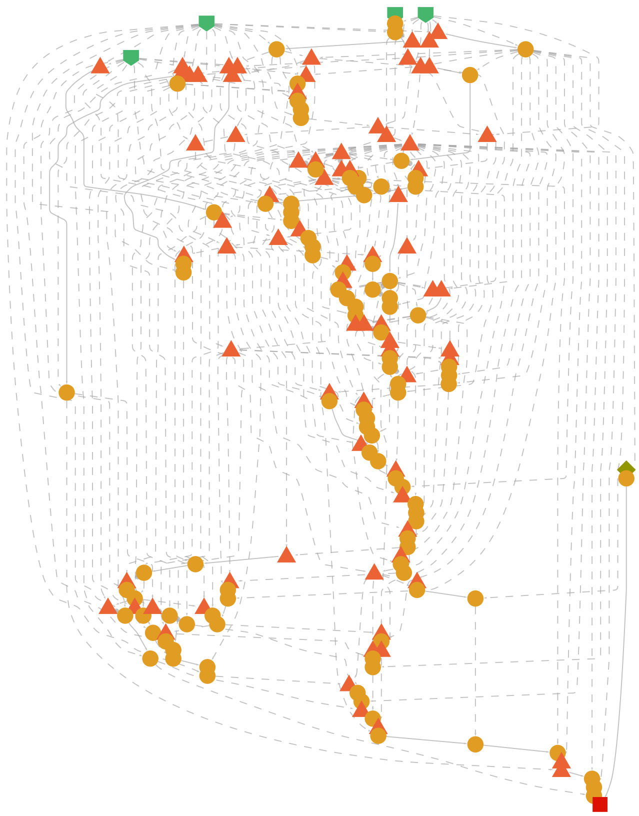
`or[and[x1_,x2_],and[x1_,x3_]]→and[x1,or[x2,x3]]`

which follows from Substitution Lemma 107.

large output | show less | show more | show all | set size limit...

In[∘]:= `proofAxB6fromHunt["ProofGraph"]`

Out[ ]=



In[ ]:= Clear[proofAxB6fromHunt]

In[∘]:=
```
proofAxB7fromHunt["ProofNotebook"]
```

Out[∘]=

Axiom 1

We are given that:

`or[x1,not[x1]]==1`

Hypothesis 1

We would like to show that:

`or[a,not[a]]==1`

Conclusion 1

We obtain the conclusion:

`True`

Proof

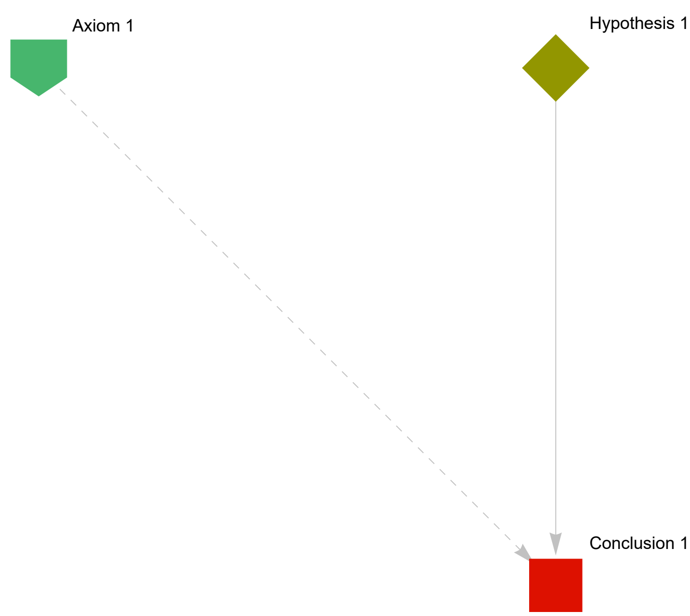
Take Hypothesis 1, and apply the substitution:

`or[x1_,not[x1_]]→1`

which follows from Axiom 1.

In[∘]:=
```
proofAxB7fromHunt["ProofGraph"]
```

Out[∘]=



In[∘]:=
```
Clear[proofAxB7fromHunt]
```

In[∘]:= `proofAxB8fromHunt["ProofNotebook"]`

Out[∘]=

Axiom 1

We are given that:

`0==and[x1,not[x1]]`

Hypothesis 1

We would like to show that:

`0==and[a,not[a]]`

Conclusion 1

We obtain the conclusion:

`True`

Proof

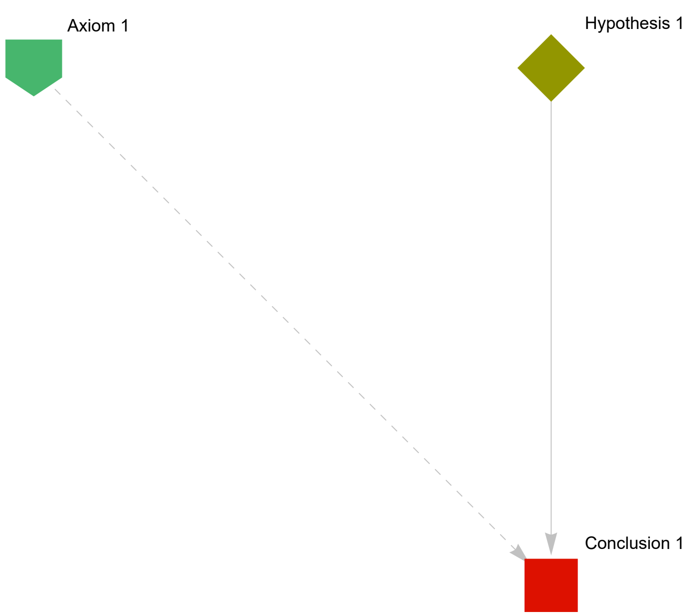
Take Hypothesis 1, and apply the substitution:

`and[x1_,not[x1_]]→0`

which follows from Axiom 1.

In[∘]:= `proofAxB8fromHunt["ProofGraph"]`

Out[∘]=



In[∘]:= `Clear[proofAxB8fromHunt]`

Appendix 5. Derivation of AxH3 from Robbins logic + winker1.

In[∘]:= proofAxH3fromRobbinsw1["ProofNotebook"]

Axiom 1

We are given that:

`x1==not[or[not[or[x1,x2]],not[or[x1,not[x2]]]]]`

Axiom 2

We are given that:

`or[x1,x2]==or[x2,x1]`

Axiom 3

We are given that:

`or[x1,or[x2,x3]]==or[or[x1,x2],x3]`

Axiom 4

We are given that:

`or[c,d]==d`

Hypothesis 1

We would like to show that:

`or[not[or[not[a],b]],not[or[not[a],not[b]]]]==a`

Critical Pair Lemma 1

The following expressions are equivalent:

`not[or[x1,x2]]==not[or[x1,not[or[not[or[x1,x2]],not[not[or[x1,not[x2]]]]]]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[x1_,not[x2_]]]]]→x1`

contains a subpattern of the form:

`not[or[x1_,x2_]]`

which can be unified with the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[x1_,not[x2_]]]]]→x1`

where these rules follow from Axiom 1 and Axiom 1 respectively.

Critical Pair Lemma 2

The following expressions are equivalent:

`not[or[x1,x2]]==not[or[not[or[not[or[x1,x2]],or[x1,not[x2]]]],x1]]`

Proof

Note that the input for the rule:

Note that the input for the rule:

```
not[or[not[or[x1_,x2_]],not[or[x1_,not[x2_]]]]]→x1
```

contains a subpattern of the form:

```
not[or[x1_,not[x2_]]]
```

which can be unified with the input for the rule:

```
not[or[not[or[x1_,x2_]],not[or[x1_,not[x2_]]]]]→x1
```

where these rules follow from Axiom 1 and Axiom 1 respectively.

Critical Pair Lemma 3

The following expressions are equivalent:

```
x1⩵not[or[not[or[x1,or[not[or[x2,x3]],not[or[x2,not[x3]]]]]],not[or[x1,x2]]]]
```

Proof

Note that the input for the rule:

```
not[or[not[or[x1_,x2_]],not[or[x1_,not[x2_]]]]]→x1
```

contains a subpattern of the form:

```
not[x2_]
```

which can be unified with the input for the rule:

```
not[or[not[or[x1_,x2_]],not[or[x1_,not[x2_]]]]]→x1
```

where these rules follow from Axiom 1 and Axiom 1 respectively.

Critical Pair Lemma 4

The following expressions are equivalent:

```
x1⩵not[or[not[or[x2,x1]],not[or[x1,not[x2]]]]]
```

Proof

Note that the input for the rule:

```
not[or[not[or[x1_,x2_]],not[or[x1_,not[x2_]]]]]→x1
```

contains a subpattern of the form:

```
or[x1_,x2_]
```

which can be unified with the input for the rule:

```
or[x1_,x2_]↔or[x2_,x1_]
```

where these rules follow from Axiom 1 and Axiom 2 respectively.

Critical Pair Lemma 5

The following expressions are equivalent:

```
or[x1,or[x2,x3]]⩵or[x3,or[x1,x2]]
```

Proof

Note that the input for the rule:

```
or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]
```

contains a subpattern of the form:

```
or[or[x1_,x2_],x3_]
```

which can be unified with the input for the rule:

```
or[x1_,x2_]↔or[x2_,x1_]
```

where these rules follow from Axiom 3 and Axiom 2 respectively.

Critical Pair Lemma 6

The following expressions are equivalent:

`or[x1,or[x2,x3]]==or[or[x2,x1],x3]`

Proof

Note that the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Axiom 3 and Axiom 2 respectively.

Substitution Lemma 1

It can be shown that:

`or[x1,or[x2,x3]]==or[x2,or[x1,x3]]`

Proof

We start by taking Critical Pair Lemma 6, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Critical Pair Lemma 7

The following expressions are equivalent:

`or[x1,x2]==not[or[not[or[x1,or[x2,x3]]],not[or[or[x1,x2],not[x3]]]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[x1_,not[x2_]]]]]→x1`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

where these rules follow from Axiom 1 and Axiom 3 respectively.

Substitution Lemma 2

It can be shown that:

`or[x1,x2]==not[or[not[or[x1,or[x2,x3]]],not[or[x1,or[x2,not[x3]]]]]]`

Proof

We start by taking Critical Pair Lemma 7, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Critical Pair Lemma 8

Critical Pair Lemma 8

The following expressions are equivalent:

`or[c,or[d,x1]]==or[d,x1]`

Proof

Note that the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[c,d]→d`

where these rules follow from Axiom 3 and Axiom 4 respectively.

Critical Pair Lemma 9

The following expressions are equivalent:

`c==not[or[not[d],not[or[c,not[d]]]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[x1_,not[x2_]]]]]→x1`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[c,d]→d`

where these rules follow from Axiom 1 and Axiom 4 respectively.

Critical Pair Lemma 10

The following expressions are equivalent:

`or[d,x1]==or[c,or[x1,d]]`

Proof

Note that the input for the rule:

`or[c,or[d,x1_]]→or[d,x1]`

contains a subpattern of the form:

`or[d,x1_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Critical Pair Lemma 8 and Axiom 2 respectively.

Critical Pair Lemma 11

The following expressions are equivalent:

`c==not[or[not[or[d,x1]],not[or[c,not[or[d,x1]]]]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[x1_,not[x2_]]]]]→x1`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[c,or[d,x1_]]→or[d,x1]`

where these rules follow from Axiom 1 and Critical Pair Lemma 8 respectively.

Critical Pair Lemma 12

The following expressions are equivalent:

`or[d,or[x1,x2]]==or[c,or[x1,or[x2,d]]]`

Proof

Note that the input for the rule:

`or[c,or[x1_,d]]→or[d,x1]`

contains a subpattern of the form:

`or[x1_,d]`

which can be unified with the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

where these rules follow from Critical Pair Lemma 10 and Axiom 3 respectively.

Critical Pair Lemma 13

The following expressions are equivalent:

`or[c,or[or[x1,d],x2]]==or[or[d,x1],x2]`

Proof

Note that the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[c,or[x1_,d]]→or[d,x1]`

where these rules follow from Axiom 3 and Critical Pair Lemma 10 respectively.

Substitution Lemma 3

It can be shown that:

`or[c,or[x1,or[d,x2]]]==or[or[d,x1],x2]`

Proof

We start by taking Critical Pair Lemma 13, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Substitution Lemma 4

It can be shown that:

`or[c,or[x1,or[d,x2]]]==or[d,or[x1,x2]]`

PROOF

We start by taking Substitution Lemma 3, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Critical Pair Lemma 14

The following expressions are equivalent:

`x1==not[or[not[or[x1,or[not[d],not[or[c,not[d]]]]]],not[or[x1,c]]]]`

PROOF

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[x1_,not[x2_]]]]]→x1`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[or[not[d],not[or[c,not[d]]]]]→c`

where these rules follow from Axiom 1 and Critical Pair Lemma 9 respectively.

Critical Pair Lemma 15

The following expressions are equivalent:

`or[d,or[or[x1,x2],x3]]==or[c,or[x1,or[x2,or[x3,d]]]]`

PROOF

Note that the input for the rule:

`or[c,or[x1_,or[x2_,d]]]→or[d,or[x1,x2]]`

contains a subpattern of the form:

`or[x1_,or[x2_,d]]`

which can be unified with the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

where these rules follow from Critical Pair Lemma 12 and Axiom 3 respectively.

Substitution Lemma 5

It can be shown that:

`or[d,or[x1,or[x2,x3]]]==or[c,or[x1,or[x2,or[x3,d]]]]`

PROOF

We start by taking Critical Pair Lemma 15, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Critical Pair Lemma 16

The following expressions are equivalent:

`or[d,or[c,x1]]==or[c,or[d,x1]]`

PROOF

Note that the input for the rule:

Note that the input for the rule:

`or[c,or[x1_,or[x2_,d]]]→or[d,or[x1,x2]]`

contains a subpattern of the form:

`or[x1_,or[x2_,d]]`

which can be unified with the input for the rule:

`or[c,or[x1_,d]]→or[d,x1]`

where these rules follow from Critical Pair Lemma 12 and Critical Pair Lemma 10 respectively.

Substitution Lemma 6

It can be shown that:

`or[d,or[c,x1]]==or[d,x1]`

Proof

We start by taking Critical Pair Lemma 16, and apply the substitution:

`or[c,or[d,x1_]]→or[d,x1]`

which follows from Critical Pair Lemma 8.

Critical Pair Lemma 17

The following expressions are equivalent:

`or[d,or[x1,c]]==or[c,or[x1,d]]`

Proof

Note that the input for the rule:

`or[c,or[x1_,or[x2_,d]]]→or[d,or[x1,x2]]`

contains a subpattern of the form:

`or[x2_,d]`

which can be unified with the input for the rule:

`or[c,d]→d`

where these rules follow from Critical Pair Lemma 12 and Axiom 4 respectively.

Substitution Lemma 7

It can be shown that:

`or[d,or[x1,c]]==or[d,x1]`

Proof

We start by taking Critical Pair Lemma 17, and apply the substitution:

`or[c,or[x1_,d]]→or[d,x1]`

which follows from Critical Pair Lemma 10.

Critical Pair Lemma 18

The following expressions are equivalent:

`d==not[or[not[or[d,x1]],not[or[d,not[or[c,x1]]]]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[x1_,not[x2_]]]]]→x1`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[d,or[c,x1_]]→or[d,x1]`

where these rules follow from Axiom 1 and Substitution Lemma 6 respectively.

Critical Pair Lemma 19

The following expressions are equivalent:

`or[d,or[or[x1,c],x2]]==or[or[d,x1],x2]`

Proof

Note that the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[d,or[x1_,c]]→or[d,x1]`

where these rules follow from Axiom 3 and Substitution Lemma 7 respectively.

Substitution Lemma 8

It can be shown that:

`or[d,or[x1,or[c,x2]]]==or[or[d,x1],x2]`

Proof

We start by taking Critical Pair Lemma 19, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Substitution Lemma 9

It can be shown that:

`or[d,or[x1,or[c,x2]]]==or[d,or[x1,x2]]`

Proof

We start by taking Substitution Lemma 8, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Critical Pair Lemma 20

The following expressions are equivalent:

`x1==not[or[not[or[x2,or[x3,x1]]],not[or[x1,not[or[x2,x3]]]]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[x2_,not[x1_]]]]]→x2`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

where these rules follow from Critical Pair Lemma 4 and Axiom 3 respectively.

Critical Pair Lemma 21

The following expressions are equivalent:

`d==not[or[not[d],not[or[d,not[c]]]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[x2_,not[x1_]]]]]→x2`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[c,d]→d`

where these rules follow from Critical Pair Lemma 4 and Axiom 4 respectively.

Critical Pair Lemma 22

The following expressions are equivalent:

`or[x1,d]==not[or[not[or[d,x1]],not[or[or[x1,d],not[c]]]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[x2_,not[x1_]]]]]→x2`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[c,or[x1_,d]]→or[d,x1]`

where these rules follow from Critical Pair Lemma 4 and Critical Pair Lemma 10 respectively.

Substitution Lemma 10

It can be shown that:

`or[x1,d]==not[or[not[or[d,x1]],not[or[x1,or[d,not[c]]]]]]`

Proof

We start by taking Critical Pair Lemma 22, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Critical Pair Lemma 23

The following expressions are equivalent:

`x1==not[or[not[or[x2,x1]],not[or[not[x2],x1]]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[x2_,not[x1_]]]]]→x2`

`not[or[not[or[x1_,x2_]],not[or[x2_,not[x1_]]]]]→x2`

contains a subpattern of the form:

`or[x2_,not[x1_]]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Critical Pair Lemma 4 and Axiom 2 respectively.

Critical Pair Lemma 24

The following expressions are equivalent:

`x1==not[or[not[or[x1,or[not[or[x2,x3]],not[or[x3,not[x2]]]]]],not[or[x1,x3]]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[x1_,not[x2_]]]]]→x1`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[x2_,not[x1_]]]]]→x2`

where these rules follow from Axiom 1 and Critical Pair Lemma 4 respectively.

Critical Pair Lemma 25

The following expressions are equivalent:

`not[d]==not[or[not[or[or[d,not[c]],not[d]]],d]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[x2_,not[x1_]]]]]→x2`

contains a subpattern of the form:

`not[or[x2_,not[x1_]]]`

which can be unified with the input for the rule:

`not[or[not[d],not[or[d,not[c]]]]]→d`

where these rules follow from Critical Pair Lemma 4 and Critical Pair Lemma 21 respectively.

Substitution Lemma 11

It can be shown that:

`not[d]==not[or[not[or[d,or[not[c],not[d]]]],d]]`

Proof

We start by taking Critical Pair Lemma 25, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Critical Pair Lemma 26

The following expressions are equivalent:

`x1==not[or[not[or[x2,or[x3,x1]]],not[or[not[or[x2,x3]],x1]]]]`

PROOF

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[not[x1_],x2_]]]]→x2`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

where these rules follow from Critical Pair Lemma 23 and Axiom 3 respectively.

Critical Pair Lemma 27

The following expressions are equivalent:

`or[x1,d]⩵not[or[not[or[d,x1]],not[or[not[c],or[x1,d]]]]]`

PROOF

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[not[x1_],x2_]]]]→x2`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[c,or[x1_,d]]→or[d,x1]`

where these rules follow from Critical Pair Lemma 23 and Critical Pair Lemma 10 respectively.

Critical Pair Lemma 28

The following expressions are equivalent:

`not[or[x1,not[x2]]]⩵not[or[not[or[or[x2,x1],not[or[x1,not[x2]]]]],x1]]`

PROOF

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[not[x1_],x2_]]]]→x2`

contains a subpattern of the form:

`not[or[not[x1_],x2_]]`

which can be unified with the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[x2_,not[x1_]]]]]→x2`

where these rules follow from Critical Pair Lemma 23 and Critical Pair Lemma 4 respectively.

Substitution Lemma 12

It can be shown that:

`not[or[x1,not[x2]]]⩵not[or[not[or[x2,or[x1,not[or[x1,not[x2]]]]]],x1]]`

PROOF

We start by taking Critical Pair Lemma 28, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Critical Pair Lemma 29

The following expressions are equivalent:

`not[or[c,not[d]]]==not[or[not[or[d,not[or[c,not[d]]]]],c]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[not[x1_],x2_]]]]→x2`

contains a subpattern of the form:

`not[or[not[x1_],x2_]]`

which can be unified with the input for the rule:

`not[or[not[d],not[or[c,not[d]]]]]→c`

where these rules follow from Critical Pair Lemma 23 and Critical Pair Lemma 9 respectively.

Critical Pair Lemma 30

The following expressions are equivalent:

`not[or[not[x1],x2]]==not[or[not[or[or[x1,x2],not[or[not[x1],x2]]]],x2]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[not[x1_],x2_]]]]→x2`

contains a subpattern of the form:

`not[or[not[x1_],x2_]]`

which can be unified with the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[not[x1_],x2_]]]]→x2`

where these rules follow from Critical Pair Lemma 23 and Critical Pair Lemma 23 respectively.

Substitution Lemma 13

It can be shown that:

`not[or[not[x1],x2]]==not[or[not[or[x1,or[x2,not[or[not[x1],x2]]]]],x2]]`

Proof

We start by taking Critical Pair Lemma 30, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Critical Pair Lemma 31

The following expressions are equivalent:

`x1==not[or[not[or[or[not[or[x2,x3]],not[or[not[x2],x3]]],x1]],not[or[x3,x1]]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[not[x1_],x2_]]]]→x2`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[not[x1_],x2_]]]]→x2`

where these rules follow from Critical Pair Lemma 23 and Critical Pair Lemma 23 respectively.

Substitution Lemma 14

It can be shown that:

`x1==not[or[not[or[not[or[x2,x3]],or[not[or[not[x2],x3]],x1]]],not[or[x3,x1]]]]`

Proof

We start by taking Critical Pair Lemma 31, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Substitution Lemma 15

It can be shown that:

`not[or[x1,x2]]==not[or[x1,not[or[not[or[x1,x2]],or[x1,not[x2]]]]]]`

Proof

We start by taking Critical Pair Lemma 2, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Substitution Lemma 16

It can be shown that:

`not[or[x1,x2]]==not[or[x1,not[or[or[x1,not[x2]],not[or[x1,x2]]]]]]`

Proof

We start by taking Substitution Lemma 15, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Substitution Lemma 17

It can be shown that:

`not[or[x1,x2]]==not[or[x1,not[or[x1,or[not[x2],not[or[x1,x2]]]]]]]`

Proof

We start by taking Substitution Lemma 16, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Critical Pair Lemma 32

The following expressions are equivalent:

`not[or[not[or[x1,x2]],not[or[x2,not[x1]]]]]==not[or[not[or[x1,x2]],not[or[not[or[x1,x2`

Proof

Note that the input for the rule:

`not[or[x1_,not[or[x1_,or[not[x2_],not[or[x1_,x2_]]]]]]]→not[or[x1,x2]]`

contains a subpattern of the form:

`not[or[x1_,x2_]]`

which can be unified with the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[x2_,not[x1_]]]]]→x2`

where these rules follow from Substitution Lemma 17 and Critical Pair Lemma 4 respectively.

Substitution Lemma 18

It can be shown that:

`x1==not[or[not[or[x2,x1]],not[or[not[or[x2,x1]],or[not[not[or[x1,not[x2]]]],x1]]]]]`

Proof

We start by taking Critical Pair Lemma 32, and apply the substitution:

`not[or[not[or[x1_,x2_]],not[or[x2_,not[x1_]]]]]→x2`

which follows from Critical Pair Lemma 4.

Critical Pair Lemma 33

The following expressions are equivalent:

`x1==not[or[not[or[or[x2,not[or[x2,or[not[x3],not[or[x2,x3]]]]]],x1]],not[or[x1,not[or[`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[x2_,not[x1_]]]]]→x2`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[or[x1_,not[or[x1_,or[not[x2_],not[or[x1_,x2_]]]]]]]→not[or[x1,x2]]`

where these rules follow from Critical Pair Lemma 4 and Substitution Lemma 17 respectively.

Substitution Lemma 19

It can be shown that:

`x1==not[or[not[or[x2,or[not[or[x2,or[not[x3],not[or[x2,x3]]]]],x1]]],not[or[x1,not[or[`

Proof

We start by taking Critical Pair Lemma 33, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Substitution Lemma 20

It can be shown that:

`not[d]==not[or[d,not[or[d,or[not[c],not[d]]]]]]`

Proof

We start by taking Substitution Lemma 11, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Substitution Lemma 21

It can be shown that:

`not[or[c,not[d]]]==not[or[c,not[or[d,not[or[c,not[d]]]]]]]`

Proof

We start by taking Critical Pair Lemma 29, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Critical Pair Lemma 34

The following expressions are equivalent:

`c==not[or[not[or[c,or[d,not[or[c,not[d]]]]]],not[or[c,not[d]]]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[x1_,not[x2_]]]]]→x1`

contains a subpattern of the form:

`not[or[x1_,not[x2_]]]`

which can be unified with the input for the rule:

`not[or[c,not[or[d,not[or[c,not[d]]]]]]]→not[or[c,not[d]]]`

where these rules follow from Axiom 1 and Substitution Lemma 21 respectively.

Substitution Lemma 22

It can be shown that:

`c==not[or[not[or[d,not[or[c,not[d]]]]],not[or[c,not[d]]]]]`

Proof

We start by taking Critical Pair Lemma 34, and apply the substitution:

`or[c,or[d,x1_]]→or[d,x1]`

which follows from Critical Pair Lemma 8.

Critical Pair Lemma 35

The following expressions are equivalent:

`x1==not[or[not[or[x1,or[c,not[or[d,not[or[c,not[d]]]]]]]],not[or[x1,not[or[c,not[d]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[x1_,not[x2_]]]]]→x1`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[or[c,not[or[d,not[or[c,not[d]]]]]]]→not[or[c,not[d]]]`

where these rules follow from Axiom 1 and Substitution Lemma 21 respectively.

Substitution Lemma 23

It can be shown that:

`x1==not[or[not[or[x1,x2]],not[or[x1,or[not[or[x2,x3]],not[or[x2,not[x3]]]]]]]]`

PROOF

We start by taking Critical Pair Lemma 3, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Critical Pair Lemma 36

The following expressions are equivalent:

`c==not[or[not[d],not[or[c,or[not[or[d,x1]],not[or[d,not[x1]]]]]]]]`

PROOF

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[x1_,or[not[or[x2_,x3_]],not[or[x2_,not[x3_]]]]]]]]→x1`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[c,d]→d`

where these rules follow from Substitution Lemma 23 and Axiom 4 respectively.

Critical Pair Lemma 37

The following expressions are equivalent:

`c==not[or[not[or[x1,d]],not[or[c,not[or[d,x1]]]]]]`

PROOF

Note that the input for the rule:

`not[or[not[or[d,x1_]],not[or[c,not[or[d,x1_]]]]]]→c`

contains a subpattern of the form:

`or[d,x1_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Critical Pair Lemma 11 and Axiom 2 respectively.

Critical Pair Lemma 38

The following expressions are equivalent:

`d==not[or[not[or[x1,d]],not[or[d,not[or[c,x1]]]]]]`

PROOF

Note that the input for the rule:

`not[or[not[or[d,x1_]],not[or[d,not[or[c,x1_]]]]]]→d`

contains a subpattern of the form:

`or[d,x1_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Critical Pair Lemma 18 and Axiom 2 respectively.

Critical Pair Lemma 39

The following expressions are equivalent:

`not[or[d,x1]]==not[or[not[or[not[or[d,x1]],or[x1,or[d,not[c]]]]],or[x1,d]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[x1_,not[x2_]]]]]→x1`

contains a subpattern of the form:

`not[or[x1_,not[x2_]]]`

which can be unified with the input for the rule:

`not[or[not[or[d,x1_]],not[or[x1_,or[d,not[c]]]]]]→or[x1,d]`

where these rules follow from Axiom 1 and Substitution Lemma 10 respectively.

Critical Pair Lemma 40

The following expressions are equivalent:

`or[x1,or[x2,x3]]==or[x3,or[x2,x1]]`

Proof

Note that the input for the rule:

`or[x1_,or[x2_,x3_]]↔or[x3_,or[x1_,x2_]]`

contains a subpattern of the form:

`or[x2_,x3_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Critical Pair Lemma 5 and Axiom 2 respectively.

Substitution Lemma 24

It can be shown that:

`or[x1,d]==not[or[not[or[d,x1]],not[or[d,or[not[c],x1]]]]]`

Proof

We start by taking Critical Pair Lemma 27, and apply the substitution:

`or[x1_,or[x2_,x3_]]→or[x3,or[x1,x2]]`

which follows from Critical Pair Lemma 5.

Substitution Lemma 25

It can be shown that:

`not[or[x1,not[x2]]]==not[or[x1,not[or[x2,or[x1,not[or[x1,not[x2]]]]]]]]`

Proof

We start by taking Substitution Lemma 12, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Critical Pair Lemma 41

The following expressions are equivalent:

`x1==not[or[not[or[or[x2,or[x1,not[or[x1,not[x2]]]]],x1]],not[or[x1,not[x2]]]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[x2_,not[x1_]]]]]→x2`

contains a subpattern of the form:

`not[or[x2_,not[x1_]]]`

which can be unified with the input for the rule:

`not[or[x1_,not[or[x2_,or[x1_,not[or[x1_,not[x2_]]]]]]]]→not[or[x1,not[x2]]]`

where these rules follow from Critical Pair Lemma 4 and Substitution Lemma 25 respectively.

Substitution Lemma 26

It can be shown that:

`x1==not[or[not[or[x2,or[or[x1,not[or[x1,not[x2]]]],x1]]],not[or[x1,not[x2]]]]]`

Proof

We start by taking Critical Pair Lemma 41, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Substitution Lemma 27

It can be shown that:

`x1==not[or[not[or[x2,or[x1,or[not[or[x1,not[x2]]],x1]]]],not[or[x1,not[x2]]]]]`

Proof

We start by taking Substitution Lemma 26, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Substitution Lemma 28

It can be shown that:

`not[or[not[x1],x2]]==not[or[x2,not[or[x1,or[x2,not[or[not[x1],x2]]]]]]]`

Proof

We start by taking Substitution Lemma 13, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Critical Pair Lemma 42

The following expressions are equivalent:

`not[or[not[or[x1,x2]],not[or[not[x1],x2]]]]==not[or[not[or[not[x1],x2]],not[or[or[x1,`

Proof

Note that the input for the rule:

`not[or[x1_,not[or[x2_,or[x1_,not[or[not[x2_],x1_]]]]]]]→not[or[not[x2],x1]]`

contains a subpattern of the form:

`not[or[not[x2_],x1_]]`

which can be unified with the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[not[x1_],x2_]]]]→x2`

where these rules follow from Substitution Lemma 28 and Critical Pair Lemma 23 respectively.

Substitution Lemma 29

It can be shown that:

`x1==not[or[not[or[not[x2],x1]],not[or[or[x2,x1],or[not[or[not[x2],x1]],x1]]]]]`

Proof

We start by taking Critical Pair Lemma 42, and apply the substitution:

`not[or[not[or[x1_,x2_]],not[or[not[x1_],x2_]]]]→x2`

which follows from Critical Pair Lemma 23.

Substitution Lemma 30

It can be shown that:

`x1==not[or[not[or[not[x2],x1]],not[or[x2,or[x1,or[not[or[not[x2],x1]],x1]]]]]]`

Proof

We start by taking Substitution Lemma 29, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Critical Pair Lemma 43

The following expressions are equivalent:

`not[or[c,not[or[d,x1]]]]==not[or[not[or[or[x1,d],not[or[c,not[or[d,x1]]]]]],c]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[not[x1_],x2_]]]]→x2`

contains a subpattern of the form:

`not[or[not[x1_],x2_]]`

which can be unified with the input for the rule:

`not[or[not[or[x1_,d]],not[or[c,not[or[d,x1_]]]]]]→c`

where these rules follow from Critical Pair Lemma 23 and Critical Pair Lemma 37 respectively.

Substitution Lemma 31

It can be shown that:

`not[or[c,not[or[d,x1]]]]==not[or[not[or[x1,or[d,not[or[c,not[or[d,x1]]]]]]],c]]`

Proof

We start by taking Critical Pair Lemma 43, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Critical Pair Lemma 44

The following expressions are equivalent:

`not[or[x1,d]]==not[or[not[or[not[or[x1,d]],or[d,not[or[c,x1]]]]],d]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[x1_,not[x2_]]]]]→x1`

contains a subpattern of the form:

`not[or[x1_,not[x2_]]]`

which can be unified with the input for the rule:

`not[or[not[or[x1_,d]],not[or[d,not[or[c,x1_]]]]]]→d`

where these rules follow from Axiom 1 and Critical Pair Lemma 38 respectively.

Critical Pair Lemma 45

The following expressions are equivalent:

`not[or[d,not[or[c,x1]]]]==not[or[not[or[or[x1,d],not[or[d,not[or[c,x1]]]]]],d]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[not[x1_],x2_]]]]→x2`

contains a subpattern of the form:

`not[or[not[x1_],x2_]]`

which can be unified with the input for the rule:

`not[or[not[or[x1_,d]],not[or[d,not[or[c,x1_]]]]]]→d`

where these rules follow from Critical Pair Lemma 23 and Critical Pair Lemma 38 respectively.

Substitution Lemma 32

It can be shown that:

`not[or[d,not[or[c,x1]]]]==not[or[not[or[x1,or[d,not[or[d,not[or[c,x1]]]]]]],d]]`

Proof

We start by taking Critical Pair Lemma 45, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Critical Pair Lemma 46

The following expressions are equivalent:

`or[x1,x2]==not[or[not[or[x3,or[x1,x2]]],not[or[x1,or[not[x3],x2]]]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[not[x1_],x2_]]]]→x2`

contains a subpattern of the form:

`or[not[x1_],x2_]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,x3_]]↔or[x2_,or[x1_,x3_]]`

where these rules follow from Critical Pair Lemma 23 and Substitution Lemma 1 respectively.

Critical Pair Lemma 47

The following expressions are equivalent:

`or[x1,x2]==not[or[not[or[x1,or[x3,x2]]],not[or[or[x1,x2],not[x3]]]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[x2_,not[x1_]]]]]→x2`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,x3_]]↔or[x2_,or[x1_,x3_]]`

where these rules follow from Critical Pair Lemma 4 and Substitution Lemma 1 respectively.

Substitution Lemma 33

It can be shown that:

`or[x1,x2]==not[or[not[or[x1,or[x3,x2]]],not[or[x1,or[x2,not[x3]]]]]]`

Proof

We start by taking Critical Pair Lemma 47, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Critical Pair Lemma 48

The following expressions are equivalent:

`or[or[x1,x2],d]==not[or[not[or[x1,or[d,x2]]],not[or[d,or[not[c],or[x1,x2]]]]]]`

Proof

Note that the input for the rule:

`not[or[not[or[d,x1_]],not[or[d,or[not[c],x1_]]]]]→or[x1,d]`

contains a subpattern of the form:

`or[d,x1_]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,x3_]]↔or[x2_,or[x1_,x3_]]`

where these rules follow from Substitution Lemma 24 and Substitution Lemma 1 respectively.

Substitution Lemma 34

It can be shown that:

`or[x1,or[x2,d]]==not[or[not[or[x1,or[d,x2]]],not[or[d,or[not[c],or[x1,x2]]]]]]`

Proof

We start by taking Critical Pair Lemma 48, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Critical Pair Lemma 49

Critical Pair Lemma 49

The following expressions are equivalent:

`or[c,x1]==not[or[not[or[d,or[x1,x2]]],not[or[c,or[x1,not[or[d,x2]]]]]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,or[x2_,x3_]]],not[or[x1_,or[x2_,not[x3_]]]]]]→or[x1,x2]`

contains a subpattern of the form:

`or[x1_,or[x2_,x3_]]`

which can be unified with the input for the rule:

`or[c,or[x1_,or[d,x2_]]]→or[d,or[x1,x2]]`

where these rules follow from Substitution Lemma 2 and Substitution Lemma 4 respectively.

Critical Pair Lemma 50

The following expressions are equivalent:

`or[x1,c]==not[or[not[or[x1,or[d,x2]]],not[or[x1,or[c,not[or[d,x2]]]]]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,or[x2_,x3_]]],not[or[x1_,or[x2_,not[x3_]]]]]]→or[x1,x2]`

contains a subpattern of the form:

`or[x2_,x3_]`

which can be unified with the input for the rule:

`or[c,or[d,x1_]]→or[d,x1]`

where these rules follow from Substitution Lemma 2 and Critical Pair Lemma 8 respectively.

Substitution Lemma 35

It can be shown that:

`x1==not[or[not[or[x1,c]],not[or[x1,or[not[d],not[or[c,not[d]]]]]]]]`

Proof

We start by taking Critical Pair Lemma 14, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Critical Pair Lemma 51

The following expressions are equivalent:

`x1==not[or[not[or[c,x1]],not[or[x1,or[not[d],not[or[c,not[d]]]]]]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,c]],not[or[x1_,or[not[d],not[or[c,not[d]]]]]]]]→x1`

contains a subpattern of the form:

`or[x1_,c]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Substitution Lemma 35 and Axiom 2 respectively.

Substitution Lemma 36

It can be shown that:

`c==not[or[not[or[c,not[d]]],not[or[d,not[or[c,not[d]]]]]]]`

Proof

We start by taking Substitution Lemma 22, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Critical Pair Lemma 52

The following expressions are equivalent:

`not[or[d,not[or[c,not[d]]]]]==not[or[not[or[or[c,not[d]],not[or[d,not[or[c,not[d]]]]]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[not[x1_],x2_]]]]→x2`

contains a subpattern of the form:

`not[or[not[x1_],x2_]]`

which can be unified with the input for the rule:

`not[or[not[or[c,not[d]]],not[or[d,not[or[c,not[d]]]]]]]→c`

where these rules follow from Critical Pair Lemma 23 and Substitution Lemma 36 respectively.

Substitution Lemma 37

It can be shown that:

`not[or[d,not[or[c,not[d]]]]]==not[or[not[or[c,or[not[d],not[or[d,not[or[c,not[d]]]]]]]`

Proof

We start by taking Critical Pair Lemma 52, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Critical Pair Lemma 53

The following expressions are equivalent:

`or[c,x1]==not[or[not[or[d,or[x2,x1]]],not[or[or[c,x1],not[or[d,x2]]]]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,or[x2_,x3_]]],not[or[x3_,not[or[x1_,x2_]]]]]]→x3`

contains a subpattern of the form:

`or[x1_,or[x2_,x3_]]`

which can be unified with the input for the rule:

`or[d,or[x1_,or[c,x2_]]]→or[d,or[x1,x2]]`

where these rules follow from Critical Pair Lemma 20 and Substitution Lemma 9 respectively.

Substitution Lemma 38

It can be shown that:

`or[c,x1]==not[or[not[or[d,or[x2,x1]]],not[or[c,or[x1,not[or[d,x2]]]]]]]`

Proof

We start by taking Critical Pair Lemma 53, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Critical Pair Lemma 54

The following expressions are equivalent:

`x1==not[or[not[or[x1,or[x2,x3]]],not[or[x1,not[or[x3,x2]]]]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,or[x2_,x3_]]],not[or[x3_,not[or[x1_,x2_]]]]]]→x3`

contains a subpattern of the form:

`or[x1_,or[x2_,x3_]]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,x3_]]↔or[x3_,or[x2_,x1_]]`

where these rules follow from Critical Pair Lemma 20 and Critical Pair Lemma 40 respectively.

Critical Pair Lemma 55

The following expressions are equivalent:

`or[d,x1]==not[or[not[or[d,or[x2,x1]]],not[or[not[or[c,x2]],or[d,x1]]]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,or[x2_,x3_]]],not[or[not[or[x1_,x2_]],x3_]]]]→x3`

contains a subpattern of the form:

`or[x1_,or[x2_,x3_]]`

which can be unified with the input for the rule:

`or[c,or[x1_,or[d,x2_]]]→or[d,or[x1,x2]]`

where these rules follow from Critical Pair Lemma 26 and Substitution Lemma 4 respectively.

Substitution Lemma 39

It can be shown that:

`not[or[c,not[or[d,x1]]]]==not[or[c,not[or[x1,or[d,not[or[c,not[or[d,x1]]]]]]]]]`

Proof

We start by taking Substitution Lemma 31, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Substitution Lemma 40

It can be shown that:

```
not[or[x1,d]]==not[or[d,not[or[not[or[x1,d]],or[d,not[or[c,x1]]]]]]]
```

PROOF

We start by taking Critical Pair Lemma 44, and apply the substitution:

```
or[x1_,x2_]→or[x2,x1]
```

which follows from Axiom 2.

Substitution Lemma 41

It can be shown that:

```
not[or[x1,d]]==not[or[d,not[or[d,or[not[or[x1,d]],not[or[c,x1]]]]]]]
```

PROOF

We start by taking Substitution Lemma 40, and apply the substitution:

```
or[x1_,or[x2_,x3_]]→or[x2,or[x1,x3]]
```

which follows from Substitution Lemma 1.

Substitution Lemma 42

It can be shown that:

```
not[or[d,not[or[c,x1]]]]==not[or[d,not[or[x1,or[d,not[or[d,not[or[c,x1]]]]]]]]]
```

PROOF

We start by taking Substitution Lemma 32, and apply the substitution:

```
or[x1_,x2_]→or[x2,x1]
```

which follows from Axiom 2.

Substitution Lemma 43

It can be shown that:

```
not[or[d,not[or[c,not[d]]]]]==not[or[c,not[or[c,or[not[d],not[or[d,not[or[c,not[d]]]]
```

PROOF

We start by taking Substitution Lemma 37, and apply the substitution:

```
or[x1_,x2_]→or[x2,x1]
```

which follows from Axiom 2.

Substitution Lemma 44

It can be shown that:

```
not[or[d,x1]]==not[or[d,or[x1,not[or[not[or[d,x1]],or[x1,or[d,not[c]]]]]]]]
```

PROOF

We start by taking Critical Pair Lemma 39, and apply the substitution:

```
or[x1_,or[x2_,x3_]]→or[x3,or[x2,x1]]
```

which follows from Critical Pair Lemma 40.

Substitution Lemma 45

It can be shown that:

`not[or[d,x1]]==not[or[d,or[x1,not[or[or[d,not[c]],or[x1,not[or[d,x1]]]]]]]]`

PROOF

We start by taking Substitution Lemma 44, and apply the substitution:

`or[x1_,or[x2_,x3_]]→or[x3,or[x2,x1]]`

which follows from Critical Pair Lemma 40.

Substitution Lemma 46

It can be shown that:

`not[or[d,x1]]==not[or[d,or[x1,not[or[d,or[not[c],or[x1,not[or[d,x1]]]]]]]]]`

PROOF

We start by taking Substitution Lemma 45, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Substitution Lemma 47

It can be shown that:

`x1==not[or[not[or[x1,x2]],not[or[x1,or[not[or[x3,x2]],not[or[x2,not[x3]]]]]]]]`

PROOF

We start by taking Critical Pair Lemma 24, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Critical Pair Lemma 56

The following expressions are equivalent:

`or[c,x1]==not[or[not[or[d,or[x1,x2]]],not[or[x1,or[c,not[or[d,x2]]]]]]]`

PROOF

Note that the input for the rule:

`not[or[not[or[d,or[x1_,x2_]]],not[or[c,or[x1_,not[or[d,x2_]]]]]]]→or[c,x1]`

contains a subpattern of the form:

`or[c,or[x1_,not[or[d,x2_]]]]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,x3_]]↔or[x2_,or[x1_,x3_]]`

where these rules follow from Critical Pair Lemma 49 and Substitution Lemma 1 respectively.

Critical Pair Lemma 57

The following expressions are equivalent:

`or[c,c]==not[or[not[or[d,x1]],not[or[c,or[c,not[or[d,x1]]]]]]]`

PROOF

Note that the input for the rule:

`not[or[not[or[x1_,or[d,x2_]]],not[or[x1_,or[c,not[or[d,x2_]]]]]]]→or[x1,c]`

contains a subpattern of the form:

`or[x1_,or[d,x2_]]`

which can be unified with the input for the rule:

`or[c,or[d,x1_]]→or[d,x1]`

where these rules follow from Critical Pair Lemma 50 and Critical Pair Lemma 8 respectively.

Critical Pair Lemma 58

The following expressions are equivalent:

`or[x1,c]==not[or[not[or[x2,or[d,x1]]],not[or[x1,or[c,not[or[d,x2]]]]]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,or[d,x2_]]],not[or[x1_,or[c,not[or[d,x2_]]]]]]]→or[x1,c]`

contains a subpattern of the form:

`or[x1_,or[d,x2_]]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,x3_]]↔or[x3_,or[x2_,x1_]]`

where these rules follow from Critical Pair Lemma 50 and Critical Pair Lemma 40 respectively.

Substitution Lemma 48

It can be shown that:

`or[d,x1]==not[or[not[or[d,or[x2,x1]]],not[or[d,or[not[or[c,x2]],x1]]]]]`

Proof

We start by taking Critical Pair Lemma 55, and apply the substitution:

`or[x1_,or[x2_,x3_]]→or[x2,or[x1,x3]]`

which follows from Substitution Lemma 1.

Critical Pair Lemma 59

The following expressions are equivalent:

`or[d,x1]==not[or[not[or[d,or[x2,x1]]],not[or[d,or[not[or[x2,c]],x1]]]]]`

Proof

Note that the input for the rule:

`not[or[not[or[d,or[x1_,x2_]]],not[or[d,or[not[or[c,x1_]],x2_]]]]]→or[d,x2]`

contains a subpattern of the form:

`or[c,x1_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Substitution Lemma 48 and Axiom 2 respectively.

Substitution Lemma 49

It can be shown that:

`x1==not[or[not[or[x2,x1]],not[or[not[or[x3,x2]],or[not[or[not[x3],x2]],x1]]]]]`

Proof

We start by taking Substitution Lemma 14, and apply the substitution:

We start by taking Substitution Lemma 14, and apply the substitution:

```
or[x1_,x2_]→or[x2,x1]
```

which follows from Axiom 2.

Substitution Lemma 50

It can be shown that:

```
x1==not[or[not[or[x2,x1]],not[or[x1,or[not[not[or[x1,not[x2]]]],not[or[x2,x1]]]]]]]
```

Proof

We start by taking Substitution Lemma 18, and apply the substitution:

```
or[x1_,or[x2_,x3_]]→or[x3,or[x2,x1]]
```

which follows from Critical Pair Lemma 40.

Substitution Lemma 51

It can be shown that:

```
x1==not[or[not[or[x2,x1]],not[or[x1,or[not[or[x2,x1]],not[not[or[x1,not[x2]]]]]]]]]
```

Proof

We start by taking Substitution Lemma 50, and apply the substitution:

```
or[x1_,x2_]→or[x2,x1]
```

which follows from Axiom 2.

Substitution Lemma 52

It can be shown that:

```
x1==not[or[not[or[x1,not[x2]]],not[or[x2,or[x1,or[not[or[x1,not[x2]]],x1]]]]]]
```

Proof

We start by taking Substitution Lemma 27, and apply the substitution:

```
or[x1_,x2_]→or[x2,x1]
```

which follows from Axiom 2.

Substitution Lemma 53

It can be shown that:

```
x1==not[or[not[or[x1,not[x2]]],not[or[x2,or[x1,or[x1,not[or[x1,not[x2]]]]]]]]]
```

Proof

We start by taking Substitution Lemma 52, and apply the substitution:

```
or[x1_,or[x2_,x3_]]→or[x3,or[x1,x2]]
```

which follows from Critical Pair Lemma 5.

Substitution Lemma 54

It can be shown that:

```
x1==not[or[not[or[not[x2],x1]],not[or[x2,or[x1,or[x1,not[or[not[x2],x1]]]]]]]]
```

Proof

We start by taking Substitution Lemma 30, and apply the substitution:

```
or[x1_,or[x2_,x3_]]→or[x3,or[x1,x2]]
```

which follows from Critical Pair Lemma 5.

which follows from Critical Pair Lemma 5.

Substitution Lemma 55

It can be shown that:

`x1==not[or[not[or[x1,not[or[x2,x3]]]],not[or[x2,or[not[or[x2,or[not[x3],not[or[x2,x3]]`

Proof

We start by taking Substitution Lemma 19, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Substitution Lemma 56

It can be shown that:

`x1==not[or[not[or[x1,not[or[c,not[d]]]]],not[or[x1,or[c,not[or[d,not[or[c,not[d]]]]]]`

Proof

We start by taking Critical Pair Lemma 35, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Critical Pair Lemma 60

The following expressions are equivalent:

`not[or[c,d]]==not[or[c,not[or[not[or[c,d]],or[c,not[or[d,not[or[c,not[d]]]]]]]]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,not[or[c,not[d]]]]],not[or[x1_,or[c,not[or[d,not[or[c,not[d]]]]]]]`

contains a subpattern of the form:

`not[or[x1_,not[or[c,not[d]]]]]`

which can be unified with the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[x1_,not[x2_]]]]]→x1`

where these rules follow from Substitution Lemma 56 and Axiom 1 respectively.

Substitution Lemma 57

It can be shown that:

`not[d]==not[or[c,not[or[not[or[c,d]],or[c,not[or[d,not[or[c,not[d]]]]]]]]]]`

Proof

We start by taking Critical Pair Lemma 60, and apply the substitution:

`or[c,d]→d`

which follows from Axiom 4.

Substitution Lemma 58

It can be shown that:

`not[d]==not[or[c,not[or[not[d],or[c,not[or[d,not[or[c,not[d]]]]]]]]]]`

Proof

We start by taking Substitution Lemma 57, and apply the substitution:

`or[c,d]→d`

which follows from Axiom 4.

Substitution Lemma 59

It can be shown that:

`not[d]==not[or[c,not[or[c,or[not[d],not[or[d,not[or[c,not[d]]]]]]]]]]`

Proof

We start by taking Substitution Lemma 58, and apply the substitution:

`or[x1_,or[x2_,x3_]]→or[x2,or[x1,x3]]`

which follows from Substitution Lemma 1.

Substitution Lemma 60

It can be shown that:

`not[d]==not[or[d,not[or[c,not[d]]]]]`

Proof

We start by taking Substitution Lemma 59, and apply the substitution:

`not[or[c,not[or[c,or[not[d],not[or[d,not[or[c,not[d]]]]]]]]]]→not[or[d,not[or[c,not[`

which follows from Substitution Lemma 43.

Critical Pair Lemma 61

The following expressions are equivalent:

`d==not[or[not[or[or[c,not[d]],d]],not[d]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[x2_,not[x1_]]]]]→x2`

contains a subpattern of the form:

`not[or[x2_,not[x1_]]]`

which can be unified with the input for the rule:

`not[or[d,not[or[c,not[d]]]]]→not[d]`

where these rules follow from Critical Pair Lemma 4 and Substitution Lemma 60 respectively.

Substitution Lemma 61

It can be shown that:

`d==not[or[not[or[c,or[not[d],d]]],not[d]]]`

Proof

We start by taking Critical Pair Lemma 61, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Substitution Lemma 62

It can be shown that:

```
d==not[or[not[or[d,not[d]]],not[d]]]
```

PROOF

We start by taking Substitution Lemma 61, and apply the substitution:

```
or[c,or[x1_,d]]→or[d,x1]
```

which follows from Critical Pair Lemma 10.

Critical Pair Lemma 62

The following expressions are equivalent:

```
or[not[or[c,not[d]]],d]==not[or[not[d],not[or[not[or[c,not[d]]],or[d,not[c]]]]]]
```

PROOF

Note that the input for the rule:

```
not[or[not[or[d,x1_]],not[or[x1_,or[d,not[c]]]]]]→or[x1,d]
```

contains a subpattern of the form:

```
not[or[d,x1_]]
```

which can be unified with the input for the rule:

```
not[or[d,not[or[c,not[d]]]]]→not[d]
```

where these rules follow from Substitution Lemma 10 and Substitution Lemma 60 respectively.

Critical Pair Lemma 63

The following expressions are equivalent:

```
d==not[or[not[or[or[c,not[d]],d]],not[or[d,or[not[or[or[c,not[d]],d]],not[not[d]]]]]]]
```

PROOF

Note that the input for the rule:

```
not[or[not[or[x1_,x2_]],not[or[x2_,or[not[or[x1_,x2_]],not[not[or[x2_,not[x1_]]]]]]]]
```

contains a subpattern of the form:

```
not[or[x2_,not[x1_]]]
```

which can be unified with the input for the rule:

```
not[or[d,not[or[c,not[d]]]]]→not[d]
```

where these rules follow from Substitution Lemma 51 and Substitution Lemma 60 respectively.

Substitution Lemma 63

It can be shown that:

```
d==not[or[not[or[c,or[not[d],d]]],not[or[d,or[not[or[or[c,not[d]],d]],not[not[d]]]]]]]
```

PROOF

We start by taking Critical Pair Lemma 63, and apply the substitution:

```
or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]
```

which follows from Axiom 3.

Substitution Lemma 64

It can be shown that:

`d==not[or[not[or[d,not[d]]],not[or[d,or[not[or[or[c,not[d]],d]],not[not[d]]]]]]]`

Proof

We start by taking Substitution Lemma 63, and apply the substitution:

`or[c,or[x1_,d]]→or[d,x1]`

which follows from Critical Pair Lemma 10.

Substitution Lemma 65

It can be shown that:

`d==not[or[not[or[d,not[d]]],not[or[d,or[not[or[c,or[not[d],d]]],not[not[d]]]]]]]`

Proof

We start by taking Substitution Lemma 64, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Substitution Lemma 66

It can be shown that:

`d==not[or[not[or[d,not[d]]],not[or[d,or[not[or[d,not[d]]],not[not[d]]]]]]]`

Proof

We start by taking Substitution Lemma 65, and apply the substitution:

`or[c,or[x1_,d]]→or[d,x1]`

which follows from Critical Pair Lemma 10.

Substitution Lemma 67

It can be shown that:

`d==not[or[not[d],not[or[d,not[d]]]]]`

Proof

We start by taking Substitution Lemma 62, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Critical Pair Lemma 64

The following expressions are equivalent:

`not[d]==not[or[d,not[or[d,or[not[or[d,or[not[not[d]],not[or[d,not[d]]]]]],not[d]]]]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,not[or[x2_,x3_]]]],not[or[x2_,or[not[or[x2_,or[not[x3_],not[or[x2_,`

contains a subpattern of the form:

`not[or[x1_,not[or[x2_,x3_]]]]`

which can be unified with the input for the rule:

`not[or[not[d],not[or[d,not[d]]]]]→d`

where these rules follow from Substitution Lemma 55 and Substitution Lemma 67

respectively.

Substitution Lemma 68

It can be shown that:

`or[d,not[or[c,not[d]]]]==not[or[not[d],not[or[not[or[c,not[d]]],or[d,not[c]]]]]]`

Proof

We start by taking Critical Pair Lemma 62, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Substitution Lemma 69

It can be shown that:

`or[d,not[or[c,not[d]]]]==not[or[not[d],not[or[not[c],or[d,not[or[c,not[d]]]]]]]]`

Proof

We start by taking Substitution Lemma 68, and apply the substitution:

`or[x1_,or[x2_,x3_]]→or[x3,or[x2,x1]]`

which follows from Critical Pair Lemma 40.

Substitution Lemma 70

It can be shown that:

`or[d,not[or[c,not[d]]]]==not[or[not[d],not[or[d,or[not[c],not[or[c,not[d]]]]]]]]`

Proof

We start by taking Substitution Lemma 69, and apply the substitution:

`or[x1_,or[x2_,x3_]]→or[x2,or[x1,x3]]`

which follows from Substitution Lemma 1.

Substitution Lemma 71

It can be shown that:

`d==not[or[not[or[d,not[d]]],not[or[d,or[not[not[d]],not[or[d,not[d]]]]]]]]`

Proof

We start by taking Substitution Lemma 66, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Critical Pair Lemma 65

The following expressions are equivalent:

`not[or[d,or[not[not[d]],not[or[d,not[d]]]]]]==not[or[not[or[or[d,not[d]],not[or[d,or[`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[not[x1_],x2_]]]]→x2`

contains a subpattern of the form:

`not[or[not[x1_],x2_]]`

which can be unified with the input for the rule:

```
not[or[not[or[d,not[d]]],not[or[d,or[not[not[d]],not[or[d,not[d]]]]]]]]→d
```

where these rules follow from Critical Pair Lemma 23 and Substitution Lemma 71 respectively.

Substitution Lemma 72

It can be shown that:

```
not[or[d,or[not[not[d]],not[or[d,not[d]]]]]]==not[or[not[or[d,or[not[d],not[or[d,or[n
```

Proof

We start by taking Critical Pair Lemma 65, and apply the substitution:

```
or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]
```

which follows from Axiom 3.

Substitution Lemma 73

It can be shown that:

```
not[d]==not[or[d,not[or[d,or[not[d],not[or[d,or[not[not[d]],not[or[d,not[d]]]]]]]]]]
```

Proof

We start by taking Critical Pair Lemma 64, and apply the substitution:

```
or[x1_,x2_]→or[x2,x1]
```

which follows from Axiom 2.

Substitution Lemma 74

It can be shown that:

```
not[or[d,or[not[not[d]],not[or[d,not[d]]]]]]==not[or[d,not[or[d,or[not[d],not[or[d,or
```

Proof

We start by taking Substitution Lemma 72, and apply the substitution:

```
or[x1_,x2_]→or[x2,x1]
```

which follows from Axiom 2.

Substitution Lemma 75

It can be shown that:

```
not[or[d,or[not[not[d]],not[or[d,not[d]]]]]]==not[d]
```

Proof

We start by taking Substitution Lemma 74, and apply the substitution:

```
not[or[d,not[or[d,or[not[d],not[or[d,or[not[not[d]],not[or[d,not[d]]]]]]]]]]→not[d]
```

which follows from Substitution Lemma 73.

Critical Pair Lemma 66

The following expressions are equivalent:

```
or[c,not[or[d,not[d]]]]==not[or[not[d],not[or[c,or[not[or[d,not[d]]],not[or[d,not[not
```

Proof

Note that the input for the rule:

Note that the input for the rule:

`not[or[not[or[d,or[x1_,x2_]]],not[or[c,or[x2_,not[or[d,x1_]]]]]]]→or[c,x2]`

contains a subpattern of the form:

`not[or[d,or[x1_,x2_]]]`

which can be unified with the input for the rule:

`not[or[d,or[not[not[d]],not[or[d,not[d]]]]]]→not[d]`

where these rules follow from Substitution Lemma 38 and Substitution Lemma 75 respectively.

Substitution Lemma 76

It can be shown that:

`or[c,not[or[d,not[d]]]]==c`

Proof

We start by taking Critical Pair Lemma 66, and apply the substitution:

`not[or[not[d],not[or[c,or[not[or[d,x1_]],not[or[d,not[x1_]]]]]]]]→c`

which follows from Critical Pair Lemma 36.

Critical Pair Lemma 67

The following expressions are equivalent:

`c==not[or[not[or[c,or[d,not[d]]]],not[c]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[x1_,not[x2_]]]]]→x1`

contains a subpattern of the form:

`or[x1_,not[x2_]]`

which can be unified with the input for the rule:

`or[c,not[or[d,not[d]]]]→c`

where these rules follow from Axiom 1 and Substitution Lemma 76 respectively.

Substitution Lemma 77

It can be shown that:

`c==not[or[not[or[d,not[d]]],not[c]]]`

Proof

We start by taking Critical Pair Lemma 67, and apply the substitution:

`or[c,or[d,x1_]]→or[d,x1]`

which follows from Critical Pair Lemma 8.

Substitution Lemma 78

It can be shown that:

`c==not[or[not[c],not[or[d,not[d]]]]]`

Proof

We start by taking Substitution Lemma 77, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Critical Pair Lemma 68

The following expressions are equivalent:

`not[or[d,not[d]]]==not[or[not[or[c,not[or[d,not[d]]]]],c]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[not[x1_],x2_]]]]→x2`

contains a subpattern of the form:

`not[or[not[x1_],x2_]]`

which can be unified with the input for the rule:

`not[or[not[c],not[or[d,not[d]]]]]→c`

where these rules follow from Critical Pair Lemma 23 and Substitution Lemma 78 respectively.

Substitution Lemma 79

It can be shown that:

`not[or[d,not[d]]]==not[or[not[c],c]]`

Proof

We start by taking Critical Pair Lemma 68, and apply the substitution:

`or[c,not[or[d,not[d]]]]→c`

which follows from Substitution Lemma 76.

Substitution Lemma 80

It can be shown that:

`not[or[d,not[d]]]==not[or[c,not[c]]]`

Proof

We start by taking Substitution Lemma 79, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Critical Pair Lemma 69

The following expressions are equivalent:

`d==not[or[not[or[c,not[c]]],not[or[d,not[or[c,not[d]]]]]]]`

Proof

Note that the input for the rule:

`not[or[not[or[d,x1_]],not[or[d,not[or[c,x1_]]]]]]→d`

contains a subpattern of the form:

`not[or[d,x1_]]`

which can be unified with the input for the rule:

`not[or[d,not[d]]]→not[or[c,not[c]]]`

where these rules follow from Critical Pair Lemma 18 and Substitution Lemma 80 respectively.

Substitution Lemma 81

It can be shown that:

`d==not[or[not[or[c,not[c]]],not[d]]]`

Proof

We start by taking Critical Pair Lemma 69, and apply the substitution:

`not[or[d,not[or[c,not[d]]]]]→not[d]`

which follows from Substitution Lemma 60.

Substitution Lemma 82

It can be shown that:

`or[c,not[or[c,not[c]]]]==c`

Proof

We start by taking Substitution Lemma 76, and apply the substitution:

`not[or[d,not[d]]]→not[or[c,not[c]]]`

which follows from Substitution Lemma 80.

Substitution Lemma 83

It can be shown that:

`not[or[not[c],not[or[c,not[c]]]]]==c`

Proof

We start by taking Substitution Lemma 78, and apply the substitution:

`not[or[d,not[d]]]→not[or[c,not[c]]]`

which follows from Substitution Lemma 80.

Critical Pair Lemma 70

The following expressions are equivalent:

`or[d,not[or[c,not[c]]]]==not[or[not[or[d,c]],not[or[d,or[not[or[c,c]],not[or[c,not[c]]`

Proof

Note that the input for the rule:

`not[or[not[or[d,or[x1_,x2_]]],not[or[d,or[not[or[x1_,c]],x2_]]]]]→or[d,x2]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[c,not[or[c,not[c]]]]→c`

where these rules follow from Critical Pair Lemma 59 and Substitution Lemma 82 respectively.

Substitution Lemma 84

It can be shown that:

`or[d,not[or[c,not[c]]]]==d`

PROOF

We start by taking Critical Pair Lemma 70, and apply the substitution:

`not[or[not[or[x1_,x2_]],not[or[x1_,or[not[or[x3_,x2_]],not[or[x2_,not[x3_]]]]]]]]→x1`

which follows from Substitution Lemma 47.

Critical Pair Lemma 71

The following expressions are equivalent:

`or[c,or[x1,not[or[c,not[c]]]]]==or[x1,c]`

PROOF

Note that the input for the rule:

`or[x1_,or[x2_,x3_]]↔or[x2_,or[x1_,x3_]]`

contains a subpattern of the form:

`or[x2_,x3_]`

which can be unified with the input for the rule:

`or[c,not[or[c,not[c]]]]→c`

where these rules follow from Substitution Lemma 1 and Substitution Lemma 82 respectively.

Critical Pair Lemma 72

The following expressions are equivalent:

`or[d,or[not[or[c,not[c]]],x1]]==or[d,x1]`

PROOF

Note that the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[d,not[or[c,not[c]]]]→d`

where these rules follow from Axiom 3 and Substitution Lemma 84 respectively.

Substitution Lemma 85

It can be shown that:

`d==not[or[not[d],not[or[c,not[c]]]]]`

PROOF

We start by taking Substitution Lemma 81, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Critical Pair Lemma 73

The following expressions are equivalent:

`not[d]==not[or[not[or[not[d],or[c,not[c]]]],d]]`

PROOF

PROOF

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[x1_,not[x2_]]]]]→x1`

contains a subpattern of the form:

`not[or[x1_,not[x2_]]]`

which can be unified with the input for the rule:

`not[or[not[d],not[or[c,not[c]]]]]→d`

where these rules follow from Axiom 1 and Substitution Lemma 85 respectively.

Critical Pair Lemma 74

The following expressions are equivalent:

`not[or[not[d],not[or[c,not[c]]]]]==not[or[not[d],not[or[or[c,not[c]],or[not[d],d]]]]]`

PROOF

Note that the input for the rule:

`not[or[x1_,not[or[x2_,or[x1_,not[or[x1_,not[x2_]]]]]]]]→not[or[x1,not[x2]]]`

contains a subpattern of the form:

`not[or[x1_,not[x2_]]]`

which can be unified with the input for the rule:

`not[or[not[d],not[or[c,not[c]]]]]→d`

where these rules follow from Substitution Lemma 25 and Substitution Lemma 85 respectively.

Substitution Lemma 86

It can be shown that:

`d==not[or[not[d],not[or[or[c,not[c]],or[not[d],d]]]]]`

PROOF

We start by taking Critical Pair Lemma 74, and apply the substitution:

`not[or[not[d],not[or[c,not[c]]]]]→d`

which follows from Substitution Lemma 85.

Substitution Lemma 87

It can be shown that:

`d==not[or[not[d],not[or[c,or[not[c],or[not[d],d]]]]]]`

PROOF

We start by taking Substitution Lemma 86, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Substitution Lemma 88

It can be shown that:

`d==not[or[not[d],not[or[d,or[not[c],not[d]]]]]]`

PROOF

We start by taking Substitution Lemma 87, and apply the substitution:

```
or[c,or[x1_,or[x2_,d]]]→or[d,or[x1,x2]]
```

which follows from Critical Pair Lemma 12.

Critical Pair Lemma 75

The following expressions are equivalent:

```
not[d]⩵not[or[d,not[or[or[c,not[c]],or[not[d],or[not[d],not[or[not[d],not[or[c,not[c]
```

Proof

Note that the input for the rule:

```
not[or[not[or[x1_,not[x2_]]],not[or[x2_,or[x1_,or[x1_,not[or[x1_,not[x2_]]]]]]]]]→x1
```

contains a subpattern of the form:

```
not[or[x1_,not[x2_]]]
```

which can be unified with the input for the rule:

```
not[or[not[d],not[or[c,not[c]]]]]→d
```

where these rules follow from Substitution Lemma 53 and Substitution Lemma 85 respectively.

Substitution Lemma 89

It can be shown that:

```
not[d]⩵not[or[d,not[or[c,or[not[c],or[not[d],or[not[d],not[or[not[d],not[or[c,not[c]]
```

Proof

We start by taking Critical Pair Lemma 75, and apply the substitution:

```
or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]
```

which follows from Axiom 3.

Substitution Lemma 90

It can be shown that:

```
not[d]⩵not[or[d,not[or[c,or[not[c],or[not[d],or[not[d],d]]]]]]]
```

Proof

We start by taking Substitution Lemma 89, and apply the substitution:

```
not[or[not[d],not[or[c,not[c]]]]]→d
```

which follows from Substitution Lemma 85.

Substitution Lemma 91

It can be shown that:

```
not[d]⩵not[or[d,not[or[d,or[not[c],or[not[d],not[d]]]]]]]
```

Proof

We start by taking Substitution Lemma 90, and apply the substitution:

```
or[c,or[x1_,or[x2_,or[x3_,d]]]]→or[d,or[x1,or[x2,x3]]]
```

which follows from Substitution Lemma 5.

Critical Pair Lemma 76

The following expressions are equivalent:

```
not[or[c,not[c]]]==not[or[not[or[not[d],not[or[c,not[c]]]]],not[or[d,or[not[or[c,not[c
```

PROOF

Note that the input for the rule:

```
not[or[not[or[not[x1_],x2_]],not[or[x1_,or[x2_,or[x2_,not[or[not[x1_],x2_]]]]]]]]→x2
```

contains a subpattern of the form:

```
or[x1_,or[x2_,or[x2_,not[or[not[x1_],x2_]]]]]
```

which can be unified with the input for the rule:

```
or[d,or[not[or[c,not[c]]],x1_]]→or[d,x1]
```

where these rules follow from Substitution Lemma 54 and Critical Pair Lemma 72 respectively.

Substitution Lemma 92

It can be shown that:

```
not[or[c,not[c]]]==not[or[d,not[or[d,or[not[or[c,not[c]]],not[or[not[d],not[or[c,not[c
```

PROOF

We start by taking Critical Pair Lemma 76, and apply the substitution:

```
not[or[not[d],not[or[c,not[c]]]]]→d
```

which follows from Substitution Lemma 85.

Substitution Lemma 93

It can be shown that:

```
not[or[c,not[c]]]==not[or[d,not[or[d,not[or[not[d],not[or[c,not[c]]]]]]]]]
```

PROOF

We start by taking Substitution Lemma 92, and apply the substitution:

```
or[d,or[not[or[c,not[c]]],x1_]]→or[d,x1]
```

which follows from Critical Pair Lemma 72.

Substitution Lemma 94

It can be shown that:

```
not[or[c,not[c]]]==not[or[d,not[or[d,d]]]]
```

PROOF

We start by taking Substitution Lemma 93, and apply the substitution:

```
not[or[not[d],not[or[c,not[c]]]]]→d
```

which follows from Substitution Lemma 85.

Critical Pair Lemma 77

The following expressions are equivalent:

```
or[c,c]==not[or[not[or[c,not[c]]],not[or[c,or[c,not[or[d,not[or[d,d]]]]]]]]]
```

PROOF

Note that the input for the rule:

`not[or[not[or[d,x1_]],not[or[c,or[c,not[or[d,x1_]]]]]]]→or[c,c]`

contains a subpattern of the form:

`not[or[d,x1_]]`

which can be unified with the input for the rule:

`not[or[d,not[or[d,d]]]]→not[or[c,not[c]]]`

where these rules follow from Critical Pair Lemma 57 and Substitution Lemma 94 respectively.

Substitution Lemma 95

It can be shown that:

`or[c,c]==not[or[not[or[c,not[c]]],not[or[c,or[c,not[or[c,not[c]]]]]]]]`

Proof

We start by taking Critical Pair Lemma 77, and apply the substitution:

`not[or[d,not[or[d,d]]]]→not[or[c,not[c]]]`

which follows from Substitution Lemma 94.

Substitution Lemma 96

It can be shown that:

`or[c,c]==not[or[not[or[c,not[c]]],not[or[c,c]]]]`

Proof

We start by taking Substitution Lemma 95, and apply the substitution:

`or[c,not[or[c,not[c]]]]→c`

which follows from Substitution Lemma 82.

Critical Pair Lemma 78

The following expressions are equivalent:

`not[or[c,not[or[d,not[or[d,d]]]]]]==not[or[c,not[or[not[or[d,d]],or[d,not[or[c,not[or`

Proof

Note that the input for the rule:

`not[or[c,not[or[x1_,or[d,not[or[c,not[or[d,x1_]]]]]]]]]→not[or[c,not[or[d,x1]]]]`

contains a subpattern of the form:

`not[or[d,x1_]]`

which can be unified with the input for the rule:

`not[or[d,not[or[d,d]]]]→not[or[c,not[c]]]`

where these rules follow from Substitution Lemma 39 and Substitution Lemma 94 respectively.

Substitution Lemma 97

It can be shown that:

`not[or[c,not[or[c,not[c]]]]]==not[or[c,not[or[not[or[d,d]],or[d,not[or[c,not[or[c,not`

Proof

We start by taking Critical Pair Lemma 78, and apply the substitution:

`not[or[d,not[or[d,d]]]]→not[or[c,not[c]]]`

which follows from Substitution Lemma 94.

Substitution Lemma 98

It can be shown that:

`not[c]==not[or[c,not[or[not[or[d,d]],or[d,not[or[c,not[or[c,not[c]]]]]]]]]]`

Proof

We start by taking Substitution Lemma 97, and apply the substitution:

`or[c,not[or[c,not[c]]]]→c`

which follows from Substitution Lemma 82.

Substitution Lemma 99

It can be shown that:

`not[c]==not[or[c,not[or[not[or[d,d]],or[d,not[c]]]]]]`

Proof

We start by taking Substitution Lemma 98, and apply the substitution:

`or[c,not[or[c,not[c]]]]→c`

which follows from Substitution Lemma 82.

Substitution Lemma 100

It can be shown that:

`not[d]==not[or[d,not[or[not[d],or[c,not[c]]]]]]`

Proof

We start by taking Critical Pair Lemma 73, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Substitution Lemma 101

It can be shown that:

`not[d]==not[or[d,not[or[not[c],or[c,not[d]]]]]]`

Proof

We start by taking Substitution Lemma 100, and apply the substitution:

`or[x1_,or[x2_,x3_]]→or[x3,or[x2,x1]]`

which follows from Critical Pair Lemma 40.

Substitution Lemma 102

It can be shown that:

`not[d]==not[or[d,not[or[c,or[not[c],not[d]]]]]]`

Proof

We start by taking Substitution Lemma 101, and apply the substitution:

`or[x1_,or[x2_,x3_]]→or[x2,or[x1,x3]]`

which follows from Substitution Lemma 1.

Substitution Lemma 103

It can be shown that:

`or[c,c]==not[or[not[or[c,c]],not[or[c,not[c]]]]]`

Proof

We start by taking Substitution Lemma 96, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Substitution Lemma 104

It can be shown that:

`or[c,c]==c`

Proof

We start by taking Substitution Lemma 103, and apply the substitution:

`not[or[not[or[x1_,x2_]],not[or[x2_,not[x1_]]]]]→x2`

which follows from Critical Pair Lemma 4.

Critical Pair Lemma 79

The following expressions are equivalent:

`or[c,or[c,x1]]==or[c,x1]`

Proof

Note that the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[c,c]→c`

where these rules follow from Axiom 3 and Substitution Lemma 104 respectively.

Critical Pair Lemma 80

The following expressions are equivalent:

`or[c,or[x1,x2]]==or[c,or[x2,or[x1,c]]]`

Proof

Note that the input for the rule:

`or[c,or[c,x1_]]→or[c,x1]`

contains a subpattern of the form:

`or[c,x1_]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,x3_]]↔or[x3_,or[x2_,x1_]]`

where these rules follow from Critical Pair Lemma 79 and Critical Pair Lemma 40 respectively.

Critical Pair Lemma 81

The following expressions are equivalent:

`or[c,or[x1,x2]]==or[c,or[x1,or[c,x2]]]`

Proof

Note that the input for the rule:

`or[c,or[c,x1_]]→or[c,x1]`

contains a subpattern of the form:

`or[c,x1_]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,x3_]]↔or[x2_,or[x1_,x3_]]`

where these rules follow from Critical Pair Lemma 79 and Substitution Lemma 1 respectively.

Critical Pair Lemma 82

The following expressions are equivalent:

`or[c,x1]==not[or[not[or[c,x1]],not[or[not[c],or[c,x1]]]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[not[x1_],x2_]]]]→x2`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[c,or[c,x1_]]→or[c,x1]`

where these rules follow from Critical Pair Lemma 23 and Critical Pair Lemma 79 respectively.

Critical Pair Lemma 83

The following expressions are equivalent:

`or[c,or[or[x1,or[x2,c]],x3]]==or[or[c,or[x2,x1]],x3]`

Proof

Note that the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[c,or[x1_,or[x2_,c]]]→or[c,or[x2,x1]]`

where these rules follow from Axiom 3 and Critical Pair Lemma 80 respectively.

Substitution Lemma 105

It can be shown that:

`or[c,or[x1,or[or[x2,c],x3]]]==or[or[c,or[x2,x1]],x3]`

Proof

We start by taking Critical Pair Lemma 83, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Substitution Lemma 106

It can be shown that:

`or[c,or[x1,or[x2,or[c,x3]]]]==or[or[c,or[x2,x1]],x3]`

Proof

We start by taking Substitution Lemma 105, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Substitution Lemma 107

It can be shown that:

`or[c,or[x1,or[x2,or[c,x3]]]]==or[c,or[or[x2,x1],x3]]`

Proof

We start by taking Substitution Lemma 106, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Substitution Lemma 108

It can be shown that:

`or[c,or[x1,or[x2,or[c,x3]]]]==or[c,or[x2,or[x1,x3]]]`

Proof

We start by taking Substitution Lemma 107, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Critical Pair Lemma 84

The following expressions are equivalent:

`not[or[d,or[not[c],not[d]]]]==not[or[not[or[d,not[or[d,or[not[c],not[d]]]]]],d]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[not[x1_],x2_]]]]→x2`

contains a subpattern of the form:

`not[or[not[x1_],x2_]]`

which can be unified with the input for the rule:

`not[or[not[d],not[or[d,or[not[c],not[d]]]]]]→d`

where these rules follow from Critical Pair Lemma 23 and Substitution Lemma 88 respectively.

Substitution Lemma 109

It can be shown that:

`not[or[d,or[not[c],not[d]]]]==not[or[not[d],d]]`

Proof

We start by taking Critical Pair Lemma 84, and apply the substitution:

`not[or[d,not[or[d,or[not[c],not[d]]]]]]→not[d]`

which follows from Substitution Lemma 20.

Substitution Lemma 110

It can be shown that:

`not[or[d,or[not[c],not[d]]]]==not[or[d,not[d]]]`

Proof

We start by taking Substitution Lemma 109, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Substitution Lemma 111

It can be shown that:

`not[or[d,or[not[c],not[d]]]]==not[or[c,not[c]]]`

Proof

We start by taking Substitution Lemma 110, and apply the substitution:

`not[or[d,not[d]]]→not[or[c,not[c]]]`

which follows from Substitution Lemma 80.

Critical Pair Lemma 85

The following expressions are equivalent:

`or[d,not[d]]==not[or[not[or[c,or[d,not[d]]]],not[or[c,not[c]]]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,or[x2_,x3_]]],not[or[x2_,or[not[x1_],x3_]]]]]→or[x2,x3]`

contains a subpattern of the form:

`not[or[x2_,or[not[x1_],x3_]]]`

which can be unified with the input for the rule:

`not[or[d,or[not[c],not[d]]]]→not[or[c,not[c]]]`

where these rules follow from Critical Pair Lemma 46 and Substitution Lemma 111 respectively.

Substitution Lemma 112

It can be shown that:

`or[d,not[d]]==not[or[not[or[d,not[d]]],not[or[c,not[c]]]]]`

Proof

We start by taking Critical Pair Lemma 85, and apply the substitution:

`or[c,or[d,x1_]]→or[d,x1]`

which follows from Critical Pair Lemma 8.

Substitution Lemma 113

It can be shown that:

```
or[d,not[d]]==not[or[not[or[c,not[c]]],not[or[c,not[c]]]]]
```

Proof

We start by taking Substitution Lemma 112, and apply the substitution:

```
not[or[d,not[d]]]→not[or[c,not[c]]]
```

which follows from Substitution Lemma 80.

Critical Pair Lemma 86

The following expressions are equivalent:

```
or[c,not[c]]==not[or[not[or[c,not[c]]],not[or[not[c],or[c,not[or[d,not[d]]]]]]]]
```

Proof

Note that the input for the rule:

```
not[or[not[or[d,or[x1_,x2_]]],not[or[x1_,or[c,not[or[d,x2_]]]]]]]→or[c,x1]
```

contains a subpattern of the form:

```
not[or[d,or[x1_,x2_]]]
```

which can be unified with the input for the rule:

```
not[or[d,or[not[c],not[d]]]]→not[or[c,not[c]]]
```

where these rules follow from Critical Pair Lemma 56 and Substitution Lemma 111 respectively.

Substitution Lemma 114

It can be shown that:

```
or[c,not[c]]==not[or[not[or[c,not[c]]],not[or[not[c],or[c,not[or[c,not[c]]]]]]]]
```

Proof

We start by taking Critical Pair Lemma 86, and apply the substitution:

```
not[or[d,not[d]]]→not[or[c,not[c]]]
```

which follows from Substitution Lemma 80.

Substitution Lemma 115

It can be shown that:

```
or[c,not[c]]==not[or[not[or[c,not[c]]],not[or[not[c],c]]]]
```

Proof

We start by taking Substitution Lemma 114, and apply the substitution:

```
or[c,not[or[c,not[c]]]]→c
```

which follows from Substitution Lemma 82.

Substitution Lemma 116

It can be shown that:

```
or[c,not[c]]==not[or[not[or[c,not[c]]],not[or[c,not[c]]]]]
```

Proof

We start by taking Substitution Lemma 115, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Substitution Lemma 117

It can be shown that:

`or[c,not[c]]==or[d,not[d]]`

Proof

We start by taking Substitution Lemma 116, and apply the substitution:

`not[or[not[or[c,not[c]]],not[or[c,not[c]]]]]→or[d,not[d]]`

which follows from Substitution Lemma 113.

Substitution Lemma 118

It can be shown that:

`not[or[not[or[c,not[c]]],not[or[c,not[c]]]]]→or[c,not[c]]`

Proof

We start by taking Substitution Lemma 113, and apply the substitution:

`or[d,not[d]]→or[c,not[c]]`

which follows from Substitution Lemma 117.

Critical Pair Lemma 87

The following expressions are equivalent:

`or[d,or[not[d],x1]]==or[or[c,not[c]],x1]`

Proof

Note that the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[d,not[d]]→or[c,not[c]]`

where these rules follow from Axiom 3 and Substitution Lemma 117 respectively.

Substitution Lemma 119

It can be shown that:

`or[d,or[not[d],x1]]==or[c,or[not[c],x1]]`

Proof

We start by taking Critical Pair Lemma 87, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Critical Pair Lemma 88

The following expressions are equivalent:

`or[d,or[x1,not[d]]]==or[c,or[x1,or[c,not[c]]]]`

Proof

Note that the input for the rule:

`or[c,or[x1_,or[d,x2_]]]→or[d,or[x1,x2]]`

contains a subpattern of the form:

`or[d,x2_]`

which can be unified with the input for the rule:

`or[d,not[d]]→or[c,not[c]]`

where these rules follow from Substitution Lemma 4 and Substitution Lemma 117 respectively.

Substitution Lemma 120

It can be shown that:

`or[d,or[x1,not[d]]]==or[c,or[x1,not[c]]]`

Proof

We start by taking Critical Pair Lemma 88, and apply the substitution:

`or[c,or[x1_,or[c,x2_]]]→or[c,or[x1,x2]]`

which follows from Critical Pair Lemma 81.

Critical Pair Lemma 89

The following expressions are equivalent:

`or[x1,c]==not[or[not[or[not[d],or[d,x1]]],not[or[x1,or[c,not[or[c,not[c]]]]]]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,or[d,x2_]]],not[or[x2_,or[c,not[or[d,x1_]]]]]]]→or[x2,c]`

contains a subpattern of the form:

`or[d,x1_]`

which can be unified with the input for the rule:

`or[d,not[d]]→or[c,not[c]]`

where these rules follow from Critical Pair Lemma 58 and Substitution Lemma 117 respectively.

Substitution Lemma 121

It can be shown that:

`or[x1,c]==not[or[not[or[not[d],or[d,x1]]],not[or[x1,c]]]]`

Proof

We start by taking Critical Pair Lemma 89, and apply the substitution:

`or[c,not[or[c,not[c]]]]→c`

which follows from Substitution Lemma 82.

Critical Pair Lemma 90

The following expressions are equivalent:

`or[c,or[not[c],or[x1,x2]]]==or[d,or[x2,or[x1,not[d]]]]`

PROOF

Note that the input for the rule:

`or[d,or[not[d],x1_]]→or[c,or[not[c],x1]]`

contains a subpattern of the form:

`or[not[d],x1_]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,x3_]]↔or[x3_,or[x2_,x1_]]`

where these rules follow from Substitution Lemma 119 and Critical Pair Lemma 40 respectively.

Critical Pair Lemma 91

The following expressions are equivalent:

`or[c,or[not[c],or[x1,x2]]]==or[d,or[x1,or[not[d],x2]]]`

PROOF

Note that the input for the rule:

`or[d,or[not[d],x1_]]→or[c,or[not[c],x1]]`

contains a subpattern of the form:

`or[not[d],x1_]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,x3_]]↔or[x2_,or[x1_,x3_]]`

where these rules follow from Substitution Lemma 119 and Substitution Lemma 1 respectively.

Critical Pair Lemma 92

The following expressions are equivalent:

`d==not[or[not[or[c,d]],not[or[c,or[not[c],not[or[c,not[d]]]]]]]]`

PROOF

Note that the input for the rule:

`not[or[not[or[c,x1_]],not[or[x1_,or[not[d],not[or[c,not[d]]]]]]]]→x1`

contains a subpattern of the form:

`or[x1_,or[not[d],not[or[c,not[d]]]]]`

which can be unified with the input for the rule:

`or[d,or[not[d],x1_]]→or[c,or[not[c],x1]]`

where these rules follow from Critical Pair Lemma 51 and Substitution Lemma 119 respectively.

Substitution Lemma 122

It can be shown that:

`d==not[or[not[d],not[or[c,or[not[c],not[or[c,not[d]]]]]]]]`

PROOF

We start by taking Critical Pair Lemma 92, and apply the substitution:

```
or[c,d]→d
```

which follows from Axiom 4.

Substitution Lemma 123

It can be shown that:

```
not[or[c,or[not[c],not[c]]]]==not[or[c,not[c]]]
```

Proof

We start by taking Substitution Lemma 111, and apply the substitution:

```
or[d,or[x1_,not[d]]]→or[c,or[x1,not[c]]]
```

which follows from Substitution Lemma 120.

Critical Pair Lemma 93

The following expressions are equivalent:

```
d==not[or[not[or[or[not[c],not[c]],d]],not[or[d,not[or[c,not[c]]]]]]]
```

Proof

Note that the input for the rule:

```
not[or[not[or[x1_,d]],not[or[d,not[or[c,x1_]]]]]]→d
```

contains a subpattern of the form:

```
not[or[c,x1_]]
```

which can be unified with the input for the rule:

```
not[or[c,or[not[c],not[c]]]]→not[or[c,not[c]]]
```

where these rules follow from Critical Pair Lemma 38 and Substitution Lemma 123 respectively.

Substitution Lemma 124

It can be shown that:

```
d==not[or[not[or[not[c],or[not[c],d]]],not[or[d,not[or[c,not[c]]]]]]]
```

Proof

We start by taking Critical Pair Lemma 93, and apply the substitution:

```
or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]
```

which follows from Axiom 3.

Substitution Lemma 125

It can be shown that:

```
d==not[or[not[or[not[c],or[not[c],d]]],not[d]]]
```

Proof

We start by taking Substitution Lemma 124, and apply the substitution:

```
or[d,not[or[c,not[c]]]]→d
```

which follows from Substitution Lemma 84.

Critical Pair Lemma 94

The following expressions are equivalent:

The following expressions are equivalent:

```
not[or[d,not[or[c,or[not[c],not[c]]]]]]==not[or[d,not[or[or[not[c],not[c]],or[d,not[or
```

PROOF

Note that the input for the rule:

```
not[or[d,not[or[x1_,or[d,not[or[d,not[or[c,x1_]]]]]]]]]→not[or[d,not[or[c,x1]]]]
```

contains a subpattern of the form:

```
not[or[c,x1_]]
```

which can be unified with the input for the rule:

```
not[or[c,or[not[c],not[c]]]]→not[or[c,not[c]]]
```

where these rules follow from Substitution Lemma 42 and Substitution Lemma 123 respectively.

Substitution Lemma 126

It can be shown that:

```
not[or[d,not[or[c,not[c]]]]]==not[or[d,not[or[or[not[c],not[c]],or[d,not[or[d,not[or[
```

PROOF

We start by taking Critical Pair Lemma 94, and apply the substitution:

```
not[or[c,or[not[c],not[c]]]]→not[or[c,not[c]]]
```

which follows from Substitution Lemma 123.

Substitution Lemma 127

It can be shown that:

```
not[d]==not[or[d,not[or[or[not[c],not[c]],or[d,not[or[d,not[or[c,not[c]]]]]]]]]]
```

PROOF

We start by taking Substitution Lemma 126, and apply the substitution:

```
or[d,not[or[c,not[c]]]]→d
```

which follows from Substitution Lemma 84.

Substitution Lemma 128

It can be shown that:

```
not[d]==not[or[d,not[or[not[c],or[not[c],or[d,not[or[d,not[or[c,not[c]]]]]]]]]]]
```

PROOF

We start by taking Substitution Lemma 127, and apply the substitution:

```
or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]
```

which follows from Axiom 3.

Substitution Lemma 129

It can be shown that:

```
not[d]==not[or[d,not[or[not[c],or[not[c],or[d,not[d]]]]]]]
```

PROOF

We start by taking Substitution Lemma 128, and apply the substitution:

```
or[d,not[or[c,not[c]]]]→d
```

`or[d,not[or[c,not[c]]]]→d`

which follows from Substitution Lemma 84.

Substitution Lemma 130

It can be shown that:

`not[d]==not[or[d,not[or[not[c],or[not[c],or[c,not[c]]]]]]]`

Proof

We start by taking Substitution Lemma 129, and apply the substitution:

`or[d,not[d]]→or[c,not[c]]`

which follows from Substitution Lemma 117.

Critical Pair Lemma 95

The following expressions are equivalent:

`or[c,or[not[c],or[x1,or[x2,x3]]]]==or[d,or[x2,or[x3,or[x1,not[d]]]]]`

Proof

Note that the input for the rule:

`or[d,or[x1_,or[x2_,not[d]]]]→or[c,or[not[c],or[x2,x1]]]`

contains a subpattern of the form:

`or[x1_,or[x2_,not[d]]]`

which can be unified with the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

where these rules follow from Critical Pair Lemma 90 and Axiom 3 respectively.

Substitution Lemma 131

It can be shown that:

`d==not[or[not[d],not[or[not[c],or[not[c],d]]]]]`

Proof

We start by taking Substitution Lemma 125, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Substitution Lemma 132

It can be shown that:

`d==not[or[not[d],not[or[d,or[not[c],not[c]]]]]]`

Proof

We start by taking Substitution Lemma 131, and apply the substitution:

`or[x1_,or[x2_,x3_]]→or[x3,or[x2,x1]]`

which follows from Critical Pair Lemma 40.

Substitution Lemma 133

It can be shown that:

`not[c]==not[or[c,not[or[not[c],or[d,not[or[d,d]]]]]]]`

Proof

PROOF

We start by taking Substitution Lemma 99, and apply the substitution:

`or[x1_,or[x2_,x3_]]→or[x3,or[x2,x1]]`

which follows from Critical Pair Lemma 40.

Substitution Lemma 134

It can be shown that:

`not[c]==not[or[c,not[or[d,or[not[c],not[or[d,d]]]]]]]`

PROOF

We start by taking Substitution Lemma 133, and apply the substitution:

`or[x1_,or[x2_,x3_]]→or[x2,or[x1,x3]]`

which follows from Substitution Lemma 1.

Critical Pair Lemma 96

The following expressions are equivalent:

`c==not[or[not[or[c,or[d,or[not[c],not[or[d,d]]]]]],not[c]]]`

PROOF

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[x1_,not[x2_]]]]]→x1`

contains a subpattern of the form:

`not[or[x1_,not[x2_]]]`

which can be unified with the input for the rule:

`not[or[c,not[or[d,or[not[c],not[or[d,d]]]]]]]→not[c]`

where these rules follow from Axiom 1 and Substitution Lemma 134 respectively.

Substitution Lemma 135

It can be shown that:

`c==not[or[not[or[d,or[not[c],not[or[d,d]]]]],not[c]]]`

PROOF

We start by taking Critical Pair Lemma 96, and apply the substitution:

`or[c,or[d,x1_]]→or[d,x1]`

which follows from Critical Pair Lemma 8.

Substitution Lemma 136

It can be shown that:

`or[c,x1]==not[or[not[or[c,x1]],not[or[c,or[not[c],x1]]]]]`

PROOF

We start by taking Critical Pair Lemma 82, and apply the substitution:

`or[x1_,or[x2_,x3_]]→or[x2,or[x1,x3]]`

which follows from Substitution Lemma 1.

Substitution Lemma 137

It can be shown that:

It can be shown that:

`or[x1,c]==not[or[not[or[x1,c]],not[or[not[d],or[d,x1]]]]]`

PROOF

We start by taking Substitution Lemma 121, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Substitution Lemma 138

It can be shown that:

`or[x1,c]==not[or[not[or[x1,c]],not[or[d,or[not[d],x1]]]]]`

PROOF

We start by taking Substitution Lemma 137, and apply the substitution:

`or[x1_,or[x2_,x3_]]→or[x2,or[x1,x3]]`

which follows from Substitution Lemma 1.

Substitution Lemma 139

It can be shown that:

`or[x1,c]==not[or[not[or[x1,c]],not[or[c,or[not[c],x1]]]]]`

PROOF

We start by taking Substitution Lemma 138, and apply the substitution:

`or[d,or[not[d],x1_]]→or[c,or[not[c],x1]]`

which follows from Substitution Lemma 119.

Substitution Lemma 140

It can be shown that:

`not[d]==not[or[d,not[or[c,or[not[c],or[not[c],not[d]]]]]]]`

PROOF

We start by taking Substitution Lemma 91, and apply the substitution:

`or[d,or[x1_,or[not[d],x2_]]]→or[c,or[not[c],or[x1,x2]]]`

which follows from Critical Pair Lemma 91.

Critical Pair Lemma 97

The following expressions are equivalent:

`d==not[or[not[or[d,or[c,or[not[c],or[not[c],not[d]]]]]],not[d]]]`

PROOF

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[x1_,not[x2_]]]]]→x1`

contains a subpattern of the form:

`not[or[x1_,not[x2_]]]`

which can be unified with the input for the rule:

`not[or[d,not[or[c,or[not[c],or[not[c],not[d]]]]]]]→not[d]`

where these rules follow from Axiom 1 and Substitution Lemma 140 respectively.

where these rules follow from Axiom 1 and Substitution Lemma 140 respectively.

Substitution Lemma 141

It can be shown that:

`d==not[or[not[or[c,or[not[c],or[not[c],or[c,not[c]]]]]],not[d]]]`

Proof

We start by taking Critical Pair Lemma 97, and apply the substitution:

`or[d,or[x1_,or[x2_,or[x3_,not[d]]]]]->or[c,or[not[c],or[x3,or[x1,x2]]]]`

which follows from Critical Pair Lemma 95.

Substitution Lemma 142

It can be shown that:

`d==not[or[not[or[c,or[not[c],or[not[c],not[c]]]]],not[d]]]`

Proof

We start by taking Substitution Lemma 141, and apply the substitution:

`or[c,or[x1_,or[x2_,or[c,x3_]]]]->or[c,or[x2,or[x1,x3]]]`

which follows from Substitution Lemma 108.

Substitution Lemma 143

It can be shown that:

`not[d]==not[or[d,not[or[not[c],or[c,or[not[c],not[c]]]]]]]`

Proof

We start by taking Substitution Lemma 130, and apply the substitution:

`or[x1_,or[x2_,x3_]]->or[x2,or[x1,x3]]`

which follows from Substitution Lemma 1.

Substitution Lemma 144

It can be shown that:

`not[d]==not[or[d,not[or[c,or[not[c],or[not[c],not[c]]]]]]]`

Proof

We start by taking Substitution Lemma 143, and apply the substitution:

`or[x1_,or[x2_,x3_]]->or[x2,or[x1,x3]]`

which follows from Substitution Lemma 1.

Substitution Lemma 145

It can be shown that:

`c==not[or[not[c],not[or[d,or[not[c],not[or[d,d]]]]]]]`

Proof

We start by taking Substitution Lemma 135, and apply the substitution:

`or[x1_,x2_]->or[x2,x1]`

which follows from Axiom 2.

Critical Pair Lemma 98

The following expressions are equivalent:

`not[or[d,or[not[c],not[or[d,d]]]]]==not[or[not[or[c,not[or[d,or[not[c],not[or[d,d]]]]]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[not[x1_],x2_]]]]→x2`

contains a subpattern of the form:

`not[or[not[x1_],x2_]]`

which can be unified with the input for the rule:

`not[or[not[c],not[or[d,or[not[c],not[or[d,d]]]]]]]→c`

where these rules follow from Critical Pair Lemma 23 and Substitution Lemma 145 respectively.

Substitution Lemma 146

It can be shown that:

`not[or[d,or[not[c],not[or[d,d]]]]]==not[or[not[c],c]]`

Proof

We start by taking Critical Pair Lemma 98, and apply the substitution:

`not[or[c,not[or[d,or[not[c],not[or[d,d]]]]]]]→not[c]`

which follows from Substitution Lemma 134.

Substitution Lemma 147

It can be shown that:

`not[or[d,or[not[c],not[or[d,d]]]]]==not[or[c,not[c]]]`

Proof

We start by taking Substitution Lemma 146, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Critical Pair Lemma 99

The following expressions are equivalent:

`or[d,not[or[d,d]]]==not[or[not[or[c,or[d,not[or[d,d]]]]],not[or[c,not[c]]]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,or[x2_,x3_]]],not[or[x2_,or[not[x1_],x3_]]]]]→or[x2,x3]`

contains a subpattern of the form:

`not[or[x2_,or[not[x1_],x3_]]]`

which can be unified with the input for the rule:

`not[or[d,or[not[c],not[or[d,d]]]]]→not[or[c,not[c]]]`

where these rules follow from Critical Pair Lemma 46 and Substitution Lemma 147 respectively.

Substitution Lemma 148

Substitution Lemma 148

It can be shown that:

`or[d,not[or[d,d]]]==not[or[not[or[d,not[or[d,d]]]],not[or[c,not[c]]]]]`

Proof

We start by taking Critical Pair Lemma 99, and apply the substitution:

`or[c,or[d,x1_]]→or[d,x1]`

which follows from Critical Pair Lemma 8.

Substitution Lemma 149

It can be shown that:

`or[d,not[or[d,d]]]==not[or[not[or[c,not[c]]],not[or[c,not[c]]]]]`

Proof

We start by taking Substitution Lemma 148, and apply the substitution:

`not[or[d,not[or[d,d]]]]→not[or[c,not[c]]]`

which follows from Substitution Lemma 94.

Substitution Lemma 150

It can be shown that:

`or[d,not[or[d,d]]]==or[c,not[c]]`

Proof

We start by taking Substitution Lemma 149, and apply the substitution:

`not[or[not[or[c,not[c]]],not[or[c,not[c]]]]]→or[c,not[c]]`

which follows from Substitution Lemma 118.

Critical Pair Lemma 100

The following expressions are equivalent:

`or[d,or[not[or[d,d]],x1]]==or[or[c,not[c]],x1]`

Proof

Note that the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[d,not[or[d,d]]]→or[c,not[c]]`

where these rules follow from Axiom 3 and Substitution Lemma 150 respectively.

Substitution Lemma 151

It can be shown that:

`or[d,or[not[or[d,d]],x1]]==or[c,or[not[c],x1]]`

Proof

We start by taking Critical Pair Lemma 100, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Critical Pair Lemma 101

The following expressions are equivalent:

`not[or[d,d]]==not[or[d,not[or[c,or[not[c],not[or[c,d]]]]]]]`

Proof

Note that the input for the rule:

`not[or[d,not[or[d,or[not[or[x1_,d]],not[or[c,x1_]]]]]]]→not[or[x1,d]]`

contains a subpattern of the form:

`or[d,or[not[or[x1_,d]],not[or[c,x1_]]]]`

which can be unified with the input for the rule:

`or[d,or[not[or[d,d]],x1_]]→or[c,or[not[c],x1]]`

where these rules follow from Substitution Lemma 41 and Substitution Lemma 151 respectively.

Substitution Lemma 152

It can be shown that:

`not[or[d,d]]==not[or[d,not[or[c,or[not[c],not[d]]]]]]`

Proof

We start by taking Critical Pair Lemma 101, and apply the substitution:

`or[c,d]→d`

which follows from Axiom 4.

Substitution Lemma 153

It can be shown that:

`not[or[d,d]]==not[d]`

Proof

We start by taking Substitution Lemma 152, and apply the substitution:

`not[or[d,not[or[c,or[not[c],not[d]]]]]]→not[d]`

which follows from Substitution Lemma 102.

Critical Pair Lemma 102

The following expressions are equivalent:

`x1==not[or[not[or[or[d,d],x1]],not[or[not[d],x1]]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[not[x1_],x2_]]]]→x2`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[or[d,d]]→not[d]`

where these rules follow from Critical Pair Lemma 23 and Substitution Lemma 153 respectively.

Substitution Lemma 154

It can be shown that:

```
x1==not[or[not[or[d,or[d,x1]]],not[or[not[d],x1]]]]
```

Proof

We start by taking Critical Pair Lemma 102, and apply the substitution:

```
or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]
```

which follows from Axiom 3.

Substitution Lemma 155

It can be shown that:

```
x1==not[or[not[or[not[d],x1]],not[or[d,or[d,x1]]]]]
```

Proof

We start by taking Substitution Lemma 154, and apply the substitution:

```
or[x1_,x2_]→or[x2,x1]
```

which follows from Axiom 2.

Critical Pair Lemma 103

The following expressions are equivalent:

```
not[or[d,or[not[c],or[d,not[or[d,d]]]]]]==not[or[not[or[not[d],not[or[d,or[not[c],or[(
```

Proof

Note that the input for the rule:

```
not[or[not[or[not[d],x1_]],not[or[d,or[d,x1_]]]]]→x1
```

contains a subpattern of the form:

```
not[or[d,or[d,x1_]]]
```

which can be unified with the input for the rule:

```
not[or[d,or[x1_,not[or[d,or[not[c],or[x1_,not[or[d,x1_]]]]]]]]]→not[or[d,x1]]
```

where these rules follow from Substitution Lemma 155 and Substitution Lemma 46 respectively.

Substitution Lemma 156

It can be shown that:

```
not[or[d,or[not[c],or[d,not[d]]]]]==not[or[not[or[not[d],not[or[d,or[not[c],or[d,not[(
```

Proof

We start by taking Critical Pair Lemma 103, and apply the substitution:

```
not[or[d,d]]→not[d]
```

which follows from Substitution Lemma 153.

Substitution Lemma 157

It can be shown that:

```
not[or[d,or[not[c],or[c,not[c]]]]]==not[or[not[or[not[d],not[or[d,or[not[c],or[d,not[(
```

`not[or[d,or[not[c],or[c,not[c]]]]]==not[or[not[or[not[d],not[or[d,or[not[c],or[d,not[`

Proof

We start by taking Substitution Lemma 156, and apply the substitution:

`or[d,not[d]]→or[c,not[c]]`

which follows from Substitution Lemma 117.

Substitution Lemma 158

It can be shown that:

`not[or[d,or[not[c],not[c]]]]==not[or[not[or[not[d],not[or[d,or[not[c],or[d,not[or[d,d]`

Proof

We start by taking Substitution Lemma 157, and apply the substitution:

`or[d,or[x1_,or[c,x2_]]]→or[d,or[x1,x2]]`

which follows from Substitution Lemma 9.

Substitution Lemma 159

It can be shown that:

`not[or[d,or[not[c],not[c]]]]==not[or[not[or[not[d],not[or[d,or[not[c],or[d,not[d]]]]]]`

Proof

We start by taking Substitution Lemma 158, and apply the substitution:

`not[or[d,d]]→not[d]`

which follows from Substitution Lemma 153.

Substitution Lemma 160

It can be shown that:

`not[or[d,or[not[c],not[c]]]]==not[or[not[or[not[d],not[or[d,or[not[c],or[c,not[c]]]]]]`

Proof

We start by taking Substitution Lemma 159, and apply the substitution:

`or[d,not[d]]→or[c,not[c]]`

which follows from Substitution Lemma 117.

Substitution Lemma 161

It can be shown that:

`not[or[d,or[not[c],not[c]]]]==not[or[not[or[not[d],not[or[d,or[not[c],not[c]]]]]],not`

Proof

We start by taking Substitution Lemma 160, and apply the substitution:

`or[d,or[x1_,or[c,x2_]]]→or[d,or[x1,x2]]`

which follows from Substitution Lemma 9.

Substitution Lemma 162

It can be shown that:

`not[or[d,or[not[c],not[c]]]]==not[or[d,not[or[d,d]]]]`

Proof

We start by taking Substitution Lemma 161, and apply the substitution:

`not[or[not[d],not[or[d,or[not[c],not[c]]]]]]→d`

which follows from Substitution Lemma 132.

Substitution Lemma 163

It can be shown that:

`not[or[d,or[not[c],not[c]]]]==not[or[d,not[d]]]`

Proof

We start by taking Substitution Lemma 162, and apply the substitution:

`not[or[d,d]]→not[d]`

which follows from Substitution Lemma 153.

Substitution Lemma 164

It can be shown that:

`not[or[d,or[not[c],not[c]]]]==not[or[c,not[c]]]`

Proof

We start by taking Substitution Lemma 163, and apply the substitution:

`or[d,not[d]]→or[c,not[c]]`

which follows from Substitution Lemma 117.

Substitution Lemma 165

It can be shown that:

`d==not[or[not[d],not[or[c,or[not[c],or[not[c],not[c]]]]]]]`

Proof

We start by taking Substitution Lemma 142, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Critical Pair Lemma 104

The following expressions are equivalent:

`not[or[c,or[not[c],or[not[c],not[c]]]]]==not[or[not[or[d,not[or[c,or[not[c],or[not[c]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[not[x1_],x2_]]]]→x2`

contains a subpattern of the form:

`not[or[not[x1_],x2_]]`

which can be unified with the input for the rule:

`not[or[not[d],not[or[c,or[not[c],or[not[c],not[c]]]]]]]→d`

where these rules follow from Critical Pair Lemma 23 and Substitution Lemma 165 respectively.

Substitution Lemma 166

It can be shown that:

`not[or[c,or[not[c],or[not[c],not[c]]]]]==not[or[not[d],d]]`

Proof

We start by taking Critical Pair Lemma 104, and apply the substitution:

`not[or[d,not[or[c,or[not[c],or[not[c],not[c]]]]]]]→not[d]`

which follows from Substitution Lemma 144.

Substitution Lemma 167

It can be shown that:

`not[or[c,or[not[c],or[not[c],not[c]]]]]==not[or[d,not[d]]]`

Proof

We start by taking Substitution Lemma 166, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Substitution Lemma 168

It can be shown that:

`not[or[c,or[not[c],or[not[c],not[c]]]]]==not[or[c,not[c]]]`

Proof

We start by taking Substitution Lemma 167, and apply the substitution:

`or[d,not[d]]→or[c,not[c]]`

which follows from Substitution Lemma 117.

Critical Pair Lemma 105

The following expressions are equivalent:

`or[c,or[not[c],not[c]]]==not[or[not[or[c,or[not[c],not[c]]]],not[or[c,not[c]]]]]`

Proof

Note that the input for the rule:

`not[or[not[or[c,x1_]],not[or[c,or[not[c],x1_]]]]]→or[c,x1]`

contains a subpattern of the form:

`not[or[c,or[not[c],x1_]]]`

which can be unified with the input for the rule:

`not[or[c,or[not[c],or[not[c],not[c]]]]]→not[or[c,not[c]]]`

where these rules follow from Substitution Lemma 136 and Substitution Lemma 168 respectively.

Substitution Lemma 169

It can be shown that:

`or[c,or[not[c],not[c]]]==not[or[not[or[c,not[c]]],not[or[c,not[c]]]]]`

Proof

We start by taking Critical Pair Lemma 105, and apply the substitution:

```
not[or[c,or[not[c],not[c]]]]→not[or[c,not[c]]]
```

which follows from Substitution Lemma 123.

Substitution Lemma 170

It can be shown that:

```
or[c,or[not[c],not[c]]]==or[c,not[c]]
```

Proof

We start by taking Substitution Lemma 169, and apply the substitution:

```
not[or[not[or[c,not[c]]],not[or[c,not[c]]]]]→or[c,not[c]]
```

which follows from Substitution Lemma 118.

Critical Pair Lemma 106

The following expressions are equivalent:

```
or[c,or[or[not[c],not[c]],x1]]==or[or[c,not[c]],x1]
```

Proof

Note that the input for the rule:

```
or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]
```

contains a subpattern of the form:

```
or[x1_,x2_]
```

which can be unified with the input for the rule:

```
or[c,or[not[c],not[c]]]→or[c,not[c]]
```

where these rules follow from Axiom 3 and Substitution Lemma 170 respectively.

Substitution Lemma 171

It can be shown that:

```
or[c,or[not[c],or[not[c],x1]]]==or[or[c,not[c]],x1]
```

Proof

We start by taking Critical Pair Lemma 106, and apply the substitution:

```
or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]
```

which follows from Axiom 3.

Substitution Lemma 172

It can be shown that:

```
or[c,or[not[c],or[not[c],x1]]]==or[c,or[not[c],x1]]
```

Proof

We start by taking Substitution Lemma 171, and apply the substitution:

```
or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]
```

which follows from Axiom 3.

Critical Pair Lemma 107

The following expressions are equivalent:

```
or[c,or[or[not[c],not[c]],d]]==not[or[not[or[c,or[d,or[not[c],not[c]]]]],not[or[d,or[
```

Proof

Note that the input for the rule:

`not[or[not[or[x1_,or[d,x2_]]],not[or[d,or[not[c],or[x1_,x2_]]]]]]→or[x1,or[x2,d]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[c,or[not[c],not[c]]]→or[c,not[c]]`

where these rules follow from Substitution Lemma 34 and Substitution Lemma 170 respectively.

Substitution Lemma 173

It can be shown that:

`or[d,or[not[c],not[c]]]==not[or[not[or[c,or[d,or[not[c],not[c]]]]],not[or[d,or[not[c]`

Proof

We start by taking Critical Pair Lemma 107, and apply the substitution:

`or[c,or[x1_,d]]→or[d,x1]`

which follows from Critical Pair Lemma 10.

Substitution Lemma 174

It can be shown that:

`or[d,or[not[c],not[c]]]==not[or[not[or[d,or[not[c],not[c]]]],not[or[d,or[not[c],or[c,`

Proof

We start by taking Substitution Lemma 173, and apply the substitution:

`or[c,or[d,x1_]]→or[d,x1]`

which follows from Critical Pair Lemma 8.

Substitution Lemma 175

It can be shown that:

`or[d,or[not[c],not[c]]]==not[or[not[or[c,not[c]]],not[or[d,or[not[c],or[c,not[c]]]]]]]`

Proof

We start by taking Substitution Lemma 174, and apply the substitution:

`not[or[d,or[not[c],not[c]]]]→not[or[c,not[c]]]`

which follows from Substitution Lemma 164.

Substitution Lemma 176

It can be shown that:

`or[d,or[not[c],not[c]]]==not[or[not[or[c,not[c]]],not[or[d,or[not[c],not[c]]]]]]`

Proof

We start by taking Substitution Lemma 175, and apply the substitution:

`or[d,or[x1_,or[c,x2_]]]→or[d,or[x1,x2]]`

which follows from Substitution Lemma 9.

Substitution Lemma 177

Substitution Lemma 177

It can be shown that:

`or[d,or[not[c],not[c]]]==not[or[not[or[c,not[c]]],not[or[c,not[c]]]]]`

Proof

We start by taking Substitution Lemma 176, and apply the substitution:

`not[or[d,or[not[c],not[c]]]]→not[or[c,not[c]]]`

which follows from Substitution Lemma 164.

Substitution Lemma 178

It can be shown that:

`or[d,or[not[c],not[c]]]==or[c,not[c]]`

Proof

We start by taking Substitution Lemma 177, and apply the substitution:

`not[or[not[or[c,not[c]]],not[or[c,not[c]]]]]→or[c,not[c]]`

which follows from Substitution Lemma 118.

Critical Pair Lemma 108

The following expressions are equivalent:

`or[c,or[not[c],d]]==or[d,or[not[c],not[c]]]`

Proof

Note that the input for the rule:

`or[c,or[not[c],or[not[c],x1_]]]→or[c,or[not[c],x1]]`

contains a subpattern of the form:

`or[c,or[not[c],or[not[c],x1_]]]`

which can be unified with the input for the rule:

`or[c,or[x1_,or[x2_,d]]]→or[d,or[x1,x2]]`

where these rules follow from Substitution Lemma 172 and Critical Pair Lemma 12 respectively.

Substitution Lemma 179

It can be shown that:

`or[d,not[c]]==or[d,or[not[c],not[c]]]`

Proof

We start by taking Critical Pair Lemma 108, and apply the substitution:

`or[c,or[x1_,d]]→or[d,x1]`

which follows from Critical Pair Lemma 10.

Substitution Lemma 180

It can be shown that:

`or[d,not[c]]==or[c,not[c]]`

Proof

We start by taking Substitution Lemma 179, and apply the substitution:

We start by taking Substitution Lemma 179, and apply the substitution:

`or[d,or[not[c],not[c]]]→or[c,not[c]]`

which follows from Substitution Lemma 178.

Critical Pair Lemma 109

The following expressions are equivalent:

`or[d,or[not[c],x1]]⩵or[or[c,not[c]],x1]`

Proof

Note that the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[d,not[c]]→or[c,not[c]]`

where these rules follow from Axiom 3 and Substitution Lemma 180 respectively.

Substitution Lemma 181

It can be shown that:

`or[d,or[not[c],x1]]⩵or[c,or[not[c],x1]]`

Proof

We start by taking Critical Pair Lemma 109, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Critical Pair Lemma 110

The following expressions are equivalent:

`or[c,or[not[c],not[d]]]⩵or[c,or[not[c],not[c]]]`

Proof

Note that the input for the rule:

`or[d,or[not[c],x1_]]→or[c,or[not[c],x1]]`

contains a subpattern of the form:

`or[d,or[not[c],x1_]]`

which can be unified with the input for the rule:

`or[d,or[x1_,not[d]]]→or[c,or[x1,not[c]]]`

where these rules follow from Substitution Lemma 181 and Substitution Lemma 120 respectively.

Substitution Lemma 182

It can be shown that:

`or[c,or[not[c],not[d]]]⩵or[c,not[c]]`

Proof

We start by taking Critical Pair Lemma 110, and apply the substitution:

`or[c,or[not[c],not[c]]]→or[c,not[c]]`

which follows from Substitution Lemma 170.

Substitution Lemma 183

It can be shown that:

`not[or[not[d],not[or[c,or[not[c],not[or[c,not[d]]]]]]]]==or[d,not[or[c,not[d]]]]`

Proof

We start by taking Substitution Lemma 70, and apply the substitution:

`or[d,or[not[c],x1_]]→or[c,or[not[c],x1]]`

which follows from Substitution Lemma 181.

Substitution Lemma 184

It can be shown that:

`d==or[d,not[or[c,not[d]]]]`

Proof

We start by taking Substitution Lemma 183, and apply the substitution:

`not[or[not[d],not[or[c,or[not[c],not[or[c,not[d]]]]]]]]→d`

which follows from Substitution Lemma 122.

Critical Pair Lemma 111

The following expressions are equivalent:

`or[not[d],c]==not[or[not[or[not[d],c]],not[or[c,not[c]]]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,c]],not[or[c,or[not[c],x1_]]]]]→or[x1,c]`

contains a subpattern of the form:

`or[c,or[not[c],x1_]]`

which can be unified with the input for the rule:

`or[c,or[not[c],not[d]]]→or[c,not[c]]`

where these rules follow from Substitution Lemma 139 and Substitution Lemma 182 respectively.

Critical Pair Lemma 112

The following expressions are equivalent:

`or[d,or[not[or[c,not[d]]],x1]]==or[d,x1]`

Proof

Note that the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[d,not[or[c,not[d]]]]→d`

where these rules follow from Axiom 3 and Substitution Lemma 184 respectively.

Substitution Lemma 185

It can be shown that:

```
or[c,not[d]]==not[or[not[or[not[d],c]],not[or[c,not[c]]]]]
```

Proof

We start by taking Critical Pair Lemma 111, and apply the substitution:

```
or[x1_,x2_]→or[x2,x1]
```

which follows from Axiom 2.

Substitution Lemma 186

It can be shown that:

```
or[c,not[d]]==not[or[not[or[c,not[c]]],not[or[not[d],c]]]]
```

Proof

We start by taking Substitution Lemma 185, and apply the substitution:

```
or[x1_,x2_]→or[x2,x1]
```

which follows from Axiom 2.

Substitution Lemma 187

It can be shown that:

```
or[c,not[d]]==not[or[not[or[c,not[c]]],not[or[c,not[d]]]]]
```

Proof

We start by taking Substitution Lemma 186, and apply the substitution:

```
or[x1_,x2_]→or[x2,x1]
```

which follows from Axiom 2.

Critical Pair Lemma 113

The following expressions are equivalent:

```
not[or[c,not[c]]]==not[or[not[or[or[c,not[d]],not[or[c,not[c]]]]],or[c,not[d]]]]
```

Proof

Note that the input for the rule:

```
not[or[not[or[x1_,x2_]],not[or[x2_,not[x1_]]]]]→x2
```

contains a subpattern of the form:

```
not[or[x2_,not[x1_]]]
```

which can be unified with the input for the rule:

```
not[or[not[or[c,not[c]]],not[or[c,not[d]]]]]→or[c,not[d]]
```

where these rules follow from Critical Pair Lemma 4 and Substitution Lemma 187 respectively.

Substitution Lemma 188

It can be shown that:

```
not[or[c,not[c]]]==not[or[not[or[c,or[not[d],not[or[c,not[c]]]]]],or[c,not[d]]]]
```

Proof

We start by taking Critical Pair Lemma 113, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Substitution Lemma 189

It can be shown that:

`not[or[c,not[c]]]==not[or[not[or[not[d],c]],or[c,not[d]]]]`

Proof

We start by taking Substitution Lemma 188, and apply the substitution:

`or[c,or[x1_,not[or[c,not[c]]]]]→or[x1,c]`

which follows from Critical Pair Lemma 71.

Substitution Lemma 190

It can be shown that:

`not[or[c,not[c]]]==not[or[not[d],or[c,not[or[not[d],c]]]]]`

Proof

We start by taking Substitution Lemma 189, and apply the substitution:

`or[x1_,or[x2_,x3_]]→or[x3,or[x2,x1]]`

which follows from Critical Pair Lemma 40.

Substitution Lemma 191

It can be shown that:

`not[or[c,not[c]]]==not[or[c,or[not[d],not[or[not[d],c]]]]]`

Proof

We start by taking Substitution Lemma 190, and apply the substitution:

`or[x1_,or[x2_,x3_]]→or[x2,or[x1,x3]]`

which follows from Substitution Lemma 1.

Substitution Lemma 192

It can be shown that:

`not[or[c,not[c]]]==not[or[c,or[not[d],not[or[c,not[d]]]]]]`

Proof

We start by taking Substitution Lemma 191, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Critical Pair Lemma 114

The following expressions are equivalent:

`or[not[d],not[or[c,not[d]]]]==not[or[not[or[c,not[c]]],not[or[not[c],or[not[d],not[or`

Proof

Note that the input for the rule:

```
not[or[not[or[x1_,x2_]],not[or[not[x1_],x2_]]]]→x2
```

contains a subpattern of the form:

```
not[or[x1_,x2_]]
```

which can be unified with the input for the rule:

```
not[or[c,or[not[d],not[or[c,not[d]]]]]]→not[or[c,not[c]]]
```

where these rules follow from Critical Pair Lemma 23 and Substitution Lemma 192 respectively.

Substitution Lemma 193

It can be shown that:

```
or[not[d],not[or[c,not[d]]]]==not[c]
```

Proof

We start by taking Critical Pair Lemma 114, and apply the substitution:

```
not[or[not[or[c,x1_]],not[or[x1_,or[not[d],not[or[c,not[d]]]]]]]]→x1
```

which follows from Critical Pair Lemma 51.

Critical Pair Lemma 115

The following expressions are equivalent:

```
or[not[d],or[not[or[c,not[d]]],x1]]==or[not[c],x1]
```

Proof

Note that the input for the rule:

```
or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]
```

contains a subpattern of the form:

```
or[x1_,x2_]
```

which can be unified with the input for the rule:

```
or[not[d],not[or[c,not[d]]]]→not[c]
```

where these rules follow from Axiom 3 and Substitution Lemma 193 respectively.

Critical Pair Lemma 116

The following expressions are equivalent:

```
or[not[or[c,not[d]]],x1]==not[or[not[or[d,or[not[or[c,not[d]]],x1]]],not[or[not[c],x1
```

Proof

Note that the input for the rule:

```
not[or[not[or[x1_,x2_]],not[or[not[x1_],x2_]]]]→x2
```

contains a subpattern of the form:

```
or[not[x1_],x2_]
```

which can be unified with the input for the rule:

```
or[not[d],or[not[or[c,not[d]]],x1_]]→or[not[c],x1]
```

where these rules follow from Critical Pair Lemma 23 and Critical Pair Lemma 115 respectively.

Substitution Lemma 194

Substitution Lemma 194

It can be shown that:

`or[not[or[c,not[d]]],x1]==not[or[not[or[d,x1]],not[or[not[c],x1]]]]`

PROOF

We start by taking Critical Pair Lemma 116, and apply the substitution:

`or[d,or[not[or[c,not[d]]],x1_]]→or[d,x1]`

which follows from Critical Pair Lemma 112.

Critical Pair Lemma 117

The following expressions are equivalent:

`or[not[or[c,not[d]]],not[or[c,not[c]]]]==not[or[not[d],not[or[not[c],not[or[c,not[c]]`

PROOF

Note that the input for the rule:

`not[or[not[or[d,x1_]],not[or[not[c],x1_]]]]→or[not[or[c,not[d]]],x1]`

contains a subpattern of the form:

`or[d,x1_]`

which can be unified with the input for the rule:

`or[d,not[or[c,not[c]]]]→d`

where these rules follow from Substitution Lemma 194 and Substitution Lemma 84 respectively.

Substitution Lemma 195

It can be shown that:

`or[not[or[c,not[d]]],not[or[c,not[c]]]]==not[or[not[d],c]]`

PROOF

We start by taking Critical Pair Lemma 117, and apply the substitution:

`not[or[not[c],not[or[c,not[c]]]]]→c`

which follows from Substitution Lemma 83.

Substitution Lemma 196

It can be shown that:

`or[not[or[c,not[c]]],not[or[c,not[d]]]]==not[or[not[d],c]]`

PROOF

We start by taking Substitution Lemma 195, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Substitution Lemma 197

It can be shown that:

`or[not[or[c,not[c]]],not[or[c,not[d]]]]==not[or[c,not[d]]]`

PROOF

We start by taking Substitution Lemma 196, and apply the substitution:

We start by taking Substitution Lemma 196, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Critical Pair Lemma 118

The following expressions are equivalent:

`or[x1,not[or[c,not[c]]]]==not[or[not[or[x1,or[or[c,not[d]],not[or[c,not[c]]]]]],not[o`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,or[x2_,x3_]]],not[or[x1_,or[x3_,not[x2_]]]]]]→or[x1,x3]`

contains a subpattern of the form:

`or[x3_,not[x2_]]`

which can be unified with the input for the rule:

`or[not[or[c,not[c]]],not[or[c,not[d]]]]→not[or[c,not[d]]]`

where these rules follow from Substitution Lemma 33 and Substitution Lemma 197 respectively.

Substitution Lemma 198

It can be shown that:

`or[x1,not[or[c,not[c]]]]==not[or[not[or[x1,or[c,or[not[d],not[or[c,not[c]]]]]]],not[o`

Proof

We start by taking Critical Pair Lemma 118, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Substitution Lemma 199

It can be shown that:

`or[x1,not[or[c,not[c]]]]==not[or[not[or[x1,or[not[d],c]]],not[or[x1,not[or[c,not[d]]]`

Proof

We start by taking Substitution Lemma 198, and apply the substitution:

`or[c,or[x1_,not[or[c,not[c]]]]]→or[x1,c]`

which follows from Critical Pair Lemma 71.

Substitution Lemma 200

It can be shown that:

`or[x1,not[or[c,not[c]]]]==x1`

Proof

We start by taking Substitution Lemma 199, and apply the substitution:

`not[or[not[or[x1_,or[x2_,x3_]]],not[or[x1_,not[or[x3_,x2_]]]]]]→x1`

which follows from Critical Pair Lemma 54.

Critical Pair Lemma 119

The following expressions are equivalent:

The following expressions are equivalent:

`not[or[c,not[c]]]==not[or[not[x1],not[or[not[or[x2,x1]],or[not[or[not[x2],x1]],not[or`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[not[or[x3_,x1_]],or[not[or[not[x3_],x1_]],x2_]]]]]→x2`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,not[or[c,not[c]]]]→x1`

where these rules follow from Substitution Lemma 49 and Substitution Lemma 200 respectively.

Substitution Lemma 201

It can be shown that:

`not[or[c,not[c]]]==not[or[not[x1],not[or[not[or[x2,x1]],not[or[not[x2],x1]]]]]]`

Proof

We start by taking Critical Pair Lemma 119, and apply the substitution:

`or[x1_,not[or[c,not[c]]]]→x1`

which follows from Substitution Lemma 200.

Substitution Lemma 202

It can be shown that:

`not[or[c,not[c]]]==not[or[not[x1],x1]]`

Proof

We start by taking Substitution Lemma 201, and apply the substitution:

`not[or[not[or[x1_,x2_]],not[or[not[x1_],x2_]]]]→x2`

which follows from Critical Pair Lemma 23.

Substitution Lemma 203

It can be shown that:

`not[or[c,not[c]]]==not[or[x1,not[x1]]]`

Proof

We start by taking Substitution Lemma 202, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Substitution Lemma 204

It can be shown that:

`or[x1,not[or[b,not[b]]]]==x1`

Proof

We start by taking Substitution Lemma 200, and apply the substitution:

`not[or[c,not[c]]]→not[or[b,not[b]]]`

which follows from Substitution Lemma 203.

Critical Pair Lemma 120

The following expressions are equivalent:

`x1==or[not[or[b,not[b]]],x1]`

Proof

Note that the input for the rule:

`or[x1_,not[or[b,not[b]]]]→x1`

contains a subpattern of the form:

`or[x1_,not[or[b,not[b]]]]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Substitution Lemma 204 and Axiom 2 respectively.

Critical Pair Lemma 121

The following expressions are equivalent:

`not[or[b,not[b]]]==not[or[not[x1],not[or[not[or[x2,x1]],or[not[or[not[x2],x1]],not[or`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[not[or[x3_,x1_]],or[not[or[not[x3_],x1_]],x2_]]]]]→x2`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,not[or[b,not[b]]]]→x1`

where these rules follow from Substitution Lemma 49 and Substitution Lemma 204 respectively.

Substitution Lemma 205

It can be shown that:

`not[or[b,not[b]]]==not[or[not[x1],not[or[not[or[x2,x1]],not[or[not[x2],x1]]]]]]`

Proof

We start by taking Critical Pair Lemma 121, and apply the substitution:

`or[x1_,not[or[b,not[b]]]]→x1`

which follows from Substitution Lemma 204.

Substitution Lemma 206

It can be shown that:

`not[or[b,not[b]]]==not[or[not[x1],x1]]`

Proof

We start by taking Substitution Lemma 205, and apply the substitution:

`not[or[not[or[x1_,x2_]],not[or[not[x1_],x2_]]]]→x2`

which follows from Critical Pair Lemma 23.

which follows from Critical Pair Lemma 23.

Substitution Lemma 207

It can be shown that:

`not[or[b,not[b]]]==not[or[x1,not[x1]]]`

Proof

We start by taking Substitution Lemma 206, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Critical Pair Lemma 122

The following expressions are equivalent:

`x1==not[or[not[or[x1,x1]],not[or[b,not[b]]]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[x1_,not[x2_]]]]]→x1`

contains a subpattern of the form:

`not[or[x1_,not[x2_]]]`

which can be unified with the input for the rule:

`not[or[b,not[b]]]↔not[or[x1_,not[x1_]]]`

where these rules follow from Axiom 1 and Substitution Lemma 207 respectively.

Substitution Lemma 208

It can be shown that:

`x1==not[not[or[x1,x1]]]`

Proof

We start by taking Critical Pair Lemma 122, and apply the substitution:

`or[x1_,not[or[b,not[b]]]]→x1`

which follows from Substitution Lemma 204.

Critical Pair Lemma 123

The following expressions are equivalent:

`not[not[x1]]==not[or[not[or[x1,not[not[x1]]]],not[or[b,not[b]]]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[not[x1_],x2_]]]]→x2`

contains a subpattern of the form:

`not[or[not[x1_],x2_]]`

which can be unified with the input for the rule:

`not[or[b,not[b]]]↔not[or[x1_,not[x1_]]]`

where these rules follow from Critical Pair Lemma 23 and Substitution Lemma 207 respectively.

Substitution Lemma 209

It can be shown that:

`not[not[x1]]==not[not[or[x1,not[not[x1]]]]]`

Proof

We start by taking Critical Pair Lemma 123, and apply the substitution:

`or[x1_,not[or[b,not[b]]]]→x1`

which follows from Substitution Lemma 204.

Critical Pair Lemma 124

The following expressions are equivalent:

`not[or[not[or[b,not[b]]],x1]]==not[not[or[not[or[not[or[b,not[b]]],x1]],not[not[or[no`

Proof

Note that the input for the rule:

`not[or[x1_,not[or[not[or[x1_,x2_]],not[not[or[x1_,not[x2_]]]]]]]]→not[or[x1,x2]]`

contains a subpattern of the form:

`or[x1_,not[or[not[or[x1_,x2_]],not[not[or[x1_,not[x2_]]]]]]]`

which can be unified with the input for the rule:

`or[not[or[b,not[b]]],x1_]→x1`

where these rules follow from Critical Pair Lemma 1 and Critical Pair Lemma 120 respectively.

Substitution Lemma 210

It can be shown that:

`not[x1]==not[not[or[not[or[not[or[b,not[b]]],x1]],not[not[or[not[or[b,not[b]]],not[x1`

Proof

We start by taking Critical Pair Lemma 124, and apply the substitution:

`or[not[or[b,not[b]]],x1_]→x1`

which follows from Critical Pair Lemma 120.

Substitution Lemma 211

It can be shown that:

`not[x1]==not[not[or[not[x1],not[not[or[not[or[b,not[b]]],not[x1]]]]]]]`

Proof

We start by taking Substitution Lemma 210, and apply the substitution:

`or[not[or[b,not[b]]],x1_]→x1`

which follows from Critical Pair Lemma 120.

Substitution Lemma 212

It can be shown that:

`not[x1]==not[not[or[not[x1],not[not[not[x1]]]]]]`

Proof

We start by taking Substitution Lemma 211, and apply the substitution:

`or[not[or[b,not[b]]],x1_]→x1`

which follows from Critical Pair Lemma 120.

Substitution Lemma 213

It can be shown that:

`not[x1]==not[not[not[x1]]]`

Proof

We start by taking Substitution Lemma 212, and apply the substitution:

`not[not[or[x1_,not[not[x1_]]]]]→not[not[x1]]`

which follows from Substitution Lemma 209.

Critical Pair Lemma 125

The following expressions are equivalent:

`not[not[or[x1,x1]]]==not[not[x1]]`

Proof

Note that the input for the rule:

`not[not[not[x1_]]]→not[x1]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[or[x1_,x1_]]]→x1`

where these rules follow from Substitution Lemma 213 and Substitution Lemma 208 respectively.

Substitution Lemma 214

It can be shown that:

`x1==not[not[x1]]`

Proof

We start by taking Critical Pair Lemma 125, and apply the substitution:

`not[not[or[x1_,x1_]]]→x1`

which follows from Substitution Lemma 208.

Critical Pair Lemma 126

The following expressions are equivalent:

`or[not[or[x1,x2]],not[or[not[x1],x2]]]==not[x2]`

Proof

Note that the input for the rule:

`not[not[x1_]]→x1`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[not[x1_],x2_]]]]→x2`

where these rules follow from Substitution Lemma 214 and Critical Pair Lemma 23 respectively.

Substitution Lemma 215

It can be shown that:

```
or[not[or[b,not[a]]],not[or[not[a],not[b]]]]==a
```

Proof

We start by taking Hypothesis 1, and apply the substitution:

```
or[x1_,x2_]→or[x2,x1]
```

which follows from Axiom 2.

Substitution Lemma 216

It can be shown that:

```
or[not[or[b,not[a]]],not[or[not[b],not[a]]]]==a
```

Proof

We start by taking Substitution Lemma 215, and apply the substitution:

```
or[x1_,x2_]→or[x2,x1]
```

which follows from Axiom 2.

Substitution Lemma 217

It can be shown that:

```
not[not[a]]==a
```

Proof

We start by taking Substitution Lemma 216, and apply the substitution:

```
or[not[or[x1_,x2_]],not[or[not[x1_],x2_]]]→not[x2]
```

which follows from Critical Pair Lemma 126.

Conclusion 1

We obtain the conclusion:

```
True
```

Proof

Take Substitution Lemma 217, and apply the substitution:

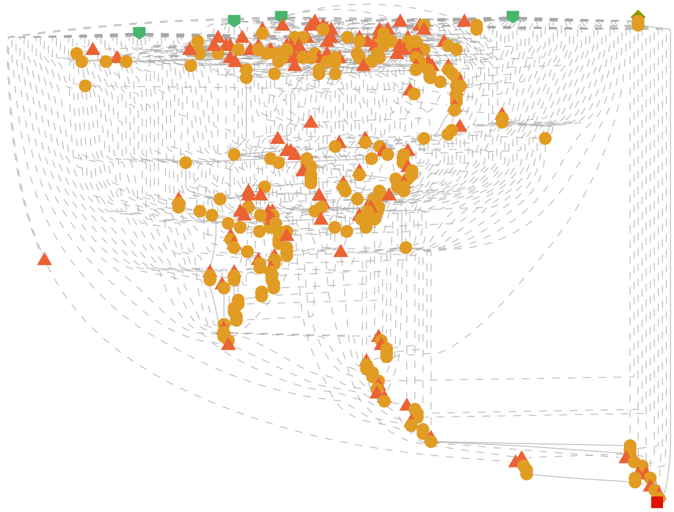
```
not[not[x1_]]→x1
```

which follows from Substitution Lemma 214.

large output | show less | show more | show all | set size limit...

In[◦]:=
```
proofAxH3fromRobbinsw1["ProofGraph"]
```

Out[•]=



In[•]:= Clear[proofAxH3fromRobbinsw1]

Appendix 6. Derivation of Wolfram logic from Wolfram “short” logic

In[ ]:=
```
proofWolframfromShort["ProofNotebook"]
```

Axiom 1

We are given that:

`x1==nand[nand[x1,x1],nand[x1,x2]]`

Axiom 2

We are given that:

`nand[x1,nand[x1,x2]]==nand[x1,nand[x2,x2]]`

Axiom 3

We are given that:

`nand[x1,nand[x1,nand[x2,x3]]]==nand[x2,nand[x2,nand[x1,x3]]]`

Hypothesis 1

We would like to show that:

`nand[nand[nand[a,b],c],nand[a,nand[nand[a,c],a]]]==c`

Critical Pair Lemma 1

The following expressions are equivalent:

`nand[x1,x1]==nand[x1,nand[nand[x1,x1],x2]]`

Proof

Note that the input for the rule:

`nand[nand[x1_,x1_],nand[x1_,x2_]]→x1`

contains a subpattern of the form:

`nand[x1_,x1_]`

which can be unified with the input for the rule:

`nand[nand[x1_,x1_],nand[x1_,x2_]]→x1`

where these rules follow from Axiom 1 and Axiom 1 respectively.

Critical Pair Lemma 2

The following expressions are equivalent:

`nand[x1,nand[x1,nand[x2,x2]]]==nand[x1,x2]`

Proof

Note that the input for the rule:

`nand[x1_,nand[x1_,x2_]]↔nand[x1_,nand[x2_,x2_]]`

contains a subpattern of the form:

`nand[x2_,x2_]`

which can be unified with the input for the rule:

nand[nand[x1_,x1_],nand[x1_,x2_]]→x1

where these rules follow from Axiom 2 and Axiom 1 respectively.

Critical Pair Lemma 3

The following expressions are equivalent:

nand[x1,nand[nand[x2,x2],nand[x2,x2]]]==nand[x1,nand[x1,nand[x1,x2]]]

Proof

Note that the input for the rule:

nand[x1_,nand[x1_,x2_]]↔nand[x1_,nand[x2_,x2_]]

contains a subpattern of the form:

nand[x1_,x2_]

which can be unified with the input for the rule:

nand[x1_,nand[x1_,x2_]]↔nand[x1_,nand[x2_,x2_]]

where these rules follow from Axiom 2 and Axiom 2 respectively.

Substitution Lemma 1

It can be shown that:

nand[x1,x2]==nand[x1,nand[x1,nand[x1,x2]]]

Proof

We start by taking Critical Pair Lemma 3, and apply the substitution:

nand[nand[x1_,x1_],nand[x1_,x2_]]→x1

which follows from Axiom 1.

Critical Pair Lemma 4

The following expressions are equivalent:

nand[x1,x2]==nand[x2,nand[x2,nand[x1,x2]]]

Proof

Note that the input for the rule:

nand[x1_,nand[x1_,nand[x2_,x2_]]]→nand[x1,x2]

contains a subpattern of the form:

nand[x1_,nand[x1_,nand[x2_,x2_]]]

which can be unified with the input for the rule:

nand[x1_,nand[x1_,nand[x2_,x3_]]]↔nand[x2_,nand[x2_,nand[x1_,x3_]]]

where these rules follow from Critical Pair Lemma 2 and Axiom 3 respectively.

Critical Pair Lemma 5

The following expressions are equivalent:

nand[x1,nand[x2,x3]]==nand[x1,nand[x2,nand[x2,nand[x1,x3]]]]

Proof

Note that the input for the rule:

nand[x1_,nand[x1_,nand[x1_,x2_]]]→nand[x1,x2]

nand[x1_,nand[x1_,nand[x1_,x2_]]]→nand[x1,x2]

contains a subpattern of the form:

nand[x1_,nand[x1_,x2_]]

which can be unified with the input for the rule:

nand[x1_,nand[x1_,nand[x2_,x3_]]]↔nand[x2_,nand[x2_,nand[x1_,x3_]]]

where these rules follow from Substitution Lemma 1 and Axiom 3 respectively.

Critical Pair Lemma 6

The following expressions are equivalent:

nand[nand[x1,x1],x1]==nand[x1,nand[x1,x1]]

Proof

Note that the input for the rule:

nand[x1_,nand[x1_,nand[x2_,x1_]]]→nand[x2,x1]

contains a subpattern of the form:

nand[x1_,nand[x2_,x1_]]

which can be unified with the input for the rule:

nand[x1_,nand[nand[x1_,x1_],x2_]]→nand[x1,x1]

where these rules follow from Critical Pair Lemma 4 and Critical Pair Lemma 1 respectively.

Critical Pair Lemma 7

The following expressions are equivalent:

nand[x1,nand[nand[x1,x1],x2]]==nand[nand[nand[x1,x1],x2],nand[nand[nand[x1,x1],x2],nand[

Proof

Note that the input for the rule:

nand[x1_,nand[x1_,nand[x2_,x1_]]]→nand[x2,x1]

contains a subpattern of the form:

nand[x2_,x1_]

which can be unified with the input for the rule:

nand[x1_,nand[nand[x1_,x1_],x2_]]→nand[x1,x1]

where these rules follow from Critical Pair Lemma 4 and Critical Pair Lemma 1 respectively.

Substitution Lemma 2

It can be shown that:

nand[x1,x1]==nand[nand[nand[x1,x1],x2],nand[nand[nand[x1,x1],x2],nand[x1,x1]]]

Proof

We start by taking Critical Pair Lemma 7, and apply the substitution:

nand[x1_,nand[nand[x1_,x1_],x2_]]→nand[x1,x1]

which follows from Critical Pair Lemma 1.

Substitution Lemma 3

It can be shown that:

nand[x1,x1]==nand[nand[nand[x1,x1],x2],x1]

PROOF

We start by taking Substitution Lemma 2, and apply the substitution:

`nand[x1_,nand[x1_,nand[x2_,x2_]]]→nand[x1,x2]`

which follows from Critical Pair Lemma 2.

Critical Pair Lemma 8

The following expressions are equivalent:

`x1==nand[nand[x1,x1],nand[x2,x1]]`

PROOF

Note that the input for the rule:

`nand[nand[x1_,x1_],nand[x1_,x2_]]→x1`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,nand[x1_,nand[x2_,x1_]]]→nand[x2,x1]`

where these rules follow from Axiom 1 and Critical Pair Lemma 4 respectively.

Critical Pair Lemma 9

The following expressions are equivalent:

`nand[x1,x1]==nand[x1,nand[x2,nand[x1,x1]]]`

PROOF

Note that the input for the rule:

`nand[x1_,nand[nand[x1_,x1_],x2_]]→nand[x1,x1]`

contains a subpattern of the form:

`nand[nand[x1_,x1_],x2_]`

which can be unified with the input for the rule:

`nand[x1_,nand[x1_,nand[x2_,x1_]]]→nand[x2,x1]`

where these rules follow from Critical Pair Lemma 1 and Critical Pair Lemma 4 respectively.

Critical Pair Lemma 10

The following expressions are equivalent:

`nand[x1,nand[x1,nand[nand[x2,x2],x2]]]==nand[nand[x2,x2],x2]`

PROOF

Note that the input for the rule:

`nand[x1_,nand[x1_,nand[x2_,x3_]]]↔nand[x2_,nand[x2_,nand[x1_,x3_]]]`

contains a subpattern of the form:

`nand[x1_,nand[x2_,x3_]]`

which can be unified with the input for the rule:

`nand[nand[x1_,x1_],nand[x2_,x1_]]→x1`

where these rules follow from Axiom 3 and Critical Pair Lemma 8 respectively.

Substitution Lemma 4

It can be shown that:

`nand[x1,nand[x1,nand[x2,nand[x2,x2]]]]==nand[nand[x2,x2],x2]`

PROOF

We start by taking Critical Pair Lemma 10, and apply the substitution:

`nand[nand[x1_,x1_],x1_]→nand[x1,nand[x1,x1]]`

which follows from Critical Pair Lemma 6.

Substitution Lemma 5

It can be shown that:

`nand[x1,nand[x1,nand[x2,nand[x2,x2]]]]==nand[x2,nand[x2,x2]]`

PROOF

We start by taking Substitution Lemma 4, and apply the substitution:

`nand[nand[x1_,x1_],x1_]→nand[x1,nand[x1,x1]]`

which follows from Critical Pair Lemma 6.

Critical Pair Lemma 11

The following expressions are equivalent:

`nand[nand[x1,x1],nand[x1,x1]]==nand[nand[x1,x2],nand[x1,x1]]`

PROOF

Note that the input for the rule:

`nand[nand[nand[x1_,x1_],x2_],x1_]→nand[x1,x1]`

contains a subpattern of the form:

`nand[x1_,x1_]`

which can be unified with the input for the rule:

`nand[nand[x1_,x1_],nand[x1_,x2_]]→x1`

where these rules follow from Substitution Lemma 3 and Axiom 1 respectively.

Substitution Lemma 6

It can be shown that:

`x1==nand[nand[x1,x2],nand[x1,x1]]`

PROOF

We start by taking Critical Pair Lemma 11, and apply the substitution:

`nand[nand[x1_,x1_],nand[x2_,x1_]]→x1`

which follows from Critical Pair Lemma 8.

Critical Pair Lemma 12

The following expressions are equivalent:

`nand[x1,nand[x2,nand[x3,nand[x1,x1]]]]==nand[x1,nand[x2,nand[x2,nand[x1,x1]]]]`

PROOF

Note that the input for the rule:

`nand[x1_,nand[x2_,nand[x2_,nand[x1_,x3_]]]]→nand[x1,nand[x2,x3]]`

contains a subpattern of the form:

`nand[x1_,x3_]`

which can be unified with the input for the rule:

`nand[x1_,nand[x2_,nand[x1_,x1_]]]→nand[x1,x1]`

where these rules follow from Critical Pair Lemma 5 and Critical Pair Lemma 9 respectively.

Substitution Lemma 7

It can be shown that:

`nand[x1,nand[x2,nand[x3,nand[x1,x1]]]]==nand[x1,nand[x2,x1]]`

Proof

We start by taking Critical Pair Lemma 12, and apply the substitution:

`nand[x1_,nand[x2_,nand[x2_,nand[x1_,x3_]]]]→nand[x1,nand[x2,x3]]`

which follows from Critical Pair Lemma 5.

Critical Pair Lemma 13

The following expressions are equivalent:

`nand[x1,nand[x2,nand[x1,nand[x3,x3]]]]==nand[x1,nand[x2,nand[x2,nand[x1,x3]]]]`

Proof

Note that the input for the rule:

`nand[x1_,nand[x2_,nand[x2_,nand[x1_,x3_]]]]→nand[x1,nand[x2,x3]]`

contains a subpattern of the form:

`nand[x1_,x3_]`

which can be unified with the input for the rule:

`nand[x1_,nand[x1_,nand[x2_,x2_]]]→nand[x1,x2]`

where these rules follow from Critical Pair Lemma 5 and Critical Pair Lemma 2 respectively.

Substitution Lemma 8

It can be shown that:

`nand[x1,nand[x2,nand[x1,nand[x3,x3]]]]==nand[x1,nand[x2,x3]]`

Proof

We start by taking Critical Pair Lemma 13, and apply the substitution:

`nand[x1_,nand[x2_,nand[x2_,nand[x1_,x3_]]]]→nand[x1,nand[x2,x3]]`

which follows from Critical Pair Lemma 5.

Critical Pair Lemma 14

The following expressions are equivalent:

`nand[x1,nand[x2,nand[x1,nand[x2,x3]]]]==nand[x1,nand[x2,nand[x2,nand[x1,x3]]]]`

Proof

Note that the input for the rule:

`nand[x1_,nand[x2_,nand[x2_,nand[x1_,x3_]]]]→nand[x1,nand[x2,x3]]`

contains a subpattern of the form:

`nand[x2_,nand[x1_,x3_]]`

which can be unified with the input for the rule:

`nand[x1_,nand[x2_,nand[x2_,nand[x1_,x3_]]]]→nand[x1,nand[x2,x3]]`

where these rules follow from Critical Pair Lemma 5 and Critical Pair Lemma 5 respectively.

Substitution Lemma 9

It can be shown that:

`nand[x1,nand[x2,nand[x1,nand[x2,x3]]]]⩵nand[x1,nand[x2,x3]]`

Proof

We start by taking Critical Pair Lemma 14, and apply the substitution:

`nand[x1_,nand[x2_,nand[x2_,nand[x1_,x3_]]]]→nand[x1,nand[x2,x3]]`

which follows from Critical Pair Lemma 5.

Critical Pair Lemma 15

The following expressions are equivalent:

`x1⩵nand[nand[x1,x1],nand[x2,nand[x2,x2]]]`

Proof

Note that the input for the rule:

`nand[nand[x1_,x1_],nand[x1_,x2_]]→x1`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,nand[x1_,nand[x2_,nand[x2_,x2_]]]]→nand[x2,nand[x2,x2]]`

where these rules follow from Axiom 1 and Substitution Lemma 5 respectively.

Critical Pair Lemma 16

The following expressions are equivalent:

`nand[x1,x1]⩵nand[x1,nand[x2,nand[x2,x2]]]`

Proof

Note that the input for the rule:

`nand[x1_,nand[nand[x1_,x1_],x2_]]→nand[x1,x1]`

contains a subpattern of the form:

`nand[nand[x1_,x1_],x2_]`

which can be unified with the input for the rule:

`nand[x1_,nand[x1_,nand[x2_,nand[x2_,x2_]]]]→nand[x2,nand[x2,x2]]`

where these rules follow from Critical Pair Lemma 1 and Substitution Lemma 5 respectively.

Critical Pair Lemma 17

The following expressions are equivalent:

`nand[x1,x1]⩵nand[x1,nand[nand[x1,x2],x2]]`

Proof

Note that the input for the rule:

`nand[x1_,x1_]↔nand[x1_,nand[x2_,nand[x2_,x2_]]]` nand[x1_,x1_]↔nand[x1_,nand[x2_,nand[x2_,x2_]]]

contains a subpattern of the form:

nand[x1_,nand[x2_,nand[x2_,x2_]]]

which can be unified with the input for the rule:

nand[x1_,nand[x2_,nand[x2_,nand[x1_,x3_]]]]→nand[x1,nand[x2,x3]]

where these rules follow from Critical Pair Lemma 16 and Critical Pair Lemma 5 respectively.

Critical Pair Lemma 18

The following expressions are equivalent:

nand[nand[x1,x2],nand[x1,nand[x1,x2]]]==nand[nand[x1,x2],nand[x1,nand[nand[x1,x2],x2]]]

Proof

Note that the input for the rule:

nand[x1_,nand[x2_,nand[x3_,nand[x1_,x1_]]]]→nand[x1,nand[x2,x1]]

contains a subpattern of the form:

nand[x2_,nand[x3_,nand[x1_,x1_]]]

which can be unified with the input for the rule:

nand[x1_,nand[x2_,nand[x2_,nand[x1_,x3_]]]]→nand[x1,nand[x2,x3]]

where these rules follow from Substitution Lemma 7 and Critical Pair Lemma 5 respectively.

Substitution Lemma 10

It can be shown that:

Out[∘]= nand[nand[x1,x2],nand[x1,nand[x1,x2]]]==nand[nand[x1,x2],nand[x1,x1]]

Proof

We start by taking Critical Pair Lemma 18, and apply the substitution:

nand[x1_,nand[nand[x1_,x2_],x2_]]→nand[x1,x1]

which follows from Critical Pair Lemma 17.

Substitution Lemma 11

It can be shown that:

nand[nand[x1,x2],nand[x1,nand[x1,x2]]]==x1

Proof

We start by taking Substitution Lemma 10, and apply the substitution:

nand[nand[x1_,x2_],nand[x1_,x1_]]→x1

which follows from Substitution Lemma 6.

Critical Pair Lemma 19

The following expressions are equivalent:

nand[x1,nand[nand[x2,x2],x1]]==nand[x1,x2]

Proof

Note that the input for the rule:

nand[x1_,nand[x2_,nand[x3_,nand[x1_,x1_]]]]→nand[x1,nand[x2,x1]]

contains a subpattern of the form:

`nand[x2_,nand[x3_,nand[x1_,x1_]]]`

which can be unified with the input for the rule:

`nand[nand[x1_,x1_],nand[x2_,nand[x2_,x2_]]]→x1`

where these rules follow from Substitution Lemma 7 and Critical Pair Lemma 15 respectively.

Critical Pair Lemma 20

The following expressions are equivalent:

`x1==nand[nand[x1,x2],nand[x1,nand[x1,nand[nand[x2,x2],x1]]]]`

Proof

Note that the input for the rule:

`nand[nand[x1_,x2_],nand[x1_,nand[x1_,x2_]]]→x1`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,nand[nand[x2_,x2_],x1_]]→nand[x1,x2]`

where these rules follow from Substitution Lemma 11 and Critical Pair Lemma 19 respectively.

Substitution Lemma 12

It can be shown that:

`x1==nand[nand[x1,x2],nand[nand[x2,x2],x1]]`

Proof

We start by taking Critical Pair Lemma 20, and apply the substitution:

`nand[x1_,nand[x1_,nand[x2_,x1_]]]→nand[x2,x1]`

which follows from Critical Pair Lemma 4.

Critical Pair Lemma 21

The following expressions are equivalent:

`x1==nand[nand[x1,x2],nand[x1,nand[x1,nand[x1,nand[x2,x2]]]]]`

Proof

Note that the input for the rule:

`nand[nand[x1_,x2_],nand[x1_,nand[x1_,x2_]]]→x1`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,nand[x1_,nand[x2_,x2_]]]→nand[x1,x2]`

where these rules follow from Substitution Lemma 11 and Critical Pair Lemma 2 respectively.

Substitution Lemma 13

It can be shown that:

`x1==nand[nand[x1,x2],nand[x1,nand[x2,x2]]]`

Proof

We start by taking Critical Pair Lemma 21, and apply the substitution:

We start by taking Critical Pair Lemma 21, and apply the substitution:

```
nand[x1_,nand[x1_,nand[x1_,x2_]]]→nand[x1,x2]
```

which follows from Substitution Lemma 1.

Critical Pair Lemma 22

The following expressions are equivalent:

```
nand[x1,nand[x1,x2]]==nand[nand[x1,x2],x1]
```

Proof

Note that the input for the rule:

```
nand[x1_,nand[x1_,nand[x2_,x1_]]]→nand[x2,x1]
```

contains a subpattern of the form:

```
nand[x1_,nand[x2_,x1_]]
```

which can be unified with the input for the rule:

```
nand[nand[x1_,x2_],nand[x1_,nand[x1_,x2_]]]→x1
```

where these rules follow from Critical Pair Lemma 4 and Substitution Lemma 11 respectively.

Critical Pair Lemma 23

The following expressions are equivalent:

```
nand[x1,nand[x1,nand[x1,nand[x2,x1]]]]==nand[nand[x2,x1],x1]
```

Proof

Note that the input for the rule:

```
nand[nand[x1_,x2_],x1_]→nand[x1,nand[x1,x2]]
```

contains a subpattern of the form:

```
nand[x1_,x2_]
```

which can be unified with the input for the rule:

```
nand[x1_,nand[x1_,nand[x2_,x1_]]]→nand[x2,x1]
```

where these rules follow from Critical Pair Lemma 22 and Critical Pair Lemma 4 respectively.

Substitution Lemma 14

It can be shown that:

```
nand[x1,nand[x2,x1]]==nand[nand[x2,x1],x1]
```

Proof

We start by taking Critical Pair Lemma 23, and apply the substitution:

```
nand[x1_,nand[x1_,nand[x1_,x2_]]]→nand[x1,x2]
```

which follows from Substitution Lemma 1.

Critical Pair Lemma 24

The following expressions are equivalent:

```
nand[x1,nand[nand[x2,x1],x1]]==nand[x1,x2]
```

Proof

Note that the input for the rule:

```
nand[x1_,nand[x2_,nand[x3_,nand[x1_,x1_]]]]→nand[x1,nand[x2,x1]]
```

contains a subpattern of the form:

contains a subpattern of the form:

`nand[x2_,nand[x3_,nand[x1_,x1_]]]`

which can be unified with the input for the rule:

`nand[nand[x1_,x2_],nand[x1_,nand[x2_,x2_]]]→x1`

where these rules follow from Substitution Lemma 7 and Substitution Lemma 13 respectively.

Substitution Lemma 15

It can be shown that:

`nand[x1,nand[x1,nand[x2,x1]]]==nand[x1,x2]`

Proof

We start by taking Critical Pair Lemma 24, and apply the substitution:

`nand[nand[x1_,x2_],x2_]→nand[x2,nand[x1,x2]]`

which follows from Substitution Lemma 14.

Substitution Lemma 16

It can be shown that:

`nand[x1,x2]==nand[x2,x1]`

Proof

We start by taking Substitution Lemma 15, and apply the substitution:

`nand[x1_,nand[x1_,nand[x2_,x1_]]]→nand[x2,x1]`

which follows from Critical Pair Lemma 4.

Critical Pair Lemma 25

The following expressions are equivalent:

`x1==nand[nand[x2,x1],nand[nand[x2,x2],x1]]`

Proof

Note that the input for the rule:

`nand[nand[x1_,x2_],nand[nand[x2_,x2_],x1_]]→x1`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,x2_]↔nand[x2_,x1_]`

where these rules follow from Substitution Lemma 12 and Substitution Lemma 16 respectively.

Critical Pair Lemma 26

The following expressions are equivalent:

`nand[x1,nand[nand[x2,x3],nand[nand[x3,x3],x2]]]==nand[x1,nand[nand[x2,x3],nand[x1,x2]]]`

Proof

Note that the input for the rule:

`nand[x1_,nand[x2_,nand[x1_,nand[x2_,x3_]]]]→nand[x1,nand[x2,x3]]`

contains a subpattern of the form:

`nand[x2_,x3_]`

which can be unified with the input for the rule:

```
nand[nand[x1_,x2_],nand[nand[x2_,x2_],x1_]]→x1
```

where these rules follow from Substitution Lemma 9 and Substitution Lemma 12 respectively.

Substitution Lemma 17

It can be shown that:

```
nand[x1,x2]==nand[x1,nand[nand[x2,x3],nand[x1,x2]]]
```

Proof

We start by taking Critical Pair Lemma 26, and apply the substitution:

```
nand[nand[x1_,x2_],nand[nand[x2_,x2_],x1_]]→x1
```

which follows from Substitution Lemma 12.

Critical Pair Lemma 27

The following expressions are equivalent:

```
nand[x1,nand[nand[x2,x3],nand[nand[x2,x2],x3]]]==nand[x1,nand[nand[x2,x3],nand[x1,x3]]]
```

Proof

Note that the input for the rule:

```
nand[x1_,nand[x2_,nand[x1_,nand[x2_,x3_]]]]→nand[x1,nand[x2,x3]]
```

contains a subpattern of the form:

```
nand[x2_,x3_]
```

which can be unified with the input for the rule:

```
nand[nand[x1_,x2_],nand[nand[x1_,x1_],x2_]]→x2
```

where these rules follow from Substitution Lemma 9 and Critical Pair Lemma 25 respectively.

Substitution Lemma 18

It can be shown that:

```
nand[x1,x2]==nand[x1,nand[nand[x3,x2],nand[x1,x2]]]
```

Proof

We start by taking Critical Pair Lemma 27, and apply the substitution:

```
nand[nand[x1_,x2_],nand[nand[x1_,x1_],x2_]]→x2
```

which follows from Critical Pair Lemma 25.

Critical Pair Lemma 28

The following expressions are equivalent:

```
nand[x1,x2]==nand[x1,nand[nand[x1,x2],nand[x2,x3]]]
```

Proof

Note that the input for the rule:

```
nand[x1_,nand[nand[x2_,x3_],nand[x1_,x2_]]]→nand[x1,x2]
```

contains a subpattern of the form:

```
nand[nand[x2_,x3_],nand[x1_,x2_]]
```

which can be unified with the input for the rule:

```
nand[x1_,x2_]↔nand[x2_,x1_]
```

nand[x1_,x2_]↔nand[x2_,x1_]

where these rules follow from Substitution Lemma 17 and Substitution Lemma 16 respectively.

Critical Pair Lemma 29

The following expressions are equivalent:

nand[x1,x2]==nand[x1,nand[nand[x1,x2],nand[x3,x2]]]

Proof

Note that the input for the rule:

nand[x1_,nand[nand[x2_,x3_],nand[x1_,x3_]]]→nand[x1,x3]

contains a subpattern of the form:

nand[nand[x2_,x3_],nand[x1_,x3_]]

which can be unified with the input for the rule:

nand[x1_,x2_]↔nand[x2_,x1_]

where these rules follow from Substitution Lemma 18 and Substitution Lemma 16 respectively.

Critical Pair Lemma 30

The following expressions are equivalent:

nand[nand[x1,x2],nand[x1,nand[x2,x2]]]==nand[nand[x1,x2],nand[x1,nand[x3,nand[x1,nand[x2

Proof

Note that the input for the rule:

nand[x1_,nand[nand[x1_,x2_],nand[x3_,x2_]]]→nand[x1,x2]

contains a subpattern of the form:

nand[x1_,x2_]

which can be unified with the input for the rule:

nand[nand[x1_,x2_],nand[x1_,nand[x2_,x2_]]]→x1

where these rules follow from Critical Pair Lemma 29 and Substitution Lemma 13 respectively.

Substitution Lemma 19

It can be shown that:

x1==nand[nand[x1,x2],nand[x1,nand[x3,nand[x1,nand[x2,x2]]]]]

Proof

We start by taking Critical Pair Lemma 30, and apply the substitution:

nand[nand[x1_,x2_],nand[x1_,nand[x2_,x2_]]]→x1

which follows from Substitution Lemma 13.

Substitution Lemma 20

It can be shown that:

x1==nand[nand[x1,x2],nand[x1,nand[x3,x2]]]

Proof

We start by taking Substitution Lemma 19, and apply the substitution:

nand[x1_,nand[x2_,nand[x1_,nand[x3_,x3_]]]]→nand[x1,nand[x2,x3]]

which follows from Substitution Lemma 8.

Critical Pair Lemma 31

The following expressions are equivalent:

`x1==nand[nand[x1,nand[x2,x3]],nand[x1,x2]]`

Proof

Note that the input for the rule:

`nand[nand[x1_,x2_],nand[x1_,nand[x3_,x2_]]]→x1`

contains a subpattern of the form:

`nand[x1_,nand[x3_,x2_]]`

which can be unified with the input for the rule:

`nand[x1_,nand[nand[x1_,x2_],nand[x2_,x3_]]]→nand[x1,x2]`

where these rules follow from Substitution Lemma 20 and Critical Pair Lemma 28 respectively.

Substitution Lemma 21

It can be shown that:

`x1==nand[nand[x1,x2],nand[x1,nand[x2,x3]]]`

Proof

We start by taking Critical Pair Lemma 31, and apply the substitution:

`nand[x1_,x2_]→nand[x2,x1]`

which follows from Substitution Lemma 16.

Substitution Lemma 22

It can be shown that:

`nand[nand[c,nand[a,b]],nand[a,nand[nand[a,c],a]]]==c`

Proof

We start by taking Hypothesis 1, and apply the substitution:

`nand[x1_,x2_]→nand[x2,x1]`

which follows from Substitution Lemma 16.

Substitution Lemma 23

It can be shown that:

`nand[nand[c,nand[a,b]],nand[a,nand[a,nand[a,c]]]]==c`

Proof

We start by taking Substitution Lemma 22, and apply the substitution:

`nand[x1_,x2_]→nand[x2,x1]`

which follows from Substitution Lemma 16.

Substitution Lemma 24

It can be shown that:

`nand[nand[c,nand[a,b]],nand[a,nand[a,nand[c,c]]]]==c`

Proof

We start by taking Substitution Lemma 23, and apply the substitution:

```
nand[x1_,nand[x1_,x2_]]→nand[x1,nand[x2,x2]]
```

which follows from Axiom 2.

Substitution Lemma 25

It can be shown that:

```
nand[nand[c,nand[a,b]],nand[a,c]]==c
```

Proof

We start by taking Substitution Lemma 24, and apply the substitution:

```
nand[x1_,nand[x1_,nand[x2_,x2_]]]→nand[x1,x2]
```

which follows from Critical Pair Lemma 2.

Substitution Lemma 26

It can be shown that:

```
nand[nand[c,nand[a,b]],nand[c,a]]==c
```

Proof

We start by taking Substitution Lemma 25, and apply the substitution:

```
nand[x1_,x2_]→nand[x2,x1]
```

which follows from Substitution Lemma 16.

Substitution Lemma 27

It can be shown that:

```
nand[nand[c,a],nand[c,nand[a,b]]]==c
```

Proof

We start by taking Substitution Lemma 26, and apply the substitution:

```
nand[x1_,x2_]→nand[x2,x1]
```

which follows from Substitution Lemma 16.

Conclusion 1

We obtain the conclusion:

```
True
```

Proof

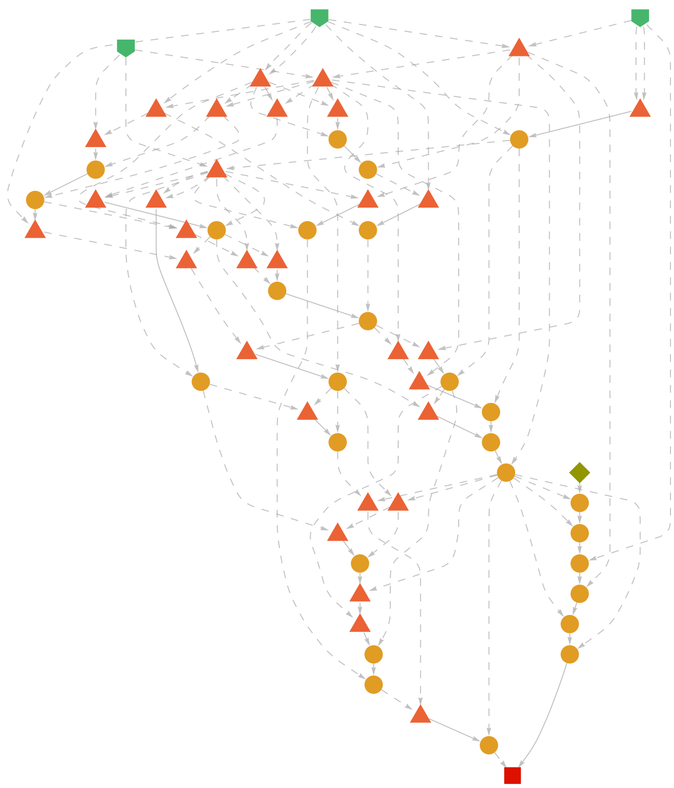
Take Substitution Lemma 27, and apply the substitution:

```
nand[nand[x1_,x2_],nand[x1_,nand[x2_,x3_]]]→x1
```

which follows from Substitution Lemma 21.

In[ ]:= proofWolframfromShort["ProofGraph"]

Out[ ]=



In[ ]:= Clear[proofWolframfromShort]

In[ ]:= proofShortfromWolfram["ProofNotebook"]

Axiom 1

We are given that:

a==nand[nand[nand[b,c],a],nand[b,nand[nand[b,a],b]]]

Hypothesis 1

We would like to show that:

nand[nand[a,a],nand[a,b]]==a

Hypothesis 2

We would like to show that:

nand[a,nand[a,b]]==nand[a,nand[b,b]]

Hypothesis 3

We would like to show that:

nand[a,nand[a,nand[b,c]]]==nand[b,nand[b,nand[a,c]]]

Critical Pair Lemma 1

The following expressions are equivalent:

nand[a,nand[nand[a,b],a]]==nand[b,nand[nand[a,c],nand[nand[nand[a,c],nand[a,nand[nand[

Proof

Note that the input for the rule:

nand[nand[nand[a_,b_],c_],nand[a_,nand[nand[a_,c_],a_]]]→c

contains a subpattern of the form:

nand[nand[a_,b_],c_]

which can be unified with the input for the rule:

nand[nand[nand[a_,b_],c_],nand[a_,nand[nand[a_,c_],a_]]]→c

where these rules follow from Axiom 1 and Axiom 1 respectively.

Critical Pair Lemma 2

The following expressions are equivalent:

a==nand[nand[b,a],nand[nand[nand[c,y1],b],nand[nand[nand[nand[c,y1],b],a],nand[nand[c,

Proof

Note that the input for the rule:

nand[nand[nand[a_,b_],c_],nand[a_,nand[nand[a_,c_],a_]]]→c

contains a subpattern of the form:

nand[a_,b_]

which can be unified with the input for the rule:

nand[nand[nand[a_,b_],c_],nand[a_,nand[nand[a_,c_],a_]]]→c

where these rules follow from Axiom 1 and Axiom 1 respectively.

Critical Pair Lemma 3

The following expressions are equivalent:

```
a==nand[nand[nand[nand[b,nand[nand[b,a],b]],c],a],nand[nand[b,nand[nand[b,a],b]],a]]
```

Proof

Note that the input for the rule:

```
nand[nand[nand[a_,b_],c_],nand[a_,nand[nand[a_,c_],a_]]]→c
```

contains a subpattern of the form:

```
nand[nand[a_,c_],a_]
```

which can be unified with the input for the rule:

```
nand[nand[nand[a_,b_],c_],nand[a_,nand[nand[a_,c_],a_]]]→c
```

where these rules follow from Axiom 1 and Axiom 1 respectively.

Critical Pair Lemma 4

The following expressions are equivalent:

```
a==nand[nand[nand[b,nand[nand[b,c],b]],a],nand[c,nand[nand[nand[nand[nand[b,y1],c],nan
```

Proof

Note that the input for the rule:

```
nand[nand[a_,b_],nand[nand[nand[c_,y1_],a_],nand[nand[nand[nand[c_,y1_],a_],b_],nand[
```

contains a subpattern of the form:

```
nand[nand[c_,y1_],a_]
```

which can be unified with the input for the rule:

```
nand[nand[nand[a_,b_],c_],nand[a_,nand[nand[a_,c_],a_]]]→c
```

where these rules follow from Critical Pair Lemma 2 and Axiom 1 respectively.

Substitution Lemma 1

It can be shown that:

```
a==nand[nand[nand[b,nand[nand[b,c],b]],a],nand[c,nand[nand[c,a],nand[nand[nand[b,y1],c
```

Proof

We start by taking Critical Pair Lemma 4, and apply the substitution:

```
nand[nand[nand[a_,b_],c_],nand[a_,nand[nand[a_,c_],a_]]]→c
```

which follows from Axiom 1.

Substitution Lemma 2

It can be shown that:

```
a==nand[nand[nand[b,nand[nand[b,c],b]],a],nand[c,nand[nand[c,a],c]]]
```

Proof

We start by taking Substitution Lemma 1, and apply the substitution:

```
nand[nand[nand[a_,b_],c_],nand[a_,nand[nand[a_,c_],a_]]]→c
```

which follows from Axiom 1.

Critical Pair Lemma 5

The following expressions are equivalent:

The following expressions are equivalent:

`a==nand[nand[b,a],nand[nand[c,b],nand[nand[nand[nand[nand[nand[y1,y2],c],nand[y1,nand[`

Proof

Note that the input for the rule:

`nand[nand[a_,b_],nand[nand[nand[c_,y1_],a_],nand[nand[nand[nand[c_,y1_],a_],b_],nand[`

contains a subpattern of the form:

`nand[c_,y1_]`

which can be unified with the input for the rule:

`nand[nand[nand[a_,b_],c_],nand[a_,nand[nand[a_,c_],a_]]]→c`

where these rules follow from Critical Pair Lemma 2 and Axiom 1 respectively.

Substitution Lemma 3

It can be shown that:

`a==nand[nand[b,a],nand[nand[c,b],nand[nand[nand[c,b],a],nand[nand[nand[nand[y1,y2],c],`

Proof

We start by taking Critical Pair Lemma 5, and apply the substitution:

`nand[nand[nand[a_,b_],c_],nand[a_,nand[nand[a_,c_],a_]]]→c`

which follows from Axiom 1.

Substitution Lemma 4

It can be shown that:

`a==nand[nand[b,a],nand[nand[c,b],nand[nand[nand[c,b],a],nand[c,b]]]]`

Proof

We start by taking Substitution Lemma 3, and apply the substitution:

`nand[nand[nand[a_,b_],c_],nand[a_,nand[nand[a_,c_],a_]]]→c`

which follows from Axiom 1.

Critical Pair Lemma 6

The following expressions are equivalent:

`a==nand[nand[nand[nand[b,nand[nand[b,c],b]],c],a],nand[c,nand[nand[c,a],c]]]`

Proof

Note that the input for the rule:

`nand[nand[nand[a_,nand[nand[a_,b_],a_]],c_],nand[b_,nand[nand[b_,c_],b_]]]→c`

contains a subpattern of the form:

`nand[nand[a_,b_],a_]`

which can be unified with the input for the rule:

`nand[nand[nand[a_,b_],c_],nand[a_,nand[nand[a_,c_],a_]]]→c`

where these rules follow from Substitution Lemma 2 and Axiom 1 respectively.

Critical Pair Lemma 7

The following expressions are equivalent:

`nand[a,nand[nand[a,b],a]]==nand[b,nand[nand[c,nand[nand[c,a],c]],nand[nand[nand[c,nand`

Proof

Note that the input for the rule:

```
nand[nand[nand[a_,b_],c_],nand[a_,nand[nand[a_,c_],a_]]]→c
```

contains a subpattern of the form:

```
nand[nand[a_,b_],c_]
```

which can be unified with the input for the rule:

```
nand[nand[nand[a_,nand[nand[a_,b_],a_]],c_],nand[b_,nand[nand[b_,c_],b_]]]→c
```

where these rules follow from Axiom 1 and Substitution Lemma 2 respectively.

Critical Pair Lemma 8

The following expressions are equivalent:

```
nand[a,nand[nand[a,b],a]]==nand[nand[b,nand[a,nand[nand[a,b],a]]],nand[nand[nand[a,c],
```

Proof

Note that the input for the rule:

```
nand[nand[a_,b_],nand[nand[c_,a_],nand[nand[nand[c_,a_],b_],nand[c_,a_]]]]→b
```

contains a subpattern of the form:

```
nand[nand[c_,a_],b_]
```

which can be unified with the input for the rule:

```
nand[nand[nand[a_,b_],c_],nand[a_,nand[nand[a_,c_],a_]]]→c
```

where these rules follow from Substitution Lemma 4 and Axiom 1 respectively.

Critical Pair Lemma 9

The following expressions are equivalent:

```
nand[a,nand[nand[a,b],a]]==nand[nand[b,nand[a,nand[nand[a,b],a]]],nand[nand[nand[c,nan
```

Proof

Note that the input for the rule:

```
nand[nand[a_,b_],nand[nand[c_,a_],nand[nand[nand[c_,a_],b_],nand[c_,a_]]]]→b
```

contains a subpattern of the form:

```
nand[nand[c_,a_],b_]
```

which can be unified with the input for the rule:

```
nand[nand[nand[a_,nand[nand[a_,b_],a_]],c_],nand[b_,nand[nand[b_,c_],b_]]]→c
```

where these rules follow from Substitution Lemma 4 and Substitution Lemma 2 respectively.

Critical Pair Lemma 10

The following expressions are equivalent:

```
nand[nand[a,b],nand[nand[nand[a,b],c],nand[a,b]]]==nand[c,nand[b,nand[nand[b,nand[nand
```

Proof

Note that the input for the rule:

```
nand[nand[nand[a_,b_],c_],nand[a_,nand[nand[a_,c_],a_]]]→c
```

contains a subpattern of the form:

```
nand[nand[a_,b_],c_]
```

which can be unified with the input for the rule:

```
nand[nand[a_,b_],nand[nand[c_,a_],nand[nand[nand[c_,a_],b_],nand[c_,a_]]]]→b
```

where these rules follow from Axiom 1 and Substitution Lemma 4 respectively.

Critical Pair Lemma 11

The following expressions are equivalent:

```
nand[nand[a,b],nand[nand[nand[a,b],c],nand[a,b]]]==nand[nand[nand[nand[b,c],y1],nand[n
```

Proof

Note that the input for the rule:

```
nand[nand[nand[a_,b_],c_],nand[a_,nand[nand[a_,c_],a_]]]→c
```

contains a subpattern of the form:

```
nand[a_,c_]
```

which can be unified with the input for the rule:

```
nand[nand[a_,b_],nand[nand[c_,a_],nand[nand[nand[c_,a_],b_],nand[c_,a_]]]]→b
```

where these rules follow from Axiom 1 and Substitution Lemma 4 respectively.

Critical Pair Lemma 12

The following expressions are equivalent:

```
nand[nand[a,b],nand[nand[nand[a,b],c],nand[a,b]]]==nand[nand[c,nand[nand[a,b],nand[nan
```

Proof

Note that the input for the rule:

```
nand[nand[a_,b_],nand[nand[c_,a_],nand[nand[nand[c_,a_],b_],nand[c_,a_]]]]→b
```

contains a subpattern of the form:

```
nand[nand[c_,a_],b_]
```

which can be unified with the input for the rule:

```
nand[nand[a_,b_],nand[nand[c_,a_],nand[nand[nand[c_,a_],b_],nand[c_,a_]]]]→b
```

where these rules follow from Substitution Lemma 4 and Substitution Lemma 4 respectively.

Critical Pair Lemma 13

The following expressions are equivalent:

```
nand[a,nand[nand[a,b],a]]==nand[b,nand[b,nand[nand[b,nand[a,nand[nand[a,b],a]]],b]]]
```

Proof

Note that the input for the rule:

```
nand[nand[nand[nand[a_,nand[nand[a_,b_],a_]],b_],c_],nand[b_,nand[nand[b_,c_],b_]]]→
```

contains a subpattern of the form:

```
nand[nand[nand[a_,nand[nand[a_,b_],a_]],b_],c_]
```

which can be unified with the input for the rule:

```
nand[nand[nand[a_,b_],c_],nand[a_,nand[nand[a_,c_],a_]]]→c
```

where these rules follow from Critical Pair Lemma 6 and Axiom 1 respectively.

Critical Pair Lemma 14

The following expressions are equivalent:

`nand[nand[a,nand[nand[a,b],a]],b]==nand[b,nand[b,nand[nand[b,nand[nand[a,nand[nand[a,b`

PROOF

Note that the input for the rule:

`nand[nand[nand[nand[a_,nand[nand[a_,b_],a_]],b_],c_],nand[b_,nand[nand[b_,c_],b_]]]→`

contains a subpattern of the form:

`nand[nand[nand[a_,nand[nand[a_,b_],a_]],b_],c_]`

which can be unified with the input for the rule:

`nand[nand[nand[nand[a_,nand[nand[a_,b_],a_]],c_],b_],nand[nand[a_,nand[nand[a_,b_],a_`

where these rules follow from Critical Pair Lemma 6 and Critical Pair Lemma 3 respectively.

Critical Pair Lemma 15

The following expressions are equivalent:

`nand[a,nand[nand[a,b],a]]==nand[b,nand[nand[nand[c,nand[nand[c,a],c]],a],nand[nand[nan`

PROOF

Note that the input for the rule:

`nand[nand[nand[a_,b_],c_],nand[a_,nand[nand[a_,c_],a_]]]→c`

contains a subpattern of the form:

`nand[nand[a_,b_],c_]`

which can be unified with the input for the rule:

`nand[nand[nand[nand[a_,nand[nand[a_,b_],a_]],b_],c_],nand[b_,nand[nand[b_,c_],b_]]]→`

where these rules follow from Axiom 1 and Critical Pair Lemma 6 respectively.

Critical Pair Lemma 16

The following expressions are equivalent:

`nand[a,nand[nand[a,a],a]]==nand[nand[a,nand[a,nand[nand[a,a],a]]],a]`

PROOF

Note that the input for the rule:

`nand[nand[a_,nand[b_,nand[nand[b_,a_],b_]]],nand[nand[nand[b_,c_],a_],nand[a_,nand[na`

contains a subpattern of the form:

`nand[nand[nand[b_,c_],a_],nand[a_,nand[nand[b_,c_],a_]]]`

which can be unified with the input for the rule:

`nand[nand[nand[a_,b_],c_],nand[a_,nand[nand[a_,c_],a_]]]→c`

where these rules follow from Critical Pair Lemma 8 and Axiom 1 respectively.

Critical Pair Lemma 17

The following expressions are equivalent:

`nand[nand[nand[a,b],c],nand[c,nand[nand[a,b],c]]]==nand[nand[a,nand[nand[a,c],a]],nand`

PROOF

Note that the input for the rule:

`nand[nand[nand[a_,b_],c_],nand[a_,nand[nand[a_,c_],a_]]]→c`

contains a subpattern of the form:

`nand[nand[a_,b_],c_]`

which can be unified with the input for the rule:

`nand[nand[a_,nand[b_,nand[nand[b_,a_],b_]]],nand[nand[nand[b_,c_],a_],nand[a_,nand[na`

where these rules follow from Axiom 1 and Critical Pair Lemma 8 respectively.

Critical Pair Lemma 18

The following expressions are equivalent:

`a==nand[nand[a,nand[nand[a,a],a]],nand[a,nand[nand[a,a],a]]]`

Proof

Note that the input for the rule:

`nand[nand[nand[a_,b_],c_],nand[a_,nand[nand[a_,c_],a_]]]→c`

contains a subpattern of the form:

`nand[nand[a_,b_],c_]`

which can be unified with the input for the rule:

`nand[nand[a_,nand[a_,nand[nand[a_,a_],a_]]],a_]→nand[a,nand[nand[a,a],a]]`

where these rules follow from Axiom 1 and Critical Pair Lemma 16 respectively.

Critical Pair Lemma 19

The following expressions are equivalent:

`a==nand[nand[nand[b,nand[b,nand[nand[b,b],b]]],a],nand[nand[b,nand[nand[b,b],b]],nand[`

Proof

Note that the input for the rule:

`nand[nand[nand[a_,nand[nand[a_,b_],a_]],c_],nand[b_,nand[nand[b_,c_],b_]]]→c`

contains a subpattern of the form:

`nand[nand[a_,b_],a_]`

which can be unified with the input for the rule:

`nand[nand[a_,nand[a_,nand[nand[a_,a_],a_]]],a_]→nand[a,nand[nand[a,a],a]]`

where these rules follow from Substitution Lemma 2 and Critical Pair Lemma 16 respectively.

Critical Pair Lemma 20

The following expressions are equivalent:

`nand[a,nand[nand[a,a],a]]==nand[nand[a,nand[a,nand[nand[a,a],a]]],nand[nand[a,nand[nan`

Proof

Note that the input for the rule:

`nand[nand[a_,nand[b_,nand[nand[b_,a_],b_]]],nand[nand[nand[b_,c_],a_],nand[a_,nand[na`

contains a subpattern of the form:

`nand[nand[b_,c_],a_]`

which can be unified with the input for the rule:

`nand[nand[a_,nand[a_,nand[nand[a_,a_],a_]]],a_]→nand[a,nand[nand[a,a],a]]`

where these rules follow from Critical Pair Lemma 8 and Critical Pair Lemma 16 respectively.

Substitution Lemma 5

It can be shown that:

```
nand[a,nand[nand[a,a],a]]==nand[nand[a,nand[a,nand[nand[a,a],a]]],nand[nand[a,nand[nan
```

PROOF

We start by taking Critical Pair Lemma 20, and apply the substitution:

```
nand[nand[a_,nand[a_,nand[nand[a_,a_],a_]]],a_]→nand[a,nand[nand[a,a],a]]
```

which follows from Critical Pair Lemma 16.

Critical Pair Lemma 21

The following expressions are equivalent:

```
nand[a,nand[nand[a,a],a]]==nand[nand[nand[nand[a,nand[nand[a,a],a]],b],nand[a,nand[nan
```

PROOF

Note that the input for the rule:

```
nand[nand[nand[a_,b_],c_],nand[a_,nand[nand[a_,c_],a_]]]→c
```

contains a subpattern of the form:

```
nand[a_,c_]
```

which can be unified with the input for the rule:

```
nand[nand[a_,nand[nand[a_,a_],a_]],nand[a_,nand[nand[a_,a_],a_]]]→a
```

where these rules follow from Axiom 1 and Critical Pair Lemma 18 respectively.

Critical Pair Lemma 22

The following expressions are equivalent:

```
nand[a,nand[nand[a,a],a]]==nand[nand[nand[nand[a,a],a],nand[a,nand[nand[a,a],a]]],nand
```

PROOF

Note that the input for the rule:

```
nand[nand[a_,b_],nand[nand[c_,a_],nand[nand[nand[c_,a_],b_],nand[c_,a_]]]]→b
```

contains a subpattern of the form:

```
nand[nand[c_,a_],b_]
```

which can be unified with the input for the rule:

```
nand[nand[a_,nand[nand[a_,a_],a_]],nand[a_,nand[nand[a_,a_],a_]]]→a
```

where these rules follow from Substitution Lemma 4 and Critical Pair Lemma 18 respectively.

Substitution Lemma 6

It can be shown that:

```
nand[a,nand[nand[a,a],a]]==nand[a,nand[nand[a,nand[nand[a,a],a]],nand[a,nand[a,nand[na
```

PROOF

We start by taking Critical Pair Lemma 22, and apply the substitution:

```
nand[nand[nand[a_,b_],c_],nand[a_,nand[nand[a_,c_],a_]]]→c
```

which follows from Axiom 1.

Critical Pair Lemma 23

The following expressions are equivalent:

```
nand[nand[a,b],nand[nand[nand[a,b],nand[a,nand[nand[a,b],a]]],nand[a,b]]]==nand[nand[a
```

PROOF

Note that the input for the rule:

```
nand[a_,nand[b_,nand[nand[b_,nand[nand[c_,b_],nand[nand[nand[c_,b_],a_],nand[c_,b_]]]
```

contains a subpattern of the form:

```
nand[b_,nand[nand[c_,b_],nand[nand[nand[c_,b_],a_],nand[c_,b_]]]]
```

which can be unified with the input for the rule:

```
nand[a_,nand[nand[b_,c_],nand[nand[nand[b_,c_],nand[b_,nand[nand[b_,a_],b_]]],nand[b_
```

where these rules follow from Critical Pair Lemma 10 and Critical Pair Lemma 1 respectively.

Critical Pair Lemma 24

The following expressions are equivalent:

```
nand[nand[a,nand[nand[a,a],a]],nand[a,nand[a,nand[nand[a,a],a]]]]==nand[nand[a,nand[na
```

PROOF

Note that the input for the rule:

```
nand[nand[nand[a_,b_],c_],nand[a_,nand[nand[a_,c_],a_]]]→c
```

contains a subpattern of the form:

```
nand[nand[a_,b_],c_]
```

which can be unified with the input for the rule:

```
nand[nand[a_,nand[a_,nand[nand[a_,a_],a_]]],nand[nand[a_,nand[nand[a_,a_],a_]],nand[a
```

where these rules follow from Axiom 1 and Substitution Lemma 5 respectively.

Substitution Lemma 7

It can be shown that:

```
nand[nand[a,nand[nand[a,a],a]],nand[a,nand[a,nand[nand[a,a],a]]]]==nand[nand[a,nand[na
```

PROOF

We start by taking Critical Pair Lemma 24, and apply the substitution:

```
nand[a_,nand[nand[a_,nand[nand[a_,a_],a_]],nand[a_,nand[a_,nand[nand[a_,a_],a_]]]]]→
```

which follows from Substitution Lemma 6.

Substitution Lemma 8

It can be shown that:

```
nand[nand[a,nand[nand[a,a],a]],nand[a,nand[a,nand[nand[a,a],a]]]]==nand[nand[a,a],nand
```

PROOF

We start by taking Substitution Lemma 7, and apply the substitution:

```
nand[nand[a_,nand[nand[a_,b_],a_]],nand[b_,nand[nand[a_,nand[nand[a_,b_],a_]],b_]]]→
```

which follows from Critical Pair Lemma 23.

Critical Pair Lemma 25

The following expressions are equivalent:

```
nand[a,nand[nand[a,a],a]]==nand[nand[nand[a,nand[a,nand[nand[a,a],a]]],nand[a,nand[nan
```

PROOF

Note that the input for the rule:

`nand[nand[nand[a_,nand[a_,nand[nand[a_,a_],a_]]],b_],nand[nand[a_,nand[nand[a_,a_],a_`

contains a subpattern of the form:

`nand[nand[a_,nand[nand[a_,a_],a_]],b_]`

which can be unified with the input for the rule:

`nand[nand[a_,nand[nand[a_,a_],a_]],nand[a_,nand[nand[a_,a_],a_]]]→a`

where these rules follow from Critical Pair Lemma 19 and Critical Pair Lemma 18 respectively.

Critical Pair Lemma 26

The following expressions are equivalent:

`nand[a,nand[nand[a,a],a]]==nand[a,nand[nand[nand[b,nand[nand[b,a],b]],a],nand[a,nand[n`

Proof

Note that the input for the rule:

`nand[a_,nand[nand[nand[b_,nand[nand[b_,c_],b_]],c_],nand[nand[nand[nand[b_,nand[nand[`

contains a subpattern of the form:

`nand[nand[nand[b_,nand[nand[b_,c_],b_]],c_],nand[c_,nand[nand[c_,a_],c_]]]`

which can be unified with the input for the rule:

`nand[nand[nand[a_,nand[nand[a_,b_],a_]],c_],nand[b_,nand[nand[b_,c_],b_]]]→c`

where these rules follow from Critical Pair Lemma 15 and Substitution Lemma 2 respectively.

Critical Pair Lemma 27

The following expressions are equivalent:

`nand[nand[nand[a,nand[nand[a,b],a]],b],nand[b,nand[nand[a,nand[nand[a,b],a]],b]]]==nan`

Proof

Note that the input for the rule:

`nand[nand[a_,nand[nand[a_,b_],a_]],nand[b_,nand[nand[b_,nand[nand[nand[a_,c_],b_],nan`

contains a subpattern of the form:

`nand[b_,nand[nand[nand[a_,c_],b_],nand[b_,nand[nand[a_,c_],b_]]]]`

which can be unified with the input for the rule:

`nand[a_,nand[nand[nand[b_,nand[nand[b_,a_],b_]],a_],nand[a_,nand[nand[b_,nand[nand[b_`

where these rules follow from Critical Pair Lemma 17 and Critical Pair Lemma 26 respectively.

Critical Pair Lemma 28

The following expressions are equivalent:

`nand[a,nand[nand[a,b],a]]==nand[nand[b,nand[a,nand[nand[a,b],a]]],nand[nand[a,nand[nan`

Proof

Note that the input for the rule:

`nand[nand[a_,nand[b_,nand[nand[b_,a_],b_]]],nand[nand[nand[b_,c_],a_],nand[a_,nand[na`

contains a subpattern of the form:

`nand[nand[nand[b_,c_],a_],nand[a_,nand[nand[b_,c_],a_]]]`

which can be unified with the input for the rule:

`nand[nand[nand[a_,nand[nand[a_,b_],a_]],b_],nand[b_,nand[nand[a_,nand[nand[a_,b_],a_]` nand[nand[nand[a_,nand[nand[a_,b_],a_]],b_],nand[b_,nand[nand[a_,nand[nand[a_,b_],a_]

where these rules follow from Critical Pair Lemma 8 and Critical Pair Lemma 27 respectively.

Critical Pair Lemma 29

The following expressions are equivalent:

`nand[a,nand[nand[a,a],a]]==nand[a,nand[nand[b,nand[nand[b,a],b]],nand[a,nand[nand[a,na`

Proof

Note that the input for the rule:

`nand[a_,nand[nand[nand[b_,nand[nand[b_,a_],b_]],a_],nand[a_,nand[nand[b_,nand[nand[b_`

contains a subpattern of the form:

`nand[nand[nand[b_,nand[nand[b_,a_],b_]],a_],nand[a_,nand[nand[b_,nand[nand[b_,a_],b_]`

which can be unified with the input for the rule:

`nand[nand[nand[a_,nand[nand[a_,b_],a_]],b_],nand[b_,nand[nand[a_,nand[nand[a_,b_],a_]`

where these rules follow from Critical Pair Lemma 26 and Critical Pair Lemma 27 respectively.

Critical Pair Lemma 30

The following expressions are equivalent:

`nand[a,nand[nand[a,a],a]]==nand[nand[a,nand[a,nand[nand[a,a],a]]],nand[nand[b,nand[nan`

Proof

Note that the input for the rule:

`nand[nand[a_,nand[b_,nand[nand[b_,a_],b_]]],nand[nand[nand[c_,nand[nand[c_,b_],c_]],a`

contains a subpattern of the form:

`nand[nand[nand[c_,nand[nand[c_,b_],c_]],a_],nand[a_,nand[nand[c_,nand[nand[c_,b_],c_]`

which can be unified with the input for the rule:

`nand[nand[nand[a_,nand[nand[a_,b_],a_]],b_],nand[b_,nand[nand[a_,nand[nand[a_,b_],a_]`

where these rules follow from Critical Pair Lemma 9 and Critical Pair Lemma 27 respectively.

Critical Pair Lemma 31

The following expressions are equivalent:

`nand[nand[a,nand[nand[a,b],a]],nand[b,nand[nand[b,nand[nand[b,b],b]],b]]]==nand[nand[b`

Proof

Note that the input for the rule:

`nand[nand[nand[a_,b_],c_],nand[a_,nand[nand[a_,c_],a_]]]→c`

contains a subpattern of the form:

`nand[nand[a_,b_],c_]`

which can be unified with the input for the rule:

`nand[nand[a_,nand[a_,nand[nand[a_,a_],a_]]],nand[nand[b_,nand[nand[b_,a_],b_]],nand[a`

where these rules follow from Axiom 1 and Critical Pair Lemma 30 respectively.

Substitution Lemma 9

It can be shown that:

`nand[nand[a,nand[nand[a,b],a]],nand[b,nand[nand[b,nand[nand[b,b],b]],b]]]==nand[nand[b`

Proof

PROOF

We start by taking Critical Pair Lemma 31, and apply the substitution:

```
nand[a_,nand[nand[b_,nand[nand[b_,a_],b_]],nand[a_,nand[nand[a_,nand[nand[a_,a_],a_]]
```

which follows from Critical Pair Lemma 29.

Substitution Lemma 10

It can be shown that:

```
nand[nand[a,nand[nand[a,b],a]],nand[b,nand[nand[b,nand[nand[b,b],b]],b]]]==nand[nand[b
```

PROOF

We start by taking Substitution Lemma 9, and apply the substitution:

```
nand[nand[a_,nand[nand[a_,b_],a_]],nand[b_,nand[nand[a_,nand[nand[a_,b_],a_]],b_]]]→
```

which follows from Critical Pair Lemma 23.

Substitution Lemma 11

It can be shown that:

```
nand[nand[a,nand[nand[a,b],a]],nand[b,nand[nand[b,nand[nand[b,b],b]],b]]]==nand[nand[b
```

PROOF

We start by taking Substitution Lemma 10, and apply the substitution:

```
nand[nand[a_,a_],nand[nand[nand[a_,a_],nand[a_,nand[nand[a_,a_],a_]]],nand[a_,a_]]]→
```

which follows from Substitution Lemma 8.

Critical Pair Lemma 32

The following expressions are equivalent:

```
nand[a,nand[nand[a,b],a]]==nand[nand[b,nand[a,nand[nand[a,b],a]]],nand[nand[b,nand[nan
```

PROOF

Note that the input for the rule:

```
nand[nand[a_,nand[b_,nand[nand[b_,a_],b_]]],nand[nand[b_,nand[nand[b_,a_],b_]],nand[a
```

contains a subpattern of the form:

```
nand[nand[b_,nand[nand[b_,a_],b_]],nand[a_,nand[nand[a_,nand[nand[a_,a_],a_]],a_]]]
```

which can be unified with the input for the rule:

```
nand[nand[a_,nand[nand[a_,b_],a_]],nand[b_,nand[nand[b_,nand[nand[b_,b_],b_]],b_]]]↔
```

where these rules follow from Critical Pair Lemma 28 and Substitution Lemma 11 respectively.

Critical Pair Lemma 33

The following expressions are equivalent:

```
nand[nand[a,b],nand[nand[nand[a,b],nand[b,nand[nand[b,b],b]]],nand[a,b]]]==nand[nand[n
```

PROOF

Note that the input for the rule:

```
nand[nand[nand[nand[a_,b_],c_],nand[nand[y1_,a_],nand[nand[nand[y1_,a_],b_],nand[y1_,
```

contains a subpattern of the form:

```
nand[nand[a_,b_],nand[b_,nand[a_,b_]]]
```

which can be unified with the input for the rule:

```
nand[nand[a_,nand[b_,nand[nand[b_,a_],b_]]],nand[nand[a_,nand[nand[a_,a_],a_]],nand[a
```

where these rules follow from Critical Pair Lemma 11 and Critical Pair Lemma 32 respectively.

Critical Pair Lemma 34

The following expressions are equivalent:

```
nand[nand[a,b],nand[nand[nand[a,b],nand[b,nand[nand[b,b],b]]],nand[a,b]]]==nand[nand[n
```

Proof

Note that the input for the rule:

```
nand[nand[a_,nand[nand[b_,c_],nand[nand[nand[b_,c_],a_],nand[b_,c_]]]],nand[nand[c_,a
```

contains a subpattern of the form:

```
nand[nand[c_,a_],nand[a_,nand[c_,a_]]]
```

which can be unified with the input for the rule:

```
nand[nand[a_,nand[b_,nand[nand[b_,a_],b_]]],nand[nand[a_,nand[nand[a_,a_],a_]],nand[a
```

where these rules follow from Critical Pair Lemma 12 and Critical Pair Lemma 32 respectively.

Critical Pair Lemma 35

The following expressions are equivalent:

```
nand[a,nand[nand[a,a],a]]==nand[nand[nand[b,a],nand[nand[nand[b,a],nand[a,nand[nand[a,
```

Proof

Note that the input for the rule:

```
nand[nand[nand[nand[a_,nand[nand[a_,a_],a_]],b_],nand[a_,nand[nand[a_,a_],a_]]],nand[
```

contains a subpattern of the form:

```
nand[nand[nand[a_,nand[nand[a_,a_],a_]],b_],nand[a_,nand[nand[a_,a_],a_]]]
```

which can be unified with the input for the rule:

```
nand[nand[nand[a_,nand[nand[a_,a_],a_]],nand[nand[b_,a_],nand[nand[nand[b_,a_],nand[a
```

where these rules follow from Critical Pair Lemma 21 and Critical Pair Lemma 34 respectively.

Critical Pair Lemma 36

The following expressions are equivalent:

```
nand[a,nand[nand[a,a],a]]==nand[nand[nand[a,nand[nand[a,a],a]],nand[a,nand[a,nand[nand
```

Proof

Note that the input for the rule:

```
nand[nand[nand[a_,b_],nand[nand[nand[a_,b_],nand[b_,nand[nand[b_,b_],b_]]],nand[a_,b_
```

contains a subpattern of the form:

```
nand[nand[a_,b_],nand[nand[nand[a_,b_],nand[b_,nand[nand[b_,b_],b_]]],nand[a_,b_]]]
```

which can be unified with the input for the rule:

```
nand[nand[a_,a_],nand[nand[nand[a_,a_],nand[a_,nand[nand[a_,a_],a_]]],nand[a_,a_]]]→
```

where these rules follow from Critical Pair Lemma 35 and Substitution Lemma 8 respectively.

Critical Pair Lemma 37

The following expressions are equivalent:

```
nand[nand[a,a],nand[nand[nand[a,a],nand[a,nand[nand[a,a],a]]],nand[a,a]]]==nand[nand[n
```

Proof

Note that the input for the rule:

```
nand[nand[nand[nand[a_,nand[a_,nand[nand[a_,a_],a_]]],b_],nand[nand[c_,a_],nand[nand[
```

contains a subpattern of the form:

```
nand[nand[c_,a_],nand[nand[nand[c_,a_],nand[a_,nand[nand[a_,a_],a_]]],nand[c_,a_]]]
```

which can be unified with the input for the rule:

```
nand[nand[a_,a_],nand[nand[nand[a_,a_],nand[a_,nand[nand[a_,a_],a_]]],nand[a_,a_]]]→
```

where these rules follow from Critical Pair Lemma 33 and Substitution Lemma 8 respectively.

Substitution Lemma 12

It can be shown that:

```
nand[nand[a,nand[nand[a,a],a]],nand[a,nand[a,nand[nand[a,a],a]]]]==nand[nand[nand[nand
```

Proof

We start by taking Critical Pair Lemma 37, and apply the substitution:

```
nand[nand[a_,a_],nand[nand[nand[a_,a_],nand[a_,nand[nand[a_,a_],a_]]],nand[a_,a_]]]→
```

which follows from Substitution Lemma 8.

Critical Pair Lemma 38

The following expressions are equivalent:

```
nand[nand[a,nand[nand[a,a],a]],nand[a,nand[a,nand[nand[a,a],a]]]]==nand[nand[a,nand[na
```

Proof

Note that the input for the rule:

```
nand[nand[nand[nand[a_,nand[a_,nand[nand[a_,a_],a_]]],b_],nand[nand[a_,nand[nand[a_,a
```

contains a subpattern of the form:

```
nand[nand[nand[a_,nand[a_,nand[nand[a_,a_],a_]]],b_],nand[nand[a_,nand[nand[a_,a_],a_
```

which can be unified with the input for the rule:

```
nand[nand[nand[a_,nand[a_,nand[nand[a_,a_],a_]]],nand[a_,nand[nand[a_,a_],a_]]],nand[
```

where these rules follow from Substitution Lemma 12 and Critical Pair Lemma 25 respectively.

Substitution Lemma 13

It can be shown that:

```
nand[nand[a,nand[nand[a,a],a]],nand[a,nand[a,nand[nand[a,a],a]]]]==a
```

Proof

We start by taking Critical Pair Lemma 38, and apply the substitution:

```
nand[nand[a_,nand[nand[a_,a_],a_]],nand[a_,nand[nand[a_,a_],a_]]]→a
```

which follows from Critical Pair Lemma 18.

Substitution Lemma 14

It can be shown that:

```
nand[a,nand[nand[a,nand[nand[a,a],a]],nand[a,nand[a,nand[nand[a,a],a]]]],nand[a,nand
```

```
nand[a,nand[nand[a,nand[nand[a,a],a]],nand[a,nand[a,nand[nand[a,a],a]]]]]==nand[a,nand
```

PROOF

We start by taking Critical Pair Lemma 36, and apply the substitution:

```
nand[nand[a_,nand[nand[a_,a_],a_]],nand[a_,nand[a_,nand[nand[a_,a_],a_]]]]→a
```

which follows from Substitution Lemma 13.

Substitution Lemma 15

It can be shown that:

```
nand[a,a]==nand[a,nand[nand[a,a],a]]
```

PROOF

We start by taking Substitution Lemma 14, and apply the substitution:

```
nand[nand[a_,nand[nand[a_,a_],a_]],nand[a_,nand[a_,nand[nand[a_,a_],a_]]]]→a
```

which follows from Substitution Lemma 13.

Substitution Lemma 16

It can be shown that:

```
nand[nand[a,nand[b,nand[nand[b,a],b]]],a]==nand[b,nand[nand[b,a],b]]
```

PROOF

We start by taking Critical Pair Lemma 32, and apply the substitution:

```
nand[nand[a_,nand[nand[a_,a_],a_]],nand[a_,nand[a_,nand[nand[a_,a_],a_]]]]→a
```

which follows from Substitution Lemma 13.

Substitution Lemma 17

It can be shown that:

```
nand[a,a]==nand[a,nand[nand[a,a],a]]
```

PROOF

We start by taking Substitution Lemma 6, and apply the substitution:

```
nand[nand[a_,nand[nand[a_,a_],a_]],nand[a_,nand[a_,nand[nand[a_,a_],a_]]]]→a
```

which follows from Substitution Lemma 13.

Critical Pair Lemma 39

The following expressions are equivalent:

```
nand[nand[a,nand[nand[a,a],a]],a]==nand[a,nand[a,nand[nand[a,nand[nand[a,a],a]],a]]]
```

PROOF

Note that the input for the rule:

```
nand[a_,nand[a_,nand[nand[a_,nand[nand[b_,nand[nand[b_,a_],b_]],a_]],a_]]]→nand[nand
```

contains a subpattern of the form:

```
nand[b_,nand[nand[b_,a_],b_]]
```

which can be unified with the input for the rule:

```
nand[a_,nand[nand[a_,a_],a_]]→nand[a,a]
```

where these rules follow from Critical Pair Lemma 14 and Substitution Lemma 15 respectively.

Substitution Lemma 18

It can be shown that:

`nand[nand[a,a],a]==nand[a,nand[a,nand[nand[a,nand[nand[a,a],a]],a]]]`

Proof

We start by taking Critical Pair Lemma 39, and apply the substitution:

`nand[a_,nand[nand[a_,a_],a_]]→nand[a,a]`

which follows from Substitution Lemma 15.

Substitution Lemma 19

It can be shown that:

`nand[nand[a,a],a]==nand[a,nand[a,nand[nand[a,a],a]]]`

Proof

We start by taking Substitution Lemma 18, and apply the substitution:

`nand[a_,nand[nand[a_,a_],a_]]→nand[a,a]`

which follows from Substitution Lemma 15.

Substitution Lemma 20

It can be shown that:

`nand[nand[a,a],a]==nand[a,nand[a,a]]`

Proof

We start by taking Substitution Lemma 19, and apply the substitution:

`nand[a_,nand[nand[a_,a_],a_]]→nand[a,a]`

which follows from Substitution Lemma 15.

Substitution Lemma 21

It can be shown that:

`nand[nand[a,a],nand[a,nand[nand[a,a],a]]]==a`

Proof

We start by taking Critical Pair Lemma 18, and apply the substitution:

`nand[a_,nand[nand[a_,a_],a_]]→nand[a,a]`

which follows from Substitution Lemma 15.

Substitution Lemma 22

It can be shown that:

`nand[nand[a,a],nand[a,a]]==a`

Proof

We start by taking Substitution Lemma 21, and apply the substitution:

`nand[a_,nand[nand[a_,a_],a_]]→nand[a,a]`

which follows from Substitution Lemma 15.

Critical Pair Lemma 40

The following expressions are equivalent:

`a==nand[nand[nand[a,b],a],nand[a,nand[a,nand[a,a]]]]`

Proof

Note that the input for the rule:

`nand[nand[nand[a_,b_],c_],nand[a_,nand[nand[a_,c_],a_]]]→c`

contains a subpattern of the form:

`nand[nand[a_,c_],a_]`

which can be unified with the input for the rule:

`nand[nand[a_,a_],a_]→nand[a,nand[a,a]]`

where these rules follow from Axiom 1 and Substitution Lemma 20 respectively.

Substitution Lemma 23

It can be shown that:

`nand[a,a]==nand[a,nand[a,nand[a,a]]]`

Proof

We start by taking Substitution Lemma 17, and apply the substitution:

`nand[nand[a_,a_],a_]→nand[a,nand[a,a]]`

which follows from Substitution Lemma 20.

Substitution Lemma 24

It can be shown that:

`a==nand[nand[nand[a,b],a],nand[a,a]]`

Proof

We start by taking Critical Pair Lemma 40, and apply the substitution:

`nand[a_,nand[a_,nand[a_,a_]]]→nand[a,a]`

which follows from Substitution Lemma 23.

Critical Pair Lemma 41

The following expressions are equivalent:

`nand[a,a]==nand[nand[nand[nand[a,a],b],nand[a,a]],a]`

Proof

Note that the input for the rule:

`nand[nand[nand[a_,b_],a_],nand[a_,a_]]→a`

contains a subpattern of the form:

`nand[a_,a_]`

which can be unified with the input for the rule:

`nand[nand[a_,a_],nand[a_,a_]]→a`

where these rules follow from Substitution Lemma 24 and Substitution Lemma 22 respectively.

Critical Pair Lemma 42

The following expressions are equivalent:

The following expressions are equivalent:

```
a==nand[nand[b,nand[nand[b,a],b]],nand[a,a]]
```

PROOF

Note that the input for the rule:

```
nand[nand[nand[a_,b_],a_],nand[a_,a_]]→a
```

contains a subpattern of the form:

```
nand[nand[a_,b_],a_]
```

which can be unified with the input for the rule:

```
nand[nand[a_,nand[b_,nand[nand[b_,a_],b_]]],a_]→nand[b,nand[nand[b,a],b]]
```

where these rules follow from Substitution Lemma 24 and Substitution Lemma 16 respectively.

Critical Pair Lemma 43

The following expressions are equivalent:

```
nand[a,a]==nand[nand[b,nand[nand[b,nand[a,a]],b]],a]
```

PROOF

Note that the input for the rule:

```
nand[nand[nand[nand[a_,a_],b_],nand[a_,a_]],a_]→nand[a,a]
```

contains a subpattern of the form:

```
nand[nand[nand[a_,a_],b_],nand[a_,a_]]
```

which can be unified with the input for the rule:

```
nand[nand[a_,nand[b_,nand[nand[b_,a_],b_]]],a_]→nand[b,nand[nand[b,a],b]]
```

where these rules follow from Critical Pair Lemma 41 and Substitution Lemma 16 respectively.

Substitution Lemma 25

It can be shown that:

```
nand[a,nand[a,nand[b,nand[nand[b,a],b]]]]==nand[b,nand[nand[b,a],b]]
```

PROOF

We start by taking Critical Pair Lemma 13, and apply the substitution:

```
nand[nand[a_,nand[b_,nand[nand[b_,a_],b_]]],a_]→nand[b,nand[nand[b,a],b]]
```

which follows from Substitution Lemma 16.

Critical Pair Lemma 44

The following expressions are equivalent:

```
nand[a,a]==nand[a,nand[b,nand[nand[b,nand[a,a]],b]]]
```

PROOF

Note that the input for the rule:

```
nand[nand[nand[a_,b_],c_],nand[a_,nand[nand[a_,c_],a_]]]→c
```

contains a subpattern of the form:

```
nand[nand[a_,b_],c_]
```

which can be unified with the input for the rule:

```
nand[nand[a_,nand[nand[a_,b_],a_]],nand[b_,b_]]→b
```

where these rules follow from Axiom 1 and Critical Pair Lemma 42 respectively.

Critical Pair Lemma 45

The following expressions are equivalent:

```
a==nand[nand[a,a],nand[b,nand[nand[b,a],b]]]
```

Proof

Note that the input for the rule:

```
nand[nand[nand[a_,b_],c_],nand[a_,nand[nand[a_,c_],a_]]]→c
```

contains a subpattern of the form:

```
nand[nand[a_,b_],c_]
```

which can be unified with the input for the rule:

```
nand[nand[a_,nand[nand[a_,nand[b_,b_]],a_]],b_]→nand[b,b]
```

where these rules follow from Axiom 1 and Critical Pair Lemma 43 respectively.

Critical Pair Lemma 46

The following expressions are equivalent:

```
nand[a,nand[nand[a,nand[nand[a,b],nand[a,b]]],a]]==nand[nand[nand[a,b],nand[a,b]],nand
```

Proof

Note that the input for the rule:

```
nand[a_,nand[nand[b_,c_],nand[nand[nand[b_,c_],nand[b_,nand[nand[b_,a_],b_]]],nand[b_
```

contains a subpattern of the form:

```
nand[nand[b_,c_],nand[b_,nand[nand[b_,a_],b_]]]
```

which can be unified with the input for the rule:

```
nand[a_,nand[b_,nand[nand[b_,nand[a_,a_]],b_]]]→nand[a,a]
```

where these rules follow from Critical Pair Lemma 1 and Critical Pair Lemma 44 respectively.

Substitution Lemma 26

It can be shown that:

```
nand[a,nand[nand[a,nand[nand[a,b],nand[a,b]]],a]]==nand[a,b]
```

Proof

We start by taking Critical Pair Lemma 46, and apply the substitution:

```
nand[nand[a_,a_],nand[b_,nand[nand[b_,a_],b_]]]→a
```

which follows from Critical Pair Lemma 45.

Critical Pair Lemma 47

The following expressions are equivalent:

```
nand[nand[nand[a,b],a],nand[a,a]]==nand[nand[nand[a,b],a],nand[nand[nand[nand[a,b],a],
```

Proof

Note that the input for the rule:

```
nand[a_,nand[nand[a_,nand[nand[a_,b_],nand[a_,b_]]],a_]]→nand[a,b]
```

contains a subpattern of the form:

`nand[a_,b_]`

which can be unified with the input for the rule:

`nand[nand[nand[a_,b_],a_],nand[a_,a_]]→a`

where these rules follow from Substitution Lemma 26 and Substitution Lemma 24 respectively.

Substitution Lemma 27

It can be shown that:

`a==nand[nand[nand[a,b],a],nand[nand[nand[nand[a,b],a],nand[a,nand[nand[nand[a,b],a],na`

Proof

We start by taking Critical Pair Lemma 47, and apply the substitution:

`nand[nand[nand[a_,b_],a_],nand[a_,a_]]→a`

which follows from Substitution Lemma 24.

Substitution Lemma 28

It can be shown that:

`a==nand[nand[nand[a,b],a],nand[nand[nand[nand[a,b],a],nand[a,a]],nand[nand[a,b],a]]]`

Proof

We start by taking Substitution Lemma 27, and apply the substitution:

`nand[nand[nand[a_,b_],a_],nand[a_,a_]]→a`

which follows from Substitution Lemma 24.

Substitution Lemma 29

It can be shown that:

`a==nand[nand[nand[a,b],a],nand[a,nand[nand[a,b],a]]]`

Proof

We start by taking Substitution Lemma 28, and apply the substitution:

`nand[nand[nand[a_,b_],a_],nand[a_,a_]]→a`

which follows from Substitution Lemma 24.

Critical Pair Lemma 48

The following expressions are equivalent:

`nand[nand[a,b],a]==nand[nand[a,nand[nand[a,b],a]],nand[nand[nand[a,b],a],nand[nand[nan`

Proof

Note that the input for the rule:

`nand[nand[nand[a_,b_],a_],nand[a_,nand[nand[a_,b_],a_]]]→a`

contains a subpattern of the form:

`nand[a_,b_]`

which can be unified with the input for the rule:

`nand[nand[nand[a_,b_],a_],nand[a_,a_]]→a`

where these rules follow from Substitution Lemma 29 and Substitution Lemma 24

respectively.

Substitution Lemma 30

It can be shown that:

`nand[nand[a,b],a]==nand[nand[a,nand[nand[a,b],a]],nand[nand[nand[a,b],a],nand[a,nand[n`

Proof

We start by taking Critical Pair Lemma 48, and apply the substitution:

`nand[nand[nand[a_,b_],a_],nand[a_,a_]]→a`

which follows from Substitution Lemma 24.

Substitution Lemma 31

It can be shown that:

`nand[nand[a,b],a]==nand[nand[a,nand[nand[a,b],a]],a]`

Proof

We start by taking Substitution Lemma 30, and apply the substitution:

`nand[nand[nand[a_,b_],a_],nand[a_,nand[nand[a_,b_],a_]]]→a`

which follows from Substitution Lemma 29.

Critical Pair Lemma 49

The following expressions are equivalent:

`nand[a,nand[nand[a,nand[nand[a,b],a]],a]]==nand[nand[nand[a,b],a],nand[nand[nand[a,b],`

Proof

Note that the input for the rule:

`nand[a_,nand[a_,nand[b_,nand[nand[b_,a_],b_]]]]→nand[b,nand[nand[b,a],b]]`

contains a subpattern of the form:

`nand[nand[b_,a_],b_]`

which can be unified with the input for the rule:

`nand[nand[a_,nand[nand[a_,b_],a_]],a_]→nand[nand[a,b],a]`

where these rules follow from Substitution Lemma 25 and Substitution Lemma 31 respectively.

Substitution Lemma 32

It can be shown that:

`nand[a,nand[nand[a,b],a]]==nand[nand[nand[a,b],a],nand[nand[nand[a,b],a],nand[a,nand[n`

Proof

We start by taking Critical Pair Lemma 49, and apply the substitution:

`nand[nand[a_,nand[nand[a_,b_],a_]],a_]→nand[nand[a,b],a]`

which follows from Substitution Lemma 31.

Substitution Lemma 33

It can be shown that:

`nand[a,nand[nand[a,b],a]]==nand[nand[nand[a,b],a],a]`

Proof

We start by taking Substitution Lemma 32, and apply the substitution:

`nand[nand[nand[a_,b_],a_],nand[a_,nand[nand[a_,b_],a_]]]→a`

which follows from Substitution Lemma 29.

Critical Pair Lemma 50

The following expressions are equivalent:

`nand[a,nand[nand[a,nand[nand[a,nand[nand[a,b],nand[a,b]]],a]],a]]==nand[nand[nand[a,b]`

Proof

Note that the input for the rule:

`nand[nand[nand[a_,b_],a_],a_]→nand[a,nand[nand[a,b],a]]`

contains a subpattern of the form:

`nand[a_,b_]`

which can be unified with the input for the rule:

`nand[a_,nand[nand[a_,nand[nand[a_,b_],nand[a_,b_]]],a_]]→nand[a,b]`

where these rules follow from Substitution Lemma 33 and Substitution Lemma 26 respectively.

Substitution Lemma 34

It can be shown that:

`nand[a,nand[nand[a,nand[nand[a,b],nand[a,b]]],a]]==nand[nand[nand[a,b],a],a]`

Proof

We start by taking Critical Pair Lemma 50, and apply the substitution:

`nand[nand[a_,nand[nand[a_,b_],a_]],a_]→nand[nand[a,b],a]`

which follows from Substitution Lemma 31.

Substitution Lemma 35

It can be shown that:

`nand[a,b]==nand[nand[nand[a,b],a],a]`

Proof

We start by taking Substitution Lemma 34, and apply the substitution:

`nand[a_,nand[nand[a_,nand[nand[a_,b_],nand[a_,b_]]],a_]]→nand[a,b]`

which follows from Substitution Lemma 26.

Substitution Lemma 36

It can be shown that:

`nand[a,b]==nand[a,nand[nand[a,b],a]]`

Proof

We start by taking Substitution Lemma 35, and apply the substitution:

`nand[nand[nand[a_,b_],a_],a_]→nand[a,nand[nand[a,b],a]]`

which follows from Substitution Lemma 33.

Substitution Lemma 37

It can be shown that:

`nand[nand[nand[a_,b_],a_],a_]→nand[a,b]`

Proof

We start by taking Substitution Lemma 33, and apply the substitution:

`nand[a_,nand[nand[a_,b_],a_]]→nand[a,b]`

which follows from Substitution Lemma 36.

Substitution Lemma 38

It can be shown that:

`nand[nand[nand[a,b],c],nand[a,c]]==c`

Proof

We start by taking Axiom 1, and apply the substitution:

`nand[a_,nand[nand[a_,b_],a_]]→nand[a,b]`

which follows from Substitution Lemma 36.

Substitution Lemma 39

It can be shown that:

`nand[nand[nand[a,b],a],nand[a,b]]==a`

Proof

We start by taking Substitution Lemma 29, and apply the substitution:

`nand[a_,nand[nand[a_,b_],a_]]→nand[a,b]`

which follows from Substitution Lemma 36.

Substitution Lemma 40

It can be shown that:

`nand[nand[a,a],nand[b,a]]==a`

Proof

We start by taking Critical Pair Lemma 45, and apply the substitution:

`nand[a_,nand[nand[a_,b_],a_]]→nand[a,b]`

which follows from Substitution Lemma 36.

Substitution Lemma 41

It can be shown that:

`nand[nand[a,b],nand[b,b]]==b`

Proof

We start by taking Critical Pair Lemma 42, and apply the substitution:

`nand[a_,nand[nand[a_,b_],a_]]→nand[a,b]`

which follows from Substitution Lemma 36.

Substitution Lemma 42

It can be shown that:

It can be shown that:

```
nand[a,nand[a,nand[b,a]]]==nand[b,nand[nand[b,a],b]]
```

PROOF

We start by taking Substitution Lemma 25, and apply the substitution:

```
nand[a_,nand[nand[a_,b_],a_]]→nand[a,b]
```

which follows from Substitution Lemma 36.

Substitution Lemma 43

It can be shown that:

```
nand[a,nand[a,nand[b,a]]]==nand[b,a]
```

PROOF

We start by taking Substitution Lemma 42, and apply the substitution:

```
nand[a_,nand[nand[a_,b_],a_]]→nand[a,b]
```

which follows from Substitution Lemma 36.

Substitution Lemma 44

It can be shown that:

```
nand[nand[a,b],nand[nand[c,a],b]]==b
```

PROOF

We start by taking Substitution Lemma 4, and apply the substitution:

```
nand[a_,nand[nand[a_,b_],a_]]→nand[a,b]
```

which follows from Substitution Lemma 36.

Substitution Lemma 45

It can be shown that:

```
nand[a,nand[nand[b,nand[nand[b,c],b]],nand[c,nand[nand[c,a],c]]]]==nand[c,nand[nand[c,
```

PROOF

We start by taking Critical Pair Lemma 7, and apply the substitution:

```
nand[a_,nand[nand[a_,b_],a_]]→nand[a,b]
```

which follows from Substitution Lemma 36.

Substitution Lemma 46

It can be shown that:

```
nand[a,nand[nand[b,c],nand[c,nand[nand[c,a],c]]]]==nand[c,nand[nand[c,a],c]]
```

PROOF

We start by taking Substitution Lemma 45, and apply the substitution:

```
nand[a_,nand[nand[a_,b_],a_]]→nand[a,b]
```

which follows from Substitution Lemma 36.

Substitution Lemma 47

It can be shown that:

```
nand[a,nand[nand[b,c],nand[c,a]]]==nand[c,nand[nand[c,a],c]]
```

Proof

We start by taking Substitution Lemma 46, and apply the substitution:

```
nand[a_,nand[nand[a_,b_],a_]]→nand[a,b]
```

which follows from Substitution Lemma 36.

Substitution Lemma 48

It can be shown that:

```
nand[a,nand[nand[b,c],nand[c,a]]]==nand[c,a]
```

Proof

We start by taking Substitution Lemma 47, and apply the substitution:

```
nand[a_,nand[nand[a_,b_],a_]]→nand[a,b]
```

which follows from Substitution Lemma 36.

Substitution Lemma 49

It can be shown that:

```
nand[a,nand[nand[b,c],nand[b,nand[nand[b,a],b]]]]==nand[b,nand[nand[b,a],b]]
```

Proof

We start by taking Critical Pair Lemma 1, and apply the substitution:

```
nand[a_,nand[nand[a_,b_],a_]]→nand[a,b]
```

which follows from Substitution Lemma 36.

Substitution Lemma 50

It can be shown that:

```
nand[a,nand[nand[b,c],nand[b,a]]]==nand[b,nand[nand[b,a],b]]
```

Proof

We start by taking Substitution Lemma 49, and apply the substitution:

```
nand[a_,nand[nand[a_,b_],a_]]→nand[a,b]
```

which follows from Substitution Lemma 36.

Substitution Lemma 51

It can be shown that:

```
nand[a,nand[nand[b,c],nand[b,a]]]==nand[b,a]
```

Proof

We start by taking Substitution Lemma 50, and apply the substitution:

```
nand[a_,nand[nand[a_,b_],a_]]→nand[a,b]
```

which follows from Substitution Lemma 36.

Substitution Lemma 52

It can be shown that:

```
nand[a,nand[b,nand[nand[c,b],nand[nand[nand[c,b],a],nand[c,b]]]]]==nand[nand[c,b],nand
```

Proof

We start by taking Critical Pair Lemma 10, and apply the substitution:

`nand[a_,nand[nand[a_,b_],a_]]→nand[a,b]`

which follows from Substitution Lemma 36.

Substitution Lemma 53

It can be shown that:

`nand[a,nand[b,nand[nand[c,b],a]]]==nand[nand[c,b],nand[nand[nand[c,b],a],nand[c,b]]]`

Proof

We start by taking Substitution Lemma 52, and apply the substitution:

`nand[a_,nand[nand[a_,b_],a_]]→nand[a,b]`

which follows from Substitution Lemma 36.

Substitution Lemma 54

It can be shown that:

`nand[a,nand[b,nand[nand[c,b],a]]]==nand[nand[c,b],a]`

Proof

We start by taking Substitution Lemma 53, and apply the substitution:

`nand[a_,nand[nand[a_,b_],a_]]→nand[a,b]`

which follows from Substitution Lemma 36.

Critical Pair Lemma 51

The following expressions are equivalent:

`a==nand[nand[a,a],nand[a,b]]`

Proof

Note that the input for the rule:

`nand[nand[a_,a_],nand[b_,a_]]→a`

contains a subpattern of the form:

`nand[b_,a_]`

which can be unified with the input for the rule:

`Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[802152,nand[a_`

where these rules follow from Substitution Lemma 40 and Substitution Lemma 37 respectively.

Critical Pair Lemma 52

The following expressions are equivalent:

`a==nand[nand[a,b],nand[a,a]]`

Proof

Note that the input for the rule:

`nand[nand[a_,b_],nand[b_,b_]]→b`

contains a subpattern of the form:

`nand[a_,b_]`

which can be unified with the input for the rule:

`Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[802152,nand[a_`

where these rules follow from Substitution Lemma 41 and Substitution Lemma 37 respectively.

Conclusion 1

We obtain the conclusion:

`True`

Proof

Take Hypothesis 1, and apply the substitution:

`nand[nand[a_,a_],nand[a_,b_]]→a`

which follows from Critical Pair Lemma 51.

Critical Pair Lemma 53

The following expressions are equivalent:

`nand[nand[a,b],a]==nand[a,nand[a,b]]`

Proof

Note that the input for the rule:

`nand[a_,nand[a_,nand[b_,a_]]]→nand[b,a]`

contains a subpattern of the form:

`nand[a_,nand[b_,a_]]`

which can be unified with the input for the rule:

`nand[a_,nand[nand[a_,b_],a_]]→nand[a,b]`

where these rules follow from Substitution Lemma 43 and Substitution Lemma 36 respectively.

Critical Pair Lemma 54

The following expressions are equivalent:

`a==nand[nand[b,a],nand[nand[b,c],a]]`

Proof

Note that the input for the rule:

`nand[nand[nand[a_,b_],c_],nand[a_,c_]]→c`

contains a subpattern of the form:

`nand[a_,b_]`

which can be unified with the input for the rule:

`nand[nand[a_,b_],nand[a_,a_]]→a`

where these rules follow from Substitution Lemma 38 and Critical Pair Lemma 52 respectively.

Substitution Lemma 55

It can be shown that:

`nand[nand[a,b],nand[nand[a,b],a]]==a`

Proof

We start by taking Substitution Lemma 39, and apply the substitution:

`nand[nand[a_,b_],a_]→nand[a,nand[a,b]]`

which follows from Critical Pair Lemma 53.

Substitution Lemma 56

It can be shown that:

`nand[nand[a,b],nand[a,nand[a,b]]]==a`

Proof

We start by taking Substitution Lemma 55, and apply the substitution:

`nand[nand[a_,b_],a_]→nand[a,nand[a,b]]`

which follows from Critical Pair Lemma 53.

Critical Pair Lemma 55

The following expressions are equivalent:

`nand[a,b]==nand[b,nand[nand[c,nand[b,b]],nand[a,b]]]`

Proof

Note that the input for the rule:

`nand[nand[a_,b_],nand[nand[c_,a_],b_]]→b`

contains a subpattern of the form:

`nand[a_,b_]`

which can be unified with the input for the rule:

`nand[nand[a_,a_],nand[b_,a_]]→a`

where these rules follow from Substitution Lemma 44 and Substitution Lemma 40 respectively.

Critical Pair Lemma 56

The following expressions are equivalent:

`nand[a,b]==nand[a,nand[nand[c,nand[a,a]],nand[a,b]]]`

Proof

Note that the input for the rule:

`nand[nand[a_,b_],nand[nand[c_,a_],b_]]→b`

contains a subpattern of the form:

`nand[a_,b_]`

which can be unified with the input for the rule:

`nand[nand[a_,a_],nand[a_,b_]]→a`

where these rules follow from Substitution Lemma 44 and Critical Pair Lemma 51 respectively.

Critical Pair Lemma 57

The following expressions are equivalent:

`nand[nand[a,b],c]==nand[nand[a,nand[nand[a,b],c]],c]`

Proof

Note that the input for the rule:

`nand[nand[a_,b_],nand[nand[a_,c_],b_]]→b`

contains a subpattern of the form:

`nand[nand[a_,c_],b_]`

which can be unified with the input for the rule:

`nand[nand[a_,b_],nand[nand[a_,c_],b_]]→b`

where these rules follow from Critical Pair Lemma 54 and Critical Pair Lemma 54 respectively.

Critical Pair Lemma 58

The following expressions are equivalent:

`nand[a,b]==nand[b,nand[nand[a,b],nand[nand[a,b],a]]]`

Proof

Note that the input for the rule:

`nand[a_,nand[nand[b_,c_],nand[c_,a_]]]→nand[c,a]`

contains a subpattern of the form:

`nand[nand[b_,c_],nand[c_,a_]]`

which can be unified with the input for the rule:

`nand[nand[a_,b_],a_]→nand[a,nand[a,b]]`

where these rules follow from Substitution Lemma 48 and Critical Pair Lemma 53 respectively.

Substitution Lemma 57

It can be shown that:

`nand[a,b]==nand[b,nand[nand[a,b],nand[a,nand[a,b]]]]`

Proof

We start by taking Critical Pair Lemma 58, and apply the substitution:

`nand[nand[a_,b_],a_]→nand[a,nand[a,b]]`

which follows from Critical Pair Lemma 53.

Substitution Lemma 58

It can be shown that:

`nand[a,b]==nand[b,a]`

Proof

We start by taking Substitution Lemma 57, and apply the substitution:

`nand[nand[a_,b_],nand[a_,nand[a_,b_]]]→a`

which follows from Substitution Lemma 56.

Critical Pair Lemma 59

The following expressions are equivalent:

`nand[nand[a,b],nand[nand[c,a],b]]==nand[nand[nand[c,a],b],nand[nand[y1,nand[a,b]],b]]`

Proof

Note that the input for the rule:

Note that the input for the rule:

`nand[a_,nand[nand[b_,c_],nand[c_,a_]]]→nand[c,a]`

contains a subpattern of the form:

`nand[c_,a_]`

which can be unified with the input for the rule:

`nand[nand[a_,b_],nand[nand[c_,a_],b_]]→b`

where these rules follow from Substitution Lemma 48 and Substitution Lemma 44 respectively.

Substitution Lemma 59

It can be shown that:

`a==nand[nand[nand[b,c],a],nand[nand[y1,nand[c,a]],a]]`

Proof

We start by taking Critical Pair Lemma 59, and apply the substitution:

`nand[nand[a_,b_],nand[nand[c_,a_],b_]]→b`

which follows from Substitution Lemma 44.

Critical Pair Lemma 60

The following expressions are equivalent:

`nand[a,b]==nand[b,nand[nand[c,a],nand[b,a]]]`

Proof

Note that the input for the rule:

`nand[a_,nand[nand[b_,c_],nand[c_,a_]]]→nand[c,a]`

contains a subpattern of the form:

`nand[c_,a_]`

which can be unified with the input for the rule:

`nand[a_,b_]↔nand[b_,a_]`

where these rules follow from Substitution Lemma 48 and Substitution Lemma 58 respectively.

Critical Pair Lemma 61

The following expressions are equivalent:

`a==nand[nand[a,b],nand[nand[b,c],a]]`

Proof

Note that the input for the rule:

`nand[nand[a_,b_],nand[nand[a_,c_],b_]]→b`

contains a subpattern of the form:

`nand[a_,b_]`

which can be unified with the input for the rule:

`nand[a_,b_]↔nand[b_,a_]`

where these rules follow from Critical Pair Lemma 54 and Substitution Lemma 58 respectively.

Critical Pair Lemma 62

The following expressions are equivalent:

`a==nand[nand[b,a],nand[a,nand[b,c]]]`

Proof

Note that the input for the rule:

`nand[nand[a_,b_],nand[nand[a_,c_],b_]]→b`

contains a subpattern of the form:

`nand[nand[a_,c_],b_]`

which can be unified with the input for the rule:

`nand[a_,b_]↔nand[b_,a_]`

where these rules follow from Critical Pair Lemma 54 and Substitution Lemma 58 respectively.

Critical Pair Lemma 63

The following expressions are equivalent:

`a==nand[nand[a,b],nand[nand[c,b],a]]`

Proof

Note that the input for the rule:

`nand[nand[a_,b_],nand[nand[c_,a_],b_]]→b`

contains a subpattern of the form:

`nand[a_,b_]`

which can be unified with the input for the rule:

`nand[a_,b_]↔nand[b_,a_]`

where these rules follow from Substitution Lemma 44 and Substitution Lemma 58 respectively.

Critical Pair Lemma 64

The following expressions are equivalent:

`a==nand[nand[b,a],nand[a,nand[c,b]]]`

Proof

Note that the input for the rule:

`nand[nand[a_,b_],nand[nand[c_,a_],b_]]→b`

contains a subpattern of the form:

`nand[nand[c_,a_],b_]`

which can be unified with the input for the rule:

`nand[a_,b_]↔nand[b_,a_]`

where these rules follow from Substitution Lemma 44 and Substitution Lemma 58 respectively.

Critical Pair Lemma 65

The following expressions are equivalent:

Out[ ]=

a==nand[nand[a,b],nand[a,nand[b,c]]]

PROOF

Note that the input for the rule:

nand[nand[a_,b_],nand[nand[b_,c_],a_]]→a

contains a subpattern of the form:

nand[nand[b_,c_],a_]

which can be unified with the input for the rule:

nand[a_,b_]↔nand[b_,a_]

where these rules follow from Critical Pair Lemma 61 and Substitution Lemma 58 respectively.

Critical Pair Lemma 66

The following expressions are equivalent:

nand[a,b]==nand[b,nand[nand[a,b],nand[nand[b,b],c]]]

PROOF

Note that the input for the rule:

nand[nand[a_,b_],nand[b_,nand[a_,c_]]]→b

contains a subpattern of the form:

nand[a_,b_]

which can be unified with the input for the rule:

nand[nand[a_,a_],nand[b_,a_]]→a

where these rules follow from Critical Pair Lemma 62 and Substitution Lemma 40 respectively.

Critical Pair Lemma 67

The following expressions are equivalent:

nand[a,b]==nand[nand[nand[b,c],nand[a,b]],a]

PROOF

Note that the input for the rule:

nand[nand[a_,b_],nand[b_,nand[a_,c_]]]→b

contains a subpattern of the form:

nand[b_,nand[a_,c_]]

which can be unified with the input for the rule:

nand[nand[a_,b_],nand[nand[b_,c_],a_]]→a

where these rules follow from Critical Pair Lemma 62 and Critical Pair Lemma 61 respectively.

Critical Pair Lemma 68

The following expressions are equivalent:

nand[a,nand[b,c]]==nand[nand[nand[a,nand[b,c]],b],a]

PROOF

Note that the input for the rule:

`nand[nand[a_,b_],nand[nand[b_,c_],a_]]→a`

contains a subpattern of the form:

`nand[nand[b_,c_],a_]`

which can be unified with the input for the rule:

`nand[nand[a_,b_],nand[b_,nand[a_,c_]]]→b`

where these rules follow from Critical Pair Lemma 61 and Critical Pair Lemma 62 respectively.

Critical Pair Lemma 69

The following expressions are equivalent:

`nand[nand[a,b],nand[b,nand[a,c]]]⩵nand[nand[b,nand[a,c]],nand[nand[y1,nand[a,b]],b]]`

Proof

Note that the input for the rule:

`nand[a_,nand[nand[b_,c_],nand[c_,a_]]]→nand[c,a]`

contains a subpattern of the form:

`nand[c_,a_]`

which can be unified with the input for the rule:

`nand[nand[a_,b_],nand[b_,nand[a_,c_]]]→b`

where these rules follow from Substitution Lemma 48 and Critical Pair Lemma 62 respectively.

Substitution Lemma 60

It can be shown that:

`a⩵nand[nand[a,nand[b,c]],nand[nand[y1,nand[b,a]],a]]`

Proof

We start by taking Critical Pair Lemma 69, and apply the substitution:

`nand[nand[a_,b_],nand[b_,nand[a_,c_]]]→b`

which follows from Critical Pair Lemma 62.

Critical Pair Lemma 70

The following expressions are equivalent:

`a⩵nand[nand[a,b],nand[a,nand[c,b]]]`

Proof

Note that the input for the rule:

`nand[nand[a_,b_],nand[nand[c_,b_],a_]]→a`

contains a subpattern of the form:

`nand[nand[c_,b_],a_]`

which can be unified with the input for the rule:

`nand[a_,b_]↔nand[b_,a_]`

where these rules follow from Critical Pair Lemma 63 and Substitution Lemma 58 respectively.

Critical Pair Lemma 71

The following expressions are equivalent:

The following expressions are equivalent:

`nand[a,b]==nand[b,nand[nand[a,b],nand[c,nand[b,b]]]]`

Proof

Note that the input for the rule:

`nand[nand[a_,b_],nand[b_,nand[c_,a_]]]→b`

contains a subpattern of the form:

`nand[a_,b_]`

which can be unified with the input for the rule:

`nand[nand[a_,a_],nand[b_,a_]]→a`

where these rules follow from Critical Pair Lemma 64 and Substitution Lemma 40 respectively.

Critical Pair Lemma 72

The following expressions are equivalent:

`nand[a,b]==nand[nand[nand[a,b],nand[b,c]],a]`

Proof

Note that the input for the rule:

`nand[nand[a_,b_],nand[a_,nand[b_,c_]]]→a`

contains a subpattern of the form:

`nand[a_,nand[b_,c_]]`

which can be unified with the input for the rule:

`nand[nand[a_,b_],nand[nand[b_,c_],a_]]→a`

where these rules follow from Critical Pair Lemma 65 and Critical Pair Lemma 61 respectively.

Critical Pair Lemma 73

The following expressions are equivalent:

`nand[nand[a,b],nand[a,nand[b,c]]]==nand[nand[a,nand[b,c]],nand[nand[y1,nand[a,b]],a]]`

Proof

Note that the input for the rule:

`nand[a_,nand[nand[b_,c_],nand[c_,a_]]]→nand[c,a]`

contains a subpattern of the form:

`nand[c_,a_]`

which can be unified with the input for the rule:

`nand[nand[a_,b_],nand[a_,nand[b_,c_]]]→a`

where these rules follow from Substitution Lemma 48 and Critical Pair Lemma 65 respectively.

Substitution Lemma 61

It can be shown that:

`a==nand[nand[a,nand[b,c]],nand[nand[y1,nand[a,b]],a]]`

Proof

We start by taking Critical Pair Lemma 73, and apply the substitution:

We start by taking Critical Pair Lemma 73, and apply the substitution:

`nand[nand[a_,b_],nand[a_,nand[b_,c_]]]→a`

which follows from Critical Pair Lemma 65.

Critical Pair Lemma 74

The following expressions are equivalent:

`nand[nand[a,b],nand[a,nand[c,b]]]==nand[nand[a,nand[c,b]],nand[nand[nand[a,b],y1],a]]`

Proof

Note that the input for the rule:

`nand[a_,nand[nand[b_,c_],nand[b_,a_]]]→nand[b,a]`

contains a subpattern of the form:

`nand[b_,a_]`

which can be unified with the input for the rule:

`nand[nand[a_,b_],nand[a_,nand[c_,b_]]]→a`

where these rules follow from Substitution Lemma 51 and Critical Pair Lemma 70 respectively.

Substitution Lemma 62

It can be shown that:

`a==nand[nand[a,nand[b,c]],nand[nand[nand[a,c],y1],a]]`

Proof

We start by taking Critical Pair Lemma 74, and apply the substitution:

`nand[nand[a_,b_],nand[a_,nand[c_,b_]]]→a`

which follows from Critical Pair Lemma 70.

Substitution Lemma 63

It can be shown that:

`nand[a,b]==nand[a,nand[nand[b,c],nand[a,b]]]`

Proof

We start by taking Critical Pair Lemma 67, and apply the substitution:

`nand[a_,b_]→nand[b,a]`

which follows from Substitution Lemma 58.

Substitution Lemma 64

It can be shown that:

`nand[a,b]==nand[a,nand[nand[a,b],nand[b,c]]]`

Proof

We start by taking Critical Pair Lemma 72, and apply the substitution:

`nand[a_,b_]→nand[b,a]`

which follows from Substitution Lemma 58.

Critical Pair Lemma 75

The following expressions are equivalent:

`nand[nand[a,nand[b,a]],b]==nand[b,b]`

PROOF

Note that the input for the rule:

`nand[a_,nand[b_,nand[nand[c_,b_],a_]]]→nand[nand[c,b],a]`

contains a subpattern of the form:

`nand[b_,nand[nand[c_,b_],a_]]`

which can be unified with the input for the rule:

`nand[nand[a_,b_],nand[nand[b_,c_],a_]]→a`

where these rules follow from Substitution Lemma 54 and Critical Pair Lemma 61 respectively.

Substitution Lemma 65

It can be shown that:

`nand[a,nand[b,nand[a,b]]]==nand[a,a]`

PROOF

We start by taking Critical Pair Lemma 75, and apply the substitution:

`nand[a_,b_]→nand[b,a]`

which follows from Substitution Lemma 58.

Critical Pair Lemma 76

The following expressions are equivalent:

`a==nand[nand[a,nand[b,nand[c,b]]],nand[nand[c,c],a]]`

PROOF

Note that the input for the rule:

`nand[nand[a_,b_],nand[nand[c_,b_],a_]]→a`

contains a subpattern of the form:

`nand[c_,b_]`

which can be unified with the input for the rule:

`nand[a_,nand[b_,nand[a_,b_]]]→nand[a,a]`

where these rules follow from Critical Pair Lemma 63 and Substitution Lemma 65 respectively.

Substitution Lemma 66

It can be shown that:

`nand[nand[a,b],c]==nand[c,nand[a,nand[nand[a,b],c]]]`

PROOF

We start by taking Critical Pair Lemma 57, and apply the substitution:

`nand[a_,b_]→nand[b,a]`

which follows from Substitution Lemma 58.

Substitution Lemma 67

It can be shown that:

It can be shown that:

`nand[a,nand[b,c]]==nand[a,nand[nand[a,nand[b,c]],b]]`

PROOF

We start by taking Critical Pair Lemma 68, and apply the substitution:

`nand[a_,b_]→nand[b,a]`

which follows from Substitution Lemma 58.

Substitution Lemma 68

It can be shown that:

`nand[a,nand[b,c]]==nand[a,nand[b,nand[a,nand[b,c]]]]`

PROOF

We start by taking Substitution Lemma 67, and apply the substitution:

`nand[a_,b_]→nand[b,a]`

which follows from Substitution Lemma 58.

Critical Pair Lemma 77

The following expressions are equivalent:

`nand[a,nand[b,a]]==nand[a,nand[b,b]]`

PROOF

Note that the input for the rule:

`nand[a_,nand[b_,nand[a_,nand[b_,c_]]]]→nand[a,nand[b,c]]`

contains a subpattern of the form:

`nand[b_,nand[a_,nand[b_,c_]]]`

which can be unified with the input for the rule:

`nand[a_,nand[b_,nand[a_,b_]]]→nand[a,a]`

where these rules follow from Substitution Lemma 68 and Substitution Lemma 65 respectively.

Substitution Lemma 69

It can be shown that:

`nand[a,nand[b,a]]==nand[a,nand[b,b]]`

PROOF

We start by taking Hypothesis 2, and apply the substitution:

`nand[a_,b_]→nand[b,a]`

which follows from Substitution Lemma 58.

Conclusion 2

We obtain the conclusion:

`True`

PROOF

Take Substitution Lemma 69, and apply the substitution:

`nand[a_,nand[b_,b_]]→nand[a,nand[b,a]]`

which follows from Critical Pair Lemma 77.

Critical Pair Lemma 78

The following expressions are equivalent:

`nand[a,nand[b,b]]==nand[a,nand[a,b]]`

Proof

Note that the input for the rule:

`nand[a_,nand[b_,a_]]↔nand[a_,nand[b_,b_]]`

contains a subpattern of the form:

`nand[b_,a_]`

which can be unified with the input for the rule:

`nand[a_,b_]↔nand[b_,a_]`

where these rules follow from Critical Pair Lemma 77 and Substitution Lemma 58 respectively.

Critical Pair Lemma 79

The following expressions are equivalent:

`nand[a,b]==nand[b,nand[b,nand[a,a]]]`

Proof

Note that the input for the rule:

`nand[a_,nand[a_,nand[b_,a_]]]→nand[b,a]`

contains a subpattern of the form:

`nand[a_,nand[b_,a_]]`

which can be unified with the input for the rule:

`nand[a_,nand[b_,a_]]↔nand[a_,nand[b_,b_]]`

where these rules follow from Substitution Lemma 43 and Critical Pair Lemma 77 respectively.

Critical Pair Lemma 80

The following expressions are equivalent:

`a==nand[nand[a,nand[b,c]],nand[a,nand[c,nand[b,b]]]]`

Proof

Note that the input for the rule:

`nand[nand[a_,b_],nand[a_,nand[c_,b_]]]→a`

contains a subpattern of the form:

`nand[c_,b_]`

which can be unified with the input for the rule:

`nand[a_,nand[b_,a_]]↔nand[a_,nand[b_,b_]]`

where these rules follow from Critical Pair Lemma 70 and Critical Pair Lemma 77 respectively.

Critical Pair Lemma 81

The following expressions are equivalent:

`nand[a,nand[b,a]]==nand[nand[b,b],a]`

Proof

Note that the input for the rule:

`nand[a_,nand[b_,a_]]↔nand[a_,nand[b_,b_]]`

contains a subpattern of the form:

`nand[a_,nand[b_,b_]]`

which can be unified with the input for the rule:

`nand[a_,b_]↔nand[b_,a_]`

where these rules follow from Critical Pair Lemma 77 and Substitution Lemma 58 respectively.

Critical Pair Lemma 82

The following expressions are equivalent:

`nand[a,nand[nand[b,b],a]]==nand[a,b]`

Proof

Note that the input for the rule:

`nand[a_,nand[b_,a_]]↔nand[a_,nand[b_,b_]]`

contains a subpattern of the form:

`nand[b_,b_]`

which can be unified with the input for the rule:

`nand[nand[a_,b_],nand[b_,b_]]→b`

where these rules follow from Critical Pair Lemma 77 and Substitution Lemma 41 respectively.

Critical Pair Lemma 83

The following expressions are equivalent:

`nand[nand[a,a],b]==nand[b,nand[b,a]]`

Proof

Note that the input for the rule:

`nand[a_,nand[a_,nand[b_,b_]]]→nand[b,a]`

contains a subpattern of the form:

`nand[b_,b_]`

which can be unified with the input for the rule:

`nand[nand[a_,b_],nand[b_,b_]]→b`

where these rules follow from Critical Pair Lemma 79 and Substitution Lemma 41 respectively.

Critical Pair Lemma 84

The following expressions are equivalent:

`a==nand[nand[a,nand[b,c]],nand[a,nand[nand[b,b],c]]]`

Proof

Note that the input for the rule:

`nand[nand[a_,b_],nand[a_,nand[c_,b_]]]→a`

contains a subpattern of the form:

`nand[c_,b_]`

which can be unified with the input for the rule:

`nand[a_,nand[b_,a_]]↔nand[nand[b_,b_],a_]`

where these rules follow from Critical Pair Lemma 70 and Critical Pair Lemma 81 respectively.

Critical Pair Lemma 85

The following expressions are equivalent:

`a==nand[nand[nand[b,c],a],nand[a,nand[nand[b,b],c]]]`

Proof

Note that the input for the rule:

`nand[nand[a_,b_],nand[b_,nand[c_,a_]]]→b`

contains a subpattern of the form:

`nand[c_,a_]`

which can be unified with the input for the rule:

`nand[a_,nand[b_,a_]]↔nand[nand[b_,b_],a_]`

where these rules follow from Critical Pair Lemma 64 and Critical Pair Lemma 81 respectively.

Critical Pair Lemma 86

The following expressions are equivalent:

`a==nand[nand[a,nand[b,c]],nand[nand[nand[b,b],c],a]]`

Proof

Note that the input for the rule:

`nand[nand[a_,b_],nand[nand[c_,b_],a_]]→a`

contains a subpattern of the form:

`nand[c_,b_]`

which can be unified with the input for the rule:

`nand[a_,nand[b_,a_]]↔nand[nand[b_,b_],a_]`

where these rules follow from Critical Pair Lemma 63 and Critical Pair Lemma 81 respectively.

Critical Pair Lemma 87

The following expressions are equivalent:

`nand[a,b]==nand[b,nand[nand[c,nand[b,c]],nand[a,b]]]`

Proof

Note that the input for the rule:

`nand[a_,nand[nand[b_,nand[a_,a_]],nand[c_,a_]]]→nand[c,a]`

contains a subpattern of the form:

`nand[b_,nand[a_,a_]]`

which can be unified with the input for the rule:

`nand[a_,nand[b_,a_]]↔nand[a_,nand[b_,b_]]` `nand[a_,nand[b_,a_]]↔nand[a_,nand[b_,b_]]`

where these rules follow from Critical Pair Lemma 55 and Critical Pair Lemma 77 respectively.

Critical Pair Lemma 88

The following expressions are equivalent:

`nand[a,b]==nand[a,nand[nand[c,nand[a,c]],nand[a,b]]]`

Proof

Note that the input for the rule:

`nand[a_,nand[nand[b_,nand[a_,a_]],nand[a_,c_]]]→nand[a,c]`

contains a subpattern of the form:

`nand[b_,nand[a_,a_]]`

which can be unified with the input for the rule:

`nand[a_,nand[b_,a_]]↔nand[a_,nand[b_,b_]]`

where these rules follow from Critical Pair Lemma 56 and Critical Pair Lemma 77 respectively.

Substitution Lemma 70

It can be shown that:

`a==nand[nand[nand[b,c],a],nand[a,nand[y1,nand[c,a]]]]`

Proof

We start by taking Substitution Lemma 59, and apply the substitution:

`nand[a_,b_]→nand[b,a]`

which follows from Substitution Lemma 58.

Critical Pair Lemma 89

The following expressions are equivalent:

`a==nand[nand[a,nand[b,nand[c,a]]],nand[nand[y1,c],a]]`

Proof

Note that the input for the rule:

`nand[nand[nand[a_,b_],c_],nand[c_,nand[y1_,nand[b_,c_]]]]→c`

contains a subpattern of the form:

`nand[nand[nand[a_,b_],c_],nand[c_,nand[y1_,nand[b_,c_]]]]`

which can be unified with the input for the rule:

`nand[a_,b_]↔nand[b_,a_]`

where these rules follow from Substitution Lemma 70 and Substitution Lemma 58 respectively.

Substitution Lemma 71

It can be shown that:

`a==nand[nand[a,nand[b,c]],nand[a,nand[y1,nand[b,a]]]]`

Proof

We start by taking Substitution Lemma 60, and apply the substitution:

`nand[a_,b_]→nand[b,a]`

which follows from Substitution Lemma 58.

Critical Pair Lemma 90

The following expressions are equivalent:

`a==nand[nand[a,nand[b,nand[c,a]]],nand[a,nand[c,y1]]]`

Proof

Note that the input for the rule:

`nand[nand[a_,nand[b_,c_]],nand[a_,nand[y1_,nand[b_,a_]]]]→a`

contains a subpattern of the form:

`nand[nand[a_,nand[b_,c_]],nand[a_,nand[y1_,nand[b_,a_]]]]`

which can be unified with the input for the rule:

`nand[a_,b_]↔nand[b_,a_]`

where these rules follow from Substitution Lemma 71 and Substitution Lemma 58 respectively.

Substitution Lemma 72

It can be shown that:

`a==nand[nand[a,nand[b,c]],nand[a,nand[y1,nand[a,b]]]]`

Proof

We start by taking Substitution Lemma 61, and apply the substitution:

`nand[a_,b_]→nand[b,a]`

which follows from Substitution Lemma 58.

Substitution Lemma 73

It can be shown that:

`a==nand[nand[a,nand[b,c]],nand[a,nand[nand[a,c],y1]]]`

Proof

We start by taking Substitution Lemma 62, and apply the substitution:

`nand[a_,b_]→nand[b,a]`

which follows from Substitution Lemma 58.

Critical Pair Lemma 91

The following expressions are equivalent:

`a==nand[nand[a,nand[nand[a,b],c]],nand[a,nand[y1,b]]]`

Proof

Note that the input for the rule:

`nand[nand[a_,nand[b_,c_]],nand[a_,nand[nand[a_,c_],y1_]]]→a`

contains a subpattern of the form:

`nand[nand[a_,nand[b_,c_]],nand[a_,nand[nand[a_,c_],y1_]]]`

which can be unified with the input for the rule:

`nand[a_,b_]↔nand[b_,a_]`

where these rules follow from Substitution Lemma 73 and Substitution Lemma 58

respectively.

Critical Pair Lemma 92

The following expressions are equivalent:

```
nand[a,nand[b,nand[b,b]]]==nand[nand[nand[a,nand[b,nand[b,b]]],nand[nand[b,b],a]],a]
```

Proof

Note that the input for the rule:

```
nand[nand[a_,nand[b_,c_]],nand[a_,nand[c_,nand[b_,b_]]]]→a
```

contains a subpattern of the form:

```
nand[a_,nand[c_,nand[b_,b_]]]
```

which can be unified with the input for the rule:

```
nand[nand[a_,nand[b_,c_]],nand[a_,nand[c_,nand[b_,b_]]]]→a
```

where these rules follow from Critical Pair Lemma 80 and Critical Pair Lemma 80 respectively.

Substitution Lemma 74

It can be shown that:

```
nand[a,nand[b,nand[b,b]]]==nand[a,a]
```

Proof

We start by taking Critical Pair Lemma 92, and apply the substitution:

```
nand[nand[a_,nand[b_,nand[c_,b_]]],nand[nand[c_,c_],a_]]→a
```

which follows from Critical Pair Lemma 76.

Critical Pair Lemma 93

The following expressions are equivalent:

```
nand[a,a]==nand[nand[b,nand[b,b]],a]
```

Proof

Note that the input for the rule:

```
nand[a_,nand[b_,nand[b_,b_]]]↔nand[a_,a_]
```

contains a subpattern of the form:

```
nand[a_,nand[b_,nand[b_,b_]]]
```

which can be unified with the input for the rule:

```
nand[a_,b_]↔nand[b_,a_]
```

where these rules follow from Substitution Lemma 74 and Substitution Lemma 58 respectively.

Critical Pair Lemma 94

The following expressions are equivalent:

```
nand[nand[a,nand[b,nand[a,b]]],nand[c,nand[c,c]]]==a
```

Proof

Note that the input for the rule:

```
nand[a_,nand[b_,nand[b_,b_]]]↔nand[a_,a_]
```

contains a subpattern of the form:

contains a subpattern of the form:

`nand[a_,a_]`

which can be unified with the input for the rule:

`nand[nand[a_,nand[b_,c_]],nand[a_,nand[y1_,nand[a_,b_]]]]→a`

where these rules follow from Substitution Lemma 74 and Substitution Lemma 72 respectively.

Substitution Lemma 75

It can be shown that:

`nand[nand[a,a],nand[b,nand[b,b]]]==a`

Proof

We start by taking Critical Pair Lemma 94, and apply the substitution:

`nand[a_,nand[b_,nand[a_,b_]]]→nand[a,a]`

which follows from Substitution Lemma 65.

Critical Pair Lemma 95

The following expressions are equivalent:

`nand[a,nand[a,b]]==nand[nand[b,nand[c,nand[c,c]]],a]`

Proof

Note that the input for the rule:

`nand[nand[a_,a_],b_]↔nand[b_,nand[b_,a_]]`

contains a subpattern of the form:

`nand[a_,a_]`

which can be unified with the input for the rule:

`nand[a_,nand[b_,nand[b_,b_]]]↔nand[a_,a_]`

where these rules follow from Critical Pair Lemma 83 and Substitution Lemma 74 respectively.

Critical Pair Lemma 96

The following expressions are equivalent:

`a==nand[nand[b,nand[b,b]],nand[a,a]]`

Proof

Note that the input for the rule:

`nand[nand[a_,a_],nand[b_,nand[b_,b_]]]→a`

contains a subpattern of the form:

`nand[nand[a_,a_],nand[b_,nand[b_,b_]]]`

which can be unified with the input for the rule:

`nand[a_,b_]↔nand[b_,a_]`

where these rules follow from Substitution Lemma 75 and Substitution Lemma 58 respectively.

Critical Pair Lemma 97

The following expressions are equivalent:

```
a==nand[nand[b,nand[b,b]],nand[nand[c,nand[c,c]],a]]
```

PROOF

Note that the input for the rule:

```
nand[nand[a_,nand[a_,a_]],nand[b_,b_]]→b
```

contains a subpattern of the form:

```
nand[b_,b_]
```

which can be unified with the input for the rule:

```
nand[a_,a_]↔nand[nand[b_,nand[b_,b_]],a_]
```

where these rules follow from Critical Pair Lemma 96 and Critical Pair Lemma 93 respectively.

Critical Pair Lemma 98

The following expressions are equivalent:

```
nand[a,nand[b,a]]==nand[nand[nand[c,nand[c,c]],b],a]
```

PROOF

Note that the input for the rule:

```
nand[a_,nand[b_,a_]]↔nand[nand[b_,b_],a_]
```

contains a subpattern of the form:

```
nand[a_,a_]
```

which can be unified with the input for the rule:

```
nand[a_,a_]↔nand[nand[b_,nand[b_,b_]],a_]
```

where these rules follow from Critical Pair Lemma 81 and Critical Pair Lemma 93 respectively.

Critical Pair Lemma 99

The following expressions are equivalent:

```
a==nand[nand[a,nand[b,nand[nand[c,nand[nand[b,b],c]],nand[y1,nand[b,b]]]]],nand[a,nand
```

PROOF

Note that the input for the rule:

```
nand[nand[a_,nand[b_,c_]],nand[a_,nand[nand[b_,b_],c_]]]→a
```

contains a subpattern of the form:

```
nand[nand[b_,b_],c_]
```

which can be unified with the input for the rule:

```
nand[a_,nand[nand[b_,nand[a_,b_]],nand[c_,a_]]]→nand[c,a]
```

where these rules follow from Critical Pair Lemma 84 and Critical Pair Lemma 87 respectively.

Substitution Lemma 76

It can be shown that:

```
a==nand[nand[a,nand[b,nand[nand[c,b],nand[y1,nand[b,b]]]]],nand[a,nand[y1,nand[b,b]]]]
```

PROOF

We start by taking Critical Pair Lemma 99, and apply the substitution:

```
nand[a_,nand[nand[b_,b_],a_]]→nand[a,b]
```

which follows from Critical Pair Lemma 82.

Substitution Lemma 77

It can be shown that:

`a==nand[nand[a,nand[b,c]],nand[a,nand[y1,nand[c,c]]]]`

Proof

We start by taking Substitution Lemma 76, and apply the substitution:

`nand[a_,nand[nand[b_,a_],nand[c_,nand[a_,a_]]]]→nand[b,a]`

which follows from Critical Pair Lemma 71.

Critical Pair Lemma 100

The following expressions are equivalent:

`a==nand[nand[nand[b,nand[nand[c,nand[nand[b,b],c]],nand[y1,nand[b,b]]]],a],nand[a,nand`

Proof

Note that the input for the rule:

`nand[nand[nand[a_,b_],c_],nand[c_,nand[nand[a_,a_],b_]]]→c`

contains a subpattern of the form:

`nand[nand[a_,a_],b_]`

which can be unified with the input for the rule:

`nand[a_,nand[nand[b_,nand[a_,b_]],nand[c_,a_]]]→nand[c,a]`

where these rules follow from Critical Pair Lemma 85 and Critical Pair Lemma 87 respectively.

Substitution Lemma 78

It can be shown that:

`a==nand[nand[nand[b,nand[nand[c,b],nand[y1,nand[b,b]]]],a],nand[a,nand[y1,nand[b,b]]]]`

Proof

We start by taking Critical Pair Lemma 100, and apply the substitution:

`nand[a_,nand[nand[b_,b_],a_]]→nand[a,b]`

which follows from Critical Pair Lemma 82.

Substitution Lemma 79

It can be shown that:

`a==nand[nand[nand[b,c],a],nand[a,nand[y1,nand[c,c]]]]`

Proof

We start by taking Substitution Lemma 78, and apply the substitution:

`nand[a_,nand[nand[b_,a_],nand[c_,nand[a_,a_]]]]→nand[b,a]`

which follows from Critical Pair Lemma 71.

Critical Pair Lemma 101

The following expressions are equivalent:

`a==nand[nand[nand[b,nand[nand[c,nand[nand[b,b],c]],nand[nand[b,b],y1]]],a],nand[a,nand`

Proof

Note that the input for the rule:

Note that the input for the rule:

`nand[nand[nand[a_,b_],c_],nand[c_,nand[nand[a_,a_],b_]]]→c`

contains a subpattern of the form:

`nand[nand[a_,a_],b_]`

which can be unified with the input for the rule:

`nand[a_,nand[nand[b_,nand[a_,b_]],nand[a_,c_]]]→nand[a,c]`

where these rules follow from Critical Pair Lemma 85 and Critical Pair Lemma 88 respectively.

Substitution Lemma 80

It can be shown that:

`a⩵nand[nand[nand[b,nand[nand[c,b],nand[nand[b,b],y1]]],a],nand[a,nand[nand[b,b],y1]]]`

Proof

We start by taking Critical Pair Lemma 101, and apply the substitution:

`nand[a_,nand[nand[b_,b_],a_]]→nand[a,b]`

which follows from Critical Pair Lemma 82.

Substitution Lemma 81

It can be shown that:

`a⩵nand[nand[nand[b,c],a],nand[a,nand[nand[c,c],y1]]]`

Proof

We start by taking Substitution Lemma 80, and apply the substitution:

`nand[a_,nand[nand[b_,a_],nand[nand[a_,a_],c_]]]→nand[b,a]`

which follows from Critical Pair Lemma 66.

Critical Pair Lemma 102

The following expressions are equivalent:

`a⩵nand[nand[a,nand[b,nand[nand[c,nand[nand[b,b],c]],nand[y1,nand[b,b]]]]],nand[nand[y`

Proof

Note that the input for the rule:

`nand[nand[a_,nand[b_,c_]],nand[nand[nand[b_,b_],c_],a_]]→a`

contains a subpattern of the form:

`nand[nand[b_,b_],c_]`

which can be unified with the input for the rule:

`nand[a_,nand[nand[b_,nand[a_,b_]],nand[c_,a_]]]→nand[c,a]`

where these rules follow from Critical Pair Lemma 86 and Critical Pair Lemma 87 respectively.

Substitution Lemma 82

It can be shown that:

`a⩵nand[nand[a,nand[b,nand[nand[c,b],nand[y1,nand[b,b]]]]],nand[nand[y1,nand[b,b]],a]]`

Proof

We start by taking Critical Pair Lemma 102, and apply the substitution:

`nand[a_,nand[nand[b_,b_],a_]]→nand[a,b]`

which follows from Critical Pair Lemma 82.

Substitution Lemma 83

It can be shown that:

`a==nand[nand[a,nand[b,c]],nand[nand[y1,nand[c,c]],a]]`

Proof

We start by taking Substitution Lemma 82, and apply the substitution:

`nand[a_,nand[nand[b_,a_],nand[c_,nand[a_,a_]]]]→nand[b,a]`

which follows from Critical Pair Lemma 71.

Critical Pair Lemma 103

The following expressions are equivalent:

`nand[nand[nand[a,nand[b,nand[c,a]]],c],a]==nand[a,a]`

Proof

Note that the input for the rule:

`nand[a_,nand[b_,nand[nand[b_,c_],a_]]]→nand[nand[b,c],a]`

contains a subpattern of the form:

`nand[b_,nand[nand[b_,c_],a_]]`

which can be unified with the input for the rule:

`nand[nand[a_,nand[b_,nand[c_,a_]]],nand[nand[y1_,c_],a_]]→a`

where these rules follow from Substitution Lemma 66 and Critical Pair Lemma 89 respectively.

Substitution Lemma 84

It can be shown that:

`nand[a,nand[nand[a,nand[b,nand[c,a]]],c]]==nand[a,a]`

Proof

We start by taking Critical Pair Lemma 103, and apply the substitution:

`nand[a_,b_]→nand[b,a]`

which follows from Substitution Lemma 58.

Substitution Lemma 85

It can be shown that:

`nand[a,nand[b,nand[a,nand[c,nand[b,a]]]]]==nand[a,a]`

Proof

We start by taking Substitution Lemma 84, and apply the substitution:

`nand[a_,b_]→nand[b,a]`

which follows from Substitution Lemma 58.

Critical Pair Lemma 104

The following expressions are equivalent:

`nand[a,nand[b,nand[c,nand[a,b]]]]==nand[a,nand[b,b]]`

Proof

Note that the input for the rule:

`nand[a_,nand[b_,nand[a_,nand[b_,c_]]]]→nand[a,nand[b,c]]`

contains a subpattern of the form:

`nand[b_,nand[a_,nand[b_,c_]]]`

which can be unified with the input for the rule:

`nand[a_,nand[b_,nand[a_,nand[c_,nand[b_,a_]]]]]→nand[a,a]`

where these rules follow from Substitution Lemma 68 and Substitution Lemma 85 respectively.

Critical Pair Lemma 105

The following expressions are equivalent:

`nand[a,nand[nand[b,b],nand[b,b]]]==nand[a,nand[nand[b,b],nand[c,nand[a,nand[b,a]]]]]`

Proof

Note that the input for the rule:

`nand[a_,nand[b_,nand[c_,nand[a_,b_]]]]→nand[a,nand[b,b]]`

contains a subpattern of the form:

`nand[a_,b_]`

which can be unified with the input for the rule:

`nand[a_,nand[b_,a_]]↔nand[a_,nand[b_,b_]]`

where these rules follow from Critical Pair Lemma 104 and Critical Pair Lemma 77 respectively.

Substitution Lemma 86

It can be shown that:

`nand[a,b]==nand[a,nand[nand[b,b],nand[c,nand[a,nand[b,a]]]]]`

Proof

We start by taking Critical Pair Lemma 105, and apply the substitution:

`nand[nand[a_,b_],nand[b_,b_]]→b`

which follows from Substitution Lemma 41.

Critical Pair Lemma 106

The following expressions are equivalent:

`nand[a,nand[nand[a,nand[b,c]],nand[c,c]]]==nand[a,a]`

Proof

Note that the input for the rule:

`nand[a_,nand[b_,nand[a_,nand[b_,c_]]]]→nand[a,nand[b,c]]`

contains a subpattern of the form:

`nand[b_,nand[a_,nand[b_,c_]]]`

which can be unified with the input for the rule:

`nand[nand[a_,nand[b_,c_]],nand[a_,nand[y1_,nand[c_,c_]]]]→a`

where these rules follow from Substitution Lemma 68 and Substitution Lemma 77

where these rules follow from Substitution Lemma 68 and Substitution Lemma 77 respectively.

Critical Pair Lemma 107

The following expressions are equivalent:

`nand[a,a]==nand[a,nand[nand[a,nand[b,nand[b,c]]],nand[nand[c,c],nand[c,c]]]]`

Proof

Note that the input for the rule:

`nand[a_,nand[nand[a_,nand[b_,c_]],nand[c_,c_]]]→nand[a,a]`

contains a subpattern of the form:

`nand[b_,c_]`

which can be unified with the input for the rule:

`nand[a_,nand[b_,b_]]↔nand[a_,nand[a_,b_]]`

where these rules follow from Critical Pair Lemma 106 and Critical Pair Lemma 78 respectively.

Substitution Lemma 87

It can be shown that:

`nand[a,a]==nand[a,nand[nand[a,nand[b,nand[b,c]]],c]]`

Proof

We start by taking Critical Pair Lemma 107, and apply the substitution:

`nand[nand[a_,b_],nand[b_,b_]]→b`

which follows from Substitution Lemma 41.

Substitution Lemma 88

It can be shown that:

`nand[a,a]==nand[a,nand[b,nand[a,nand[c,nand[c,b]]]]]`

Proof

We start by taking Substitution Lemma 87, and apply the substitution:

`nand[a_,b_]→nand[b,a]`

which follows from Substitution Lemma 58.

Critical Pair Lemma 108

The following expressions are equivalent:

`nand[a,a]==nand[a,nand[b,nand[b,nand[c,nand[a,c]]]]]`

Proof

Note that the input for the rule:

`nand[a_,nand[b_,nand[a_,nand[c_,nand[c_,b_]]]]]→nand[a,a]`

contains a subpattern of the form:

`nand[a_,nand[b_,nand[a_,nand[c_,nand[c_,b_]]]]]`

which can be unified with the input for the rule:

`nand[a_,nand[nand[b_,nand[a_,b_]],nand[a_,c_]]]→nand[a,c]`

where these rules follow from Substitution Lemma 88 and Critical Pair Lemma 88 respectively.

Critical Pair Lemma 109

The following expressions are equivalent:

`nand[a,nand[b,nand[c,nand[c,a]]]]==nand[a,nand[b,b]]`

Proof

Note that the input for the rule:

`nand[a_,nand[b_,nand[a_,nand[b_,c_]]]]→nand[a,nand[b,c]]`

contains a subpattern of the form:

`nand[b_,nand[a_,nand[b_,c_]]]`

which can be unified with the input for the rule:

`nand[a_,nand[b_,nand[a_,nand[c_,nand[c_,b_]]]]]→nand[a,a]`

where these rules follow from Substitution Lemma 68 and Substitution Lemma 88 respectively.

Critical Pair Lemma 110

The following expressions are equivalent:

`nand[a,a]==nand[a,nand[nand[b,nand[c,nand[a,b]]],b]]`

Proof

Note that the input for the rule:

`nand[a_,nand[b_,nand[b_,nand[c_,nand[a_,c_]]]]]→nand[a,a]`

contains a subpattern of the form:

`nand[b_,nand[c_,nand[a_,c_]]]`

which can be unified with the input for the rule:

`nand[nand[a_,nand[b_,nand[c_,a_]]],nand[a_,nand[c_,y1_]]]→a`

where these rules follow from Critical Pair Lemma 108 and Critical Pair Lemma 90 respectively.

Critical Pair Lemma 111

The following expressions are equivalent:

`nand[nand[a,a],nand[a,a]]==nand[nand[a,a],nand[b,nand[b,nand[c,a]]]]`

Proof

Note that the input for the rule:

`nand[a_,nand[b_,nand[b_,nand[c_,nand[a_,c_]]]]]→nand[a,a]`

contains a subpattern of the form:

`nand[c_,nand[a_,c_]]`

which can be unified with the input for the rule:

`nand[a_,nand[nand[b_,b_],a_]]→nand[a,b]`

where these rules follow from Critical Pair Lemma 108 and Critical Pair Lemma 82 respectively.

Substitution Lemma 89

Substitution Lemma 89

It can be shown that:

```
a==nand[nand[a,a],nand[b,nand[b,nand[c,a]]]]
```

PROOF

We start by taking Critical Pair Lemma 111, and apply the substitution:

```
nand[nand[a_,b_],nand[b_,b_]]→b
```

which follows from Substitution Lemma 41.

Critical Pair Lemma 112

The following expressions are equivalent:

```
nand[a,nand[a,nand[b,c]]]==nand[nand[nand[a,nand[a,nand[b,c]]],c],c]
```

PROOF

Note that the input for the rule:

```
nand[nand[a_,b_],nand[nand[c_,b_],a_]]→a
```

contains a subpattern of the form:

```
nand[nand[c_,b_],a_]
```

which can be unified with the input for the rule:

```
nand[nand[a_,a_],nand[b_,nand[b_,nand[c_,a_]]]]→a
```

where these rules follow from Critical Pair Lemma 63 and Substitution Lemma 89 respectively.

Substitution Lemma 90

It can be shown that:

```
nand[a,a]==nand[a,nand[b,nand[b,nand[c,nand[a,b]]]]]
```

PROOF

We start by taking Critical Pair Lemma 110, and apply the substitution:

```
nand[a_,b_]→nand[b,a]
```

which follows from Substitution Lemma 58.

Critical Pair Lemma 113

The following expressions are equivalent:

```
nand[nand[nand[a,b],nand[a,b]],nand[nand[b,b],c]]==nand[nand[y1,nand[y1,y1]],nand[nand
```

PROOF

Note that the input for the rule:

```
nand[nand[a_,nand[a_,a_]],nand[nand[b_,nand[b_,b_]],c_]]→c
```

contains a subpattern of the form:

```
nand[nand[b_,nand[b_,b_]],c_]
```

which can be unified with the input for the rule:

```
nand[nand[nand[a_,b_],c_],nand[c_,nand[nand[b_,b_],y1_]]]→c
```

where these rules follow from Critical Pair Lemma 97 and Substitution Lemma 81 respectively.

Substitution Lemma 91

Substitution Lemma 91

It can be shown that:

```
nand[nand[nand[a,b],nand[a,b]],nand[nand[b,b],c]]==nand[a,b]
```

Proof

We start by taking Critical Pair Lemma 113, and apply the substitution:

```
nand[nand[a_,nand[a_,a_]],nand[b_,b_]]→b
```

which follows from Critical Pair Lemma 96.

Critical Pair Lemma 114

The following expressions are equivalent:

```
a==nand[nand[a,nand[b,nand[c,y1]]],nand[nand[y1,c],a]]
```

Proof

Note that the input for the rule:

```
nand[nand[a_,nand[b_,c_]],nand[nand[y1_,nand[c_,c_]],a_]]→a
```

contains a subpattern of the form:

```
nand[y1_,nand[c_,c_]]
```

which can be unified with the input for the rule:

```
nand[a_,nand[nand[b_,c_],nand[a_,c_]]]→nand[c,a]
```

where these rules follow from Substitution Lemma 83 and Critical Pair Lemma 60 respectively.

Substitution Lemma 92

It can be shown that:

```
a==nand[nand[nand[b,c],a],nand[a,nand[y1,nand[c,b]]]]
```

Proof

We start by taking Critical Pair Lemma 114, and apply the substitution:

```
nand[a_,b_]→nand[b,a]
```

which follows from Substitution Lemma 58.

Critical Pair Lemma 115

The following expressions are equivalent:

```
nand[a,nand[a,nand[nand[b,c],nand[c,b]]]]==nand[nand[c,b],a]
```

Proof

Note that the input for the rule:

```
nand[a_,nand[a_,b_]]↔nand[nand[b_,nand[c_,nand[c_,c_]]],a_]
```

contains a subpattern of the form:

```
nand[a_,nand[b_,nand[b_,b_]]]
```

which can be unified with the input for the rule:

```
nand[nand[nand[a_,b_],c_],nand[c_,nand[y1_,nand[b_,a_]]]]→c
```

where these rules follow from Critical Pair Lemma 95 and Substitution Lemma 92 respectively.

Critical Pair Lemma 116

The following expressions are equivalent:

`nand[nand[a,b],c]==nand[c,nand[nand[nand[b,a],nand[a,b]],c]]`

Proof

Note that the input for the rule:

`nand[a_,nand[a_,nand[nand[b_,c_],nand[c_,b_]]]]→nand[nand[c,b],a]`

contains a subpattern of the form:

`nand[a_,nand[nand[b_,c_],nand[c_,b_]]]`

which can be unified with the input for the rule:

`nand[a_,b_]↔nand[b_,a_]`

where these rules follow from Critical Pair Lemma 115 and Substitution Lemma 58 respectively.

Critical Pair Lemma 117

The following expressions are equivalent:

`nand[nand[a,b],c]==nand[c,nand[nand[nand[b,a],nand[b,a]],c]]`

Proof

Note that the input for the rule:

`nand[a_,nand[nand[nand[b_,c_],nand[c_,b_]],a_]]→nand[nand[c,b],a]`

contains a subpattern of the form:

`nand[c_,b_]`

which can be unified with the input for the rule:

`nand[a_,b_]↔nand[b_,a_]`

where these rules follow from Critical Pair Lemma 116 and Substitution Lemma 58 respectively.

Substitution Lemma 93

It can be shown that:

`nand[nand[a,b],c]==nand[c,nand[b,a]]`

Proof

We start by taking Critical Pair Lemma 117, and apply the substitution:

`nand[a_,nand[nand[b_,b_],a_]]→nand[a,b]`

which follows from Critical Pair Lemma 82.

Critical Pair Lemma 118

The following expressions are equivalent:

`nand[a,nand[nand[b,c],c]]==nand[nand[c,nand[b,b]],a]`

Proof

Note that the input for the rule:

`nand[nand[a_,b_],c_]↔nand[c_,nand[b_,a_]]`

contains a subpattern of the form:

nand[a_,b_]

which can be unified with the input for the rule:

nand[a_,nand[b_,a_]]↔nand[a_,nand[b_,b_]]

where these rules follow from Substitution Lemma 93 and Critical Pair Lemma 77 respectively.

Critical Pair Lemma 119

The following expressions are equivalent:

nand[nand[a,b],c]==nand[nand[a,b],nand[nand[c,nand[b,a]],nand[c,y1]]]

Proof

Note that the input for the rule:

nand[a_,nand[nand[a_,b_],nand[b_,c_]]]→nand[a,b]

contains a subpattern of the form:

nand[a_,b_]

which can be unified with the input for the rule:

nand[nand[a_,b_],c_]↔nand[c_,nand[b_,a_]]

where these rules follow from Substitution Lemma 64 and Substitution Lemma 93 respectively.

Critical Pair Lemma 120

The following expressions are equivalent:

nand[nand[a,nand[b,c]],y1]==nand[y1,nand[a,nand[c,b]]]

Proof

Note that the input for the rule:

nand[nand[a_,b_],c_]↔nand[c_,nand[b_,a_]]

contains a subpattern of the form:

nand[b_,c_]

which can be unified with the input for the rule:

nand[nand[a_,b_],c_]↔nand[c_,nand[b_,a_]]

where these rules follow from Substitution Lemma 93 and Substitution Lemma 93 respectively.

Critical Pair Lemma 121

The following expressions are equivalent:

nand[a,nand[b,nand[c,nand[c,a]]]]==nand[a,nand[a,b]]

Proof

Note that the input for the rule:

nand[a_,nand[b_,nand[c_,nand[c_,a_]]]]↔nand[a_,nand[b_,b_]]

contains a subpattern of the form:

nand[a_,nand[b_,b_]]

which can be unified with the input for the rule:

nand[a_,nand[b_,b_]]↔nand[a_,nand[a_,b_]] `nand[a_,nand[b_,b_]]↔nand[a_,nand[a_,b_]]`

where these rules follow from Critical Pair Lemma 109 and Critical Pair Lemma 78 respectively.

Critical Pair Lemma 122

The following expressions are equivalent:

`nand[nand[a,a],nand[nand[a,a],nand[nand[b,a],c]]]==nand[nand[a,a],c]`

Proof

Note that the input for the rule:

`nand[a_,nand[b_,nand[c_,nand[c_,a_]]]]→nand[a,nand[a,b]]`

contains a subpattern of the form:

`nand[b_,nand[c_,nand[c_,a_]]]`

which can be unified with the input for the rule:

`nand[nand[nand[a_,b_],c_],nand[c_,nand[y1_,nand[b_,b_]]]]→c`

where these rules follow from Critical Pair Lemma 121 and Substitution Lemma 79 respectively.

Critical Pair Lemma 123

The following expressions are equivalent:

`nand[a,nand[a,b]]==nand[a,nand[b,nand[nand[a,a],c]]]`

Proof

Note that the input for the rule:

`nand[a_,nand[b_,nand[c_,nand[c_,a_]]]]→nand[a,nand[a,b]]`

contains a subpattern of the form:

`nand[c_,nand[c_,a_]]`

which can be unified with the input for the rule:

`nand[nand[a_,a_],b_]↔nand[b_,nand[b_,a_]]`

where these rules follow from Critical Pair Lemma 121 and Critical Pair Lemma 83 respectively.

Substitution Lemma 94

It can be shown that:

`nand[a,nand[b,nand[b,c]]]==nand[nand[b,nand[c,c]],a]`

Proof

We start by taking Critical Pair Lemma 118, and apply the substitution:

`nand[nand[a_,b_],c_]→nand[c,nand[b,a]]`

which follows from Substitution Lemma 93.

Critical Pair Lemma 124

The following expressions are equivalent:

`nand[a,nand[b,nand[c,y1]]]==nand[nand[nand[y1,c],b],a]`

Proof

Note that the input for the rule:

Note that the input for the rule:

`nand[nand[a_,nand[b_,c_]],y1_]↔nand[y1_,nand[a_,nand[c_,b_]]]`

contains a subpattern of the form:

`nand[a_,nand[b_,c_]]`

which can be unified with the input for the rule:

`nand[a_,b_]↔nand[b_,a_]`

where these rules follow from Critical Pair Lemma 120 and Substitution Lemma 58 respectively.

Critical Pair Lemma 125

The following expressions are equivalent:

`nand[nand[a,nand[b,c]],y1]==nand[y1,nand[nand[c,b],a]]`

Proof

Note that the input for the rule:

`nand[nand[a_,nand[b_,c_]],y1_]↔nand[y1_,nand[a_,nand[c_,b_]]]`

contains a subpattern of the form:

`nand[b_,nand[c_,y1_]]`

which can be unified with the input for the rule:

`nand[a_,b_]↔nand[b_,a_]`

where these rules follow from Critical Pair Lemma 120 and Substitution Lemma 58 respectively.

Substitution Lemma 95

It can be shown that:

`nand[a,nand[a,nand[b,c]]]==nand[c,nand[c,nand[nand[a,nand[b,c]],a]]]`

Proof

We start by taking Critical Pair Lemma 112, and apply the substitution:

`nand[nand[nand[a_,b_],c_],y1_]→nand[y1,nand[c,nand[b,a]]]`

which follows from Critical Pair Lemma 124.

Substitution Lemma 96

It can be shown that:

`nand[a,nand[a,nand[b,c]]]==nand[c,nand[c,nand[a,nand[nand[c,b],a]]]]`

Proof

We start by taking Substitution Lemma 95, and apply the substitution:

`nand[nand[a_,nand[b_,c_]],y1_]→nand[y1,nand[nand[c,b],a]]`

which follows from Critical Pair Lemma 125.

Critical Pair Lemma 126

The following expressions are equivalent:

`nand[a,nand[a,b]]==nand[a,nand[b,nand[c,nand[c,nand[y1,nand[a,a]]]]]]`

Proof

Note that the input for the rule:

`nand[a_,nand[b_,nand[nand[a_,a_],c_]]]→nand[a,nand[a,b]]`

contains a subpattern of the form:

`nand[nand[a_,a_],c_]`

which can be unified with the input for the rule:

`nand[a_,nand[a_,nand[b_,nand[nand[a_,c_],b_]]]]→nand[b,nand[b,nand[c,a]]]`

where these rules follow from Critical Pair Lemma 123 and Substitution Lemma 96 respectively.

Critical Pair Lemma 127

The following expressions are equivalent:

`nand[nand[a,a],b]==nand[nand[a,a],nand[nand[b,b],nand[c,a]]]`

Proof

Note that the input for the rule:

`nand[a_,nand[nand[b_,b_],nand[c_,nand[a_,nand[b_,a_]]]]]→nand[a,b]`

contains a subpattern of the form:

`nand[c_,nand[a_,nand[b_,a_]]]`

which can be unified with the input for the rule:

`nand[nand[nand[a_,b_],nand[a_,b_]],nand[nand[b_,b_],c_]]→nand[a,b]`

where these rules follow from Substitution Lemma 86 and Substitution Lemma 91 respectively.

Critical Pair Lemma 128

The following expressions are equivalent:

`nand[nand[a,a],b]==nand[nand[a,a],nand[nand[c,a],nand[nand[c,a],b]]]`

Proof

Note that the input for the rule:

`nand[nand[a_,a_],nand[nand[b_,b_],nand[c_,a_]]]→nand[nand[a,a],b]`

contains a subpattern of the form:

`nand[nand[b_,b_],nand[c_,a_]]`

which can be unified with the input for the rule:

`nand[nand[a_,a_],b_]↔nand[b_,nand[b_,a_]]`

where these rules follow from Critical Pair Lemma 127 and Critical Pair Lemma 83 respectively.

Critical Pair Lemma 129

The following expressions are equivalent:

`nand[nand[a,a],nand[nand[b,nand[a,c]],nand[b,y1]]]==nand[nand[a,a],nand[nand[a,a],nand`

Proof

Note that the input for the rule:

`nand[nand[a_,a_],nand[nand[a_,a_],nand[nand[b_,a_],c_]]]→nand[nand[a,a],c]`

contains a subpattern of the form:

contains a subpattern of the form:

`nand[nand[b_,a_],c_]`

which can be unified with the input for the rule:

`nand[nand[a_,b_],nand[nand[c_,nand[b_,a_]],nand[c_,y1_]]]→nand[nand[a,b],c]`

where these rules follow from Critical Pair Lemma 122 and Critical Pair Lemma 119 respectively.

Substitution Lemma 97

It can be shown that:

`nand[nand[a,a],nand[nand[b,nand[a,c]],nand[b,y1]]]==nand[nand[a,a],b]`

Proof

We start by taking Critical Pair Lemma 129, and apply the substitution:

`nand[nand[a_,a_],nand[nand[a_,a_],nand[nand[b_,a_],c_]]]→nand[nand[a,a],c]`

which follows from Critical Pair Lemma 122.

Critical Pair Lemma 130

The following expressions are equivalent:

`nand[a,nand[a,b]]==nand[a,nand[b,nand[nand[c,nand[nand[c,a],y1]],c]]]`

Proof

Note that the input for the rule:

`nand[a_,nand[b_,nand[c_,nand[c_,nand[y1_,nand[a_,a_]]]]]]→nand[a,nand[a,b]]`

contains a subpattern of the form:

`nand[c_,nand[y1_,nand[a_,a_]]]`

which can be unified with the input for the rule:

`nand[nand[a_,nand[nand[a_,b_],c_]],nand[a_,nand[y1_,b_]]]→a`

where these rules follow from Critical Pair Lemma 126 and Critical Pair Lemma 91 respectively.

Substitution Lemma 98

It can be shown that:

`nand[a,nand[a,b]]==nand[a,nand[b,nand[c,nand[nand[y1,nand[c,a]],c]]]]`

Proof

We start by taking Critical Pair Lemma 130, and apply the substitution:

`nand[nand[a_,nand[b_,c_]],y1_]→nand[y1,nand[nand[c,b],a]]`

which follows from Critical Pair Lemma 125.

Substitution Lemma 99

It can be shown that:

`nand[a,nand[a,b]]==nand[a,nand[b,nand[c,nand[c,nand[nand[a,c],y1]]]]]`

Proof

We start by taking Substitution Lemma 98, and apply the substitution:

`nand[nand[a_,nand[b_,c_]],y1_]→nand[y1,nand[nand[c,b],a]]`

which follows from Critical Pair Lemma 125.

which follows from Critical Pair Lemma 125.

Critical Pair Lemma 131

The following expressions are equivalent:

`nand[a,nand[a,nand[b,b]]]==nand[a,nand[nand[b,b],nand[nand[a,nand[c,b]],y1]]]`

Proof

Note that the input for the rule:

`nand[a_,nand[b_,nand[c_,nand[c_,nand[nand[a_,c_],y1_]]]]]→nand[a,nand[a,b]]`

contains a subpattern of the form:

`nand[b_,nand[c_,nand[c_,nand[nand[a_,c_],y1_]]]]`

which can be unified with the input for the rule:

`nand[nand[a_,a_],nand[nand[b_,a_],nand[nand[b_,a_],c_]]]→nand[nand[a,a],c]`

where these rules follow from Substitution Lemma 99 and Critical Pair Lemma 128 respectively.

Substitution Lemma 100

It can be shown that:

`nand[a,b]==nand[b,nand[nand[a,a],nand[nand[b,nand[c,a]],y1]]]`

Proof

We start by taking Critical Pair Lemma 131, and apply the substitution:

`nand[a_,nand[a_,nand[b_,b_]]]→nand[b,a]`

which follows from Critical Pair Lemma 79.

Critical Pair Lemma 132

The following expressions are equivalent:

`nand[a,b]==nand[b,nand[nand[c,nand[b,nand[y1,a]]],nand[a,a]]]`

Proof

Note that the input for the rule:

`nand[a_,nand[nand[b_,b_],nand[nand[a_,nand[c_,b_]],y1_]]]→nand[b,a]`

contains a subpattern of the form:

`nand[nand[b_,b_],nand[nand[a_,nand[c_,b_]],y1_]]`

which can be unified with the input for the rule:

`nand[a_,nand[b_,nand[nand[c_,b_],a_]]]→nand[nand[c,b],a]`

where these rules follow from Substitution Lemma 100 and Substitution Lemma 54 respectively.

Critical Pair Lemma 133

The following expressions are equivalent:

`nand[nand[a,nand[b,c]],c]==nand[c,nand[nand[b,b],nand[nand[a,nand[b,c]],nand[a,nand[b,`

Proof

Note that the input for the rule:

`nand[a_,nand[nand[b_,nand[a_,nand[c_,y1_]]],nand[y1_,y1_]]]→nand[y1,a]`

contains a subpattern of the form:

`nand[b_,nand[a_,nand[c_,y1_]]]`

which can be unified with the input for the rule:

`nand[a_,nand[b_,nand[b_,nand[c_,nand[a_,b_]]]]]→nand[a,a]`

where these rules follow from Critical Pair Lemma 132 and Substitution Lemma 90 respectively.

Substitution Lemma 101

It can be shown that:

`nand[nand[a,nand[b,c]],c]==nand[c,nand[nand[b,b],a]]`

Proof

We start by taking Critical Pair Lemma 133, and apply the substitution:

`nand[nand[a_,a_],nand[nand[b_,nand[a_,c_]],nand[b_,y1_]]]→nand[nand[a,a],b]`

which follows from Substitution Lemma 97.

Substitution Lemma 102

It can be shown that:

`nand[a,nand[nand[a,b],c]]==nand[a,nand[nand[b,b],c]]`

Proof

We start by taking Substitution Lemma 101, and apply the substitution:

`nand[nand[a_,nand[b_,c_]],y1_]→nand[y1,nand[nand[c,b],a]]`

which follows from Critical Pair Lemma 125.

Critical Pair Lemma 134

The following expressions are equivalent:

`nand[a,nand[nand[nand[b,b],nand[b,b]],c]]==nand[a,nand[c,nand[a,nand[a,b]]]]`

Proof

Note that the input for the rule:

`nand[a_,nand[nand[a_,b_],c_]]↔nand[a_,nand[nand[b_,b_],c_]]`

contains a subpattern of the form:

`nand[nand[a_,b_],c_]`

which can be unified with the input for the rule:

`nand[a_,nand[b_,nand[b_,c_]]]↔nand[nand[b_,nand[c_,c_]],a_]`

where these rules follow from Substitution Lemma 102 and Substitution Lemma 94 respectively.

Substitution Lemma 103

It can be shown that:

`nand[a,nand[b,c]]==nand[a,nand[c,nand[a,nand[a,b]]]]`

Proof

We start by taking Critical Pair Lemma 134, and apply the substitution:

`nand[nand[a_,b_],nand[b_,b_]]→b`

which follows from Substitution Lemma 41.

Critical Pair Lemma 135

The following expressions are equivalent:

`nand[a,nand[nand[nand[b,c],nand[a,b]],y1]]==nand[a,nand[y1,nand[a,nand[a,b]]]]`

Proof

Note that the input for the rule:

`nand[a_,nand[b_,nand[a_,nand[a_,c_]]]]→nand[a,nand[c,b]]`

contains a subpattern of the form:

`nand[a_,c_]`

which can be unified with the input for the rule:

`nand[a_,nand[nand[b_,c_],nand[a_,b_]]]→nand[a,b]`

where these rules follow from Substitution Lemma 103 and Substitution Lemma 63 respectively.

Substitution Lemma 104

It can be shown that:

`nand[a,nand[nand[nand[b,c],nand[a,b]],y1]]==nand[a,nand[b,y1]]`

Proof

We start by taking Critical Pair Lemma 135, and apply the substitution:

`nand[a_,nand[b_,nand[a_,nand[a_,c_]]]]→nand[a,nand[c,b]]`

which follows from Substitution Lemma 103.

Critical Pair Lemma 136

The following expressions are equivalent:

`nand[a,nand[b,c]]==nand[a,nand[c,nand[nand[a,b],c]]]`

Proof

Note that the input for the rule:

`nand[a_,nand[nand[nand[b_,c_],nand[a_,b_]],y1_]]→nand[a,nand[b,y1]]`

contains a subpattern of the form:

`nand[nand[nand[b_,c_],nand[a_,b_]],y1_]`

which can be unified with the input for the rule:

`nand[a_,nand[b_,a_]]↔nand[nand[nand[c_,nand[c_,c_]],b_],a_]`

where these rules follow from Substitution Lemma 104 and Critical Pair Lemma 98 respectively.

Substitution Lemma 105

It can be shown that:

`nand[a,nand[a,nand[b,c]]]==nand[c,nand[c,nand[b,a]]]`

Proof

We start by taking Substitution Lemma 96, and apply the substitution:

`nand[a_,nand[b_,nand[nand[a_,c_],b_]]]→nand[a,nand[c,b]]` `nand[a_,nand[b_,nand[nand[a_,c_],b_]]]→nand[a,nand[c,b]]`

which follows from Critical Pair Lemma 136.

Substitution Lemma 106

It can be shown that:

`nand[a,nand[a,nand[c,b]]]==nand[b,nand[b,nand[a,c]]]`

Proof

We start by taking Hypothesis 3, and apply the substitution:

`nand[a_,b_]→nand[b,a]`

which follows from Substitution Lemma 58.

Substitution Lemma 107

It can be shown that:

`nand[a,nand[a,nand[c,b]]]==nand[b,nand[b,nand[c,a]]]`

Proof

We start by taking Substitution Lemma 106, and apply the substitution:

`nand[a_,b_]→nand[b,a]`

which follows from Substitution Lemma 58.

Conclusion 3

We obtain the conclusion:

`True`

Proof

Take Substitution Lemma 107, and apply the substitution:

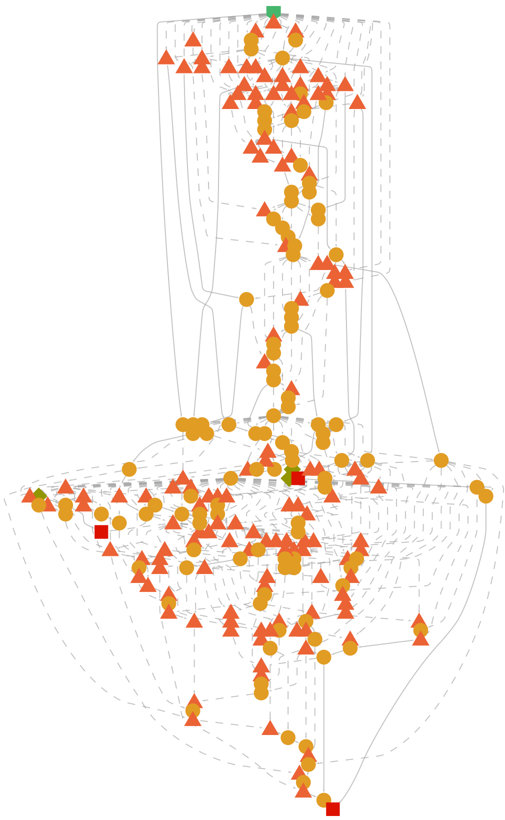
`nand[a_,nand[a_,nand[b_,c_]]]→nand[c,nand[c,nand[b,a]]]`

which follows from Substitution Lemma 105.

large output | **show less** | **show more** | **show all** | **set size limit...**

In[∘]:= proofShortfromWolfram["ProofGraph"]

Out[∘]=



In[∘]:= Clear[proofShortfromWolfram]

Appendix 7. Proof of R ⊢ winker 1

In[ ]:=
```
proofWinker1fromRobbins["ProofNotebook"]
```

Out[ ]=

Axiom 1

We are given that:

```
or[x1,x2]==not[x2]
```

Hypothesis 1

We would like to show that:

```
or[a,0]==not[0]
```

Conclusion 1

We obtain the conclusion:

```
True
```

Proof

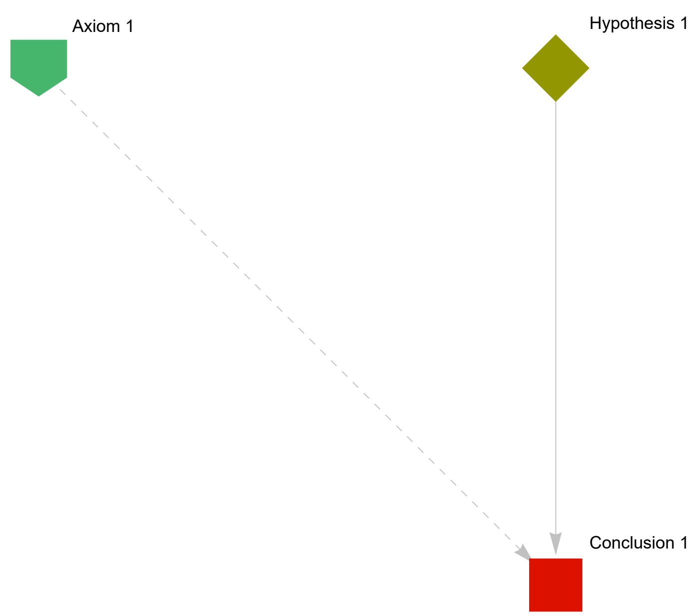
Take Hypothesis 1, and apply the substitution:

```
or[x1_,x2_]→not[x2]
```

which follows from Axiom 1.

In[ ]:=
```
proofWinker1fromRobbins["ProofGraph"]
```

Out[ ]=



In[ ]:=
```
Clear[proofWinker1fromRobbins]
```

Appendix 8. Proof of a property of “0” in Robbins logic

In[ ]:= `proofzeropropRobbins["ProofNotebook"]`

Axiom 1

We are given that:

`x1==not[or[not[or[x1,x2]],not[or[x1,not[x2]]]]]`

Axiom 2

We are given that:

`or[x1,x2]==or[x2,x1]`

Axiom 3

We are given that:

`or[x1,or[x2,x3]]==or[or[x1,x2],x3]`

Axiom 4

We are given that:

`not[or[not[x1],not[x2]]]==and[x1,x2]`

Axiom 5

We are given that:

`and[x1,not[x1]]==0`

Hypothesis 1

We would like to show that:

`or[a,0]==a`

Critical Pair Lemma 1

The following expressions are equivalent:

`or[x1,or[x2,x3]]==or[or[x2,x1],x3]`

Proof

Note that the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Axiom 3 and Axiom 2 respectively.

Substitution Lemma 1

It can be shown that:

`or[x1,or[x2,x3]]==or[x2,or[x1,x3]]`

Proof

PROOF

We start by taking Critical Pair Lemma 1, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Critical Pair Lemma 2

The following expressions are equivalent:

`and[or[not[x1],not[x2]],x3]==not[or[and[x1,x2],not[x3]]]`

PROOF

Note that the input for the rule:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

where these rules follow from Axiom 4 and Axiom 4 respectively.

Critical Pair Lemma 3

The following expressions are equivalent:

`and[x1,or[not[x2],not[x3]]]==not[or[not[x1],and[x2,x3]]]`

PROOF

Note that the input for the rule:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

where these rules follow from Axiom 4 and Axiom 4 respectively.

Substitution Lemma 2

It can be shown that:

`and[or[x1,x2],or[x1,not[x2]]]==x1`

PROOF

We start by taking Axiom 1, and apply the substitution:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

which follows from Axiom 4.

Critical Pair Lemma 4

The following expressions are equivalent:

`and[or[not[x1],not[not[x1]]],x2]==not[or[0,not[x2]]]`

PROOF

Note that the input for the rule:

`not[or[and[x1_,x2_],not[x3_]]]→and[or[not[x1],not[x2]],x3]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,not[x1_]]→0`

where these rules follow from Critical Pair Lemma 2 and Axiom 5 respectively.

Critical Pair Lemma 5

The following expressions are equivalent:

`and[x1,or[not[x2],not[not[x2]]]]==not[or[not[x1],0]]`

Proof

Note that the input for the rule:

`not[or[not[x1_],and[x2_,x3_]]]→and[x1,or[not[x2],not[x3]]]`

contains a subpattern of the form:

`and[x2_,x3_]`

which can be unified with the input for the rule:

`and[x1_,not[x1_]]→0`

where these rules follow from Critical Pair Lemma 3 and Axiom 5 respectively.

Substitution Lemma 3

It can be shown that:

`and[x1,or[not[x2],not[not[x2]]]]==not[or[0,not[x1]]]`

Proof

We start by taking Critical Pair Lemma 5, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Critical Pair Lemma 6

The following expressions are equivalent:

`not[x1]==not[or[0,not[or[not[x1],not[not[not[x1]]]]]]]`

Proof

Note that the input for the rule:

`and[or[x1_,x2_],or[x1_,not[x2_]]]→x1`

contains a subpattern of the form:

`and[or[x1_,x2_],or[x1_,not[x2_]]]`

which can be unified with the input for the rule:

`and[or[not[x1_],not[not[x1_]]],x2_]→not[or[0,not[x2]]]`

where these rules follow from Substitution Lemma 2 and Critical Pair Lemma 4 respectively.

Substitution Lemma 4

It can be shown that:

`not[x1]==not[or[0,and[x1,not[not[x1]]]]]`

PROOF

We start by taking Critical Pair Lemma 6, and apply the substitution:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

which follows from Axiom 4.

Critical Pair Lemma 7

The following expressions are equivalent:

`not[x1]⩵not[or[0,not[or[not[x1],not[x1]]]]]`

PROOF

Note that the input for the rule:

`and[or[x1_,x2_],or[x1_,not[x2_]]]→x1`

contains a subpattern of the form:

`and[or[x1_,x2_],or[x1_,not[x2_]]]`

which can be unified with the input for the rule:

`and[x1_,or[not[x2_],not[not[x2_]]]]→not[or[0,not[x1]]]`

where these rules follow from Substitution Lemma 2 and Substitution Lemma 3 respectively.

Substitution Lemma 5

It can be shown that:

`not[x1]⩵not[or[0,and[x1,x1]]]`

PROOF

We start by taking Critical Pair Lemma 7, and apply the substitution:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

which follows from Axiom 4.

Critical Pair Lemma 8

The following expressions are equivalent:

`x1⩵and[or[x1,x2],or[not[x2],x1]]`

PROOF

Note that the input for the rule:

`and[or[x1_,x2_],or[x1_,not[x2_]]]→x1`

contains a subpattern of the form:

`or[x1_,not[x2_]]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Substitution Lemma 2 and Axiom 2 respectively.

Critical Pair Lemma 9

The following expressions are equivalent:

`not[not[x1]]⩵not[or[0,not[or[not[not[x1]],x1]]]]`

PROOF

Note that the input for the rule:

`and[or[x1_,x2_],or[not[x2_],x1_]]→x1`

contains a subpattern of the form:

`and[or[x1_,x2_],or[not[x2_],x1_]]`

which can be unified with the input for the rule:

`and[x1_,or[not[x2_],not[not[x2_]]]]→not[or[0,not[x1]]]`

where these rules follow from Critical Pair Lemma 8 and Substitution Lemma 3 respectively.

Substitution Lemma 6

It can be shown that:

`not[not[x1]]==not[or[0,not[or[x1,not[not[x1]]]]]]`

Proof

We start by taking Critical Pair Lemma 9, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Critical Pair Lemma 10

The following expressions are equivalent:

`not[not[not[x1]]]==not[or[0,and[x1,not[not[x1]]]]]`

Proof

Note that the input for the rule:

`not[or[0,not[or[x1_,not[not[x1_]]]]]]→not[not[x1]]`

contains a subpattern of the form:

`not[or[x1_,not[not[x1_]]]]`

which can be unified with the input for the rule:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

where these rules follow from Substitution Lemma 6 and Axiom 4 respectively.

Substitution Lemma 7

It can be shown that:

`not[not[not[x1]]]==not[x1]`

Proof

We start by taking Critical Pair Lemma 10, and apply the substitution:

`not[or[0,and[x1_,not[not[x1_]]]]]→not[x1]`

which follows from Substitution Lemma 4.

Critical Pair Lemma 11

The following expressions are equivalent:

`not[or[not[x1],not[x2]]]==not[not[and[x1,x2]]]`

Proof

Note that the input for the rule:

`not[not[not[x1_]]]→not[x1]`

not[not[not[x1_]]]→not[x1]

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

where these rules follow from Substitution Lemma 7 and Axiom 4 respectively.

Substitution Lemma 8

It can be shown that:

`and[x1,x2]==not[not[and[x1,x2]]]`

Proof

We start by taking Critical Pair Lemma 11, and apply the substitution:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

which follows from Axiom 4.

Critical Pair Lemma 12

The following expressions are equivalent:

`and[or[x1,x2],or[not[x2],x1]]==not[not[x1]]`

Proof

Note that the input for the rule:

`not[not[and[x1_,x2_]]]→and[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[or[x1_,x2_],or[not[x2_],x1_]]→x1`

where these rules follow from Substitution Lemma 8 and Critical Pair Lemma 8 respectively.

Substitution Lemma 9

Out[]=

It can be shown that:

`x1==not[not[x1]]`

Proof

We start by taking Critical Pair Lemma 12, and apply the substitution:

`and[or[x1_,x2_],or[not[x2_],x1_]]→x1`

which follows from Critical Pair Lemma 8.

Critical Pair Lemma 13

The following expressions are equivalent:

`or[0,and[x1,x1]]==not[not[x1]]`

Proof

Note that the input for the rule:

`not[not[x1_]]→x1`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[or[0,and[x1_,x1_]]]→not[x1]`

where these rules follow from Substitution Lemma 9 and Substitution Lemma 5 respectively.

Substitution Lemma 10

It can be shown that:

`or[0,and[x1,x1]]==x1`

Proof

We start by taking Critical Pair Lemma 13, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 9.

Critical Pair Lemma 14

The following expressions are equivalent:

`or[not[x1],not[x2]]==not[and[x1,x2]]`

Proof

Note that the input for the rule:

`not[not[x1_]]→x1`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[or[not[x1_],not[x2_]]]→and[x1,x2]`

where these rules follow from Substitution Lemma 9 and Axiom 4 respectively.

Critical Pair Lemma 15

The following expressions are equivalent:

`not[and[not[x1],x2]]==or[x1,not[x2]]`

Proof

Note that the input for the rule:

`or[not[x1_],not[x2_]]→not[and[x1,x2]]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Critical Pair Lemma 14 and Substitution Lemma 9 respectively.

Critical Pair Lemma 16

The following expressions are equivalent:

`or[x1,not[not[not[x1]]]]==not[0]`

Proof

Note that the input for the rule:

`not[and[not[x1_],x2_]]→or[x1,not[x2]]`

contains a subpattern of the form:

`and[not[x1_],x2_]`

which can be unified with the input for the rule:

`and[x1_,not[x1_]]→0`

where these rules follow from Critical Pair Lemma 15 and Axiom 5 respectively.

Substitution Lemma 11

It can be shown that:

`or[x1,not[x1]]==not[0]`

Proof

We start by taking Critical Pair Lemma 16, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 9.

Critical Pair Lemma 17

The following expressions are equivalent:

`and[not[x1],x2]==not[or[x1,not[x2]]]`

Proof

Note that the input for the rule:

`not[not[x1_]]→x1`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[and[not[x1_],x2_]]→or[x1,not[x2]]`

where these rules follow from Substitution Lemma 9 and Critical Pair Lemma 15 respectively.

Critical Pair Lemma 18

The following expressions are equivalent:

`or[x1,or[x2,not[x1]]]==or[x2,not[0]]`

Proof

Note that the input for the rule:

`or[x1_,or[x2_,x3_]]↔or[x2_,or[x1_,x3_]]`

contains a subpattern of the form:

`or[x2_,x3_]`

which can be unified with the input for the rule:

`or[x1_,not[x1_]]→not[0]`

where these rules follow from Substitution Lemma 1 and Substitution Lemma 11 respectively.

Critical Pair Lemma 19

The following expressions are equivalent:

`and[not[x1],not[x2]]==not[or[x1,x2]]`

Proof

Note that the input for the rule:

`not[or[x1_,not[x2_]]]→and[not[x1],x2]`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Critical Pair Lemma 17 and Substitution Lemma 9 respectively.

Critical Pair Lemma 20

The following expressions are equivalent:

`not[x1]==or[0,not[or[x1,x1]]]`

Proof

Note that the input for the rule:

`or[0,and[x1_,x1_]]→x1`

contains a subpattern of the form:

`and[x1_,x1_]`

which can be unified with the input for the rule:

`and[not[x1_],not[x2_]]→not[or[x1,x2]]`

where these rules follow from Substitution Lemma 10 and Critical Pair Lemma 19 respectively.

Critical Pair Lemma 21

The following expressions are equivalent:

`or[0,not[0]]==or[or[x1,x1],not[x1]]`

Proof

Note that the input for the rule:

`or[x1_,or[x2_,not[x1_]]]→or[x2,not[0]]`

contains a subpattern of the form:

`or[x2_,not[x1_]]`

which can be unified with the input for the rule:

`or[0,not[or[x1_,x1_]]]→not[x1]`

where these rules follow from Critical Pair Lemma 18 and Critical Pair Lemma 20 respectively.

Substitution Lemma 12

It can be shown that:

`not[0]==or[or[x1,x1],not[x1]]`

Proof

We start by taking Critical Pair Lemma 21, and apply the substitution:

`or[x1_,not[x1_]]→not[0]`

which follows from Substitution Lemma 11.

Substitution Lemma 13

Substitution Lemma 13

It can be shown that:

`not[0]==or[x1,or[x1,not[x1]]]`

PROOF

We start by taking Substitution Lemma 12, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Substitution Lemma 14

It can be shown that:

`not[0]==or[x1,not[0]]`

PROOF

We start by taking Substitution Lemma 13, and apply the substitution:

`or[x1_,or[x2_,not[x1_]]]→or[x2,not[0]]`

which follows from Critical Pair Lemma 18.

Critical Pair Lemma 22

The following expressions are equivalent:

`not[not[0]]==or[0,not[not[0]]]`

PROOF

Note that the input for the rule:

`or[0,not[or[x1_,x1_]]]→not[x1]`

contains a subpattern of the form:

`or[x1_,x1_]`

which can be unified with the input for the rule:

`or[x1_,not[0]]→not[0]`

where these rules follow from Critical Pair Lemma 20 and Substitution Lemma 14 respectively.

Substitution Lemma 15

It can be shown that:

`0==or[0,not[not[0]]]`

PROOF

We start by taking Critical Pair Lemma 22, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 9.

Substitution Lemma 16

It can be shown that:

`0==or[0,0]`

PROOF

We start by taking Substitution Lemma 15, and apply the substitution:

`not[not[x1_]]→x1`

`not[not[x1_]]→x1`

which follows from Substitution Lemma 9.

Critical Pair Lemma 23

The following expressions are equivalent:

`or[0,or[0,x1]]⩵or[0,x1]`

Proof

Note that the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[0,0]→0`

where these rules follow from Axiom 3 and Substitution Lemma 16 respectively.

Critical Pair Lemma 24

The following expressions are equivalent:

`or[0,and[x1,x1]]⩵or[0,x1]`

Proof

Note that the input for the rule:

`or[0,or[0,x1_]]→or[0,x1]`

contains a subpattern of the form:

`or[0,x1_]`

which can be unified with the input for the rule:

`or[0,and[x1_,x1_]]→x1`

where these rules follow from Critical Pair Lemma 23 and Substitution Lemma 10 respectively.

Substitution Lemma 17

It can be shown that:

`x1⩵or[0,x1]`

Proof

We start by taking Critical Pair Lemma 24, and apply the substitution:

`or[0,and[x1_,x1_]]→x1`

which follows from Substitution Lemma 10.

Critical Pair Lemma 25

The following expressions are equivalent:

`x1⩵or[x1,0]`

Proof

Note that the input for the rule:

`or[0,x1_]→x1`

contains a subpattern of the form:

`or[0,x1_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Substitution Lemma 17 and Axiom 2 respectively.

Conclusion 1

We obtain the conclusion:

`True`

Proof

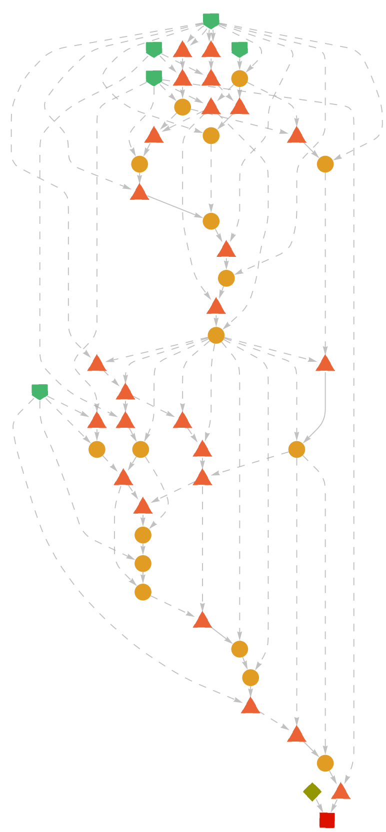
Take Hypothesis 1, and apply the substitution:

`or[x1_,0]→x1`

which follows from Critical Pair Lemma 25.

In[∘]:= `proofzeropropRobbins["ProofGraph"]`

Out[∘]=



In[∘]:= `Clear[proofzeropropRobbins]`

Appendix 9. Derivation of “short” from Sheffer logic

In[ ]:=
```
proofShortfromSheffer["ProofNotebook"]
```

Axiom 1

We are given that:

`x1==nand[nand[x1,x1],nand[x1,x1]]`

Axiom 2

We are given that:

`nand[x1,x1]==nand[x1,nand[x2,nand[x2,x2]]]`

Axiom 3

We are given that:

`nand[nand[x1,nand[x2,x3]],nand[x1,nand[x2,x3]]]==nand[nand[nand[x2,x2],x1],nand[nand[x`

Hypothesis 1

We would like to show that:

`nand[nand[a,a],nand[a,b]]==a`

Hypothesis 2

We would like to show that:

`nand[a,nand[a,b]]==nand[a,nand[b,b]]`

Hypothesis 3

We would like to show that:

`nand[a,nand[a,nand[b,c]]]==nand[b,nand[b,nand[a,c]]]`

Critical Pair Lemma 1

The following expressions are equivalent:

`nand[nand[x1,x1],nand[x2,nand[x2,x2]]]==x1`

Proof

Note that the input for the rule:

`nand[x1_,x1_]↔nand[x1_,nand[x2_,nand[x2_,x2_]]]`

contains a subpattern of the form:

`nand[x1_,x1_]`

which can be unified with the input for the rule:

`nand[nand[x1_,x1_],nand[x1_,x1_]]→x1`

where these rules follow from Axiom 2 and Axiom 1 respectively.

Critical Pair Lemma 2

The following expressions are equivalent:

`nand[x1,x1]==nand[x1,nand[nand[x2,x2],x2]]`

Proof

Note that the input for the rule:

`nand[x1_,x1_]↔nand[x1_,nand[x2_,nand[x2_,x2_]]]`

contains a subpattern of the form:

`nand[x2_,x2_]`

which can be unified with the input for the rule:

`nand[nand[x1_,x1_],nand[x1_,x1_]]→x1`

where these rules follow from Axiom 2 and Axiom 1 respectively.

Critical Pair Lemma 3

The following expressions are equivalent:

`nand[nand[nand[x1,x1],nand[x1,nand[x2,x3]]],nand[nand[nand[x2,x3],nand[x2,x3]],nand[x1`

Proof

Note that the input for the rule:

`nand[nand[x1_,nand[x2_,x3_]],nand[x1_,nand[x2_,x3_]]]↔nand[nand[nand[x2_,x2_],x1_],na`

contains a subpattern of the form:

`nand[x1_,nand[x2_,x3_]]`

which can be unified with the input for the rule:

`nand[nand[x1_,nand[x2_,x3_]],nand[x1_,nand[x2_,x3_]]]↔nand[nand[nand[x2_,x2_],x1_],na`

where these rules follow from Axiom 3 and Axiom 3 respectively.

Critical Pair Lemma 4

The following expressions are equivalent:

`nand[nand[nand[nand[x1,x1],nand[x1,x1]],nand[nand[x2,x2],x3]],nand[nand[x3,x3],nand[na`

Proof

Note that the input for the rule:

`nand[nand[x1_,nand[x2_,x3_]],nand[x1_,nand[x2_,x3_]]]↔nand[nand[nand[x2_,x2_],x1_],na`

contains a subpattern of the form:

`nand[x1_,nand[x2_,x3_]]`

which can be unified with the input for the rule:

`nand[nand[x1_,nand[x2_,x3_]],nand[x1_,nand[x2_,x3_]]]↔nand[nand[nand[x2_,x2_],x1_],na`

where these rules follow from Axiom 3 and Axiom 3 respectively.

Substitution Lemma 1

It can be shown that:

`nand[nand[x1,nand[nand[x2,x2],x3]],nand[nand[x3,x3],nand[nand[x2,x2],x3]]]==nand[nand[`

Proof

We start by taking Critical Pair Lemma 4, and apply the substitution:

```
nand[nand[x1_,x1_],nand[x1_,x1_]]→x1
```

which follows from Axiom 1.

Critical Pair Lemma 5

The following expressions are equivalent:

```
nand[nand[x1,x1],x2]==nand[nand[nand[x2,nand[x1,x1]],nand[x2,nand[x1,x1]]],nand[x3,nan
```

Proof

Note that the input for the rule:

```
nand[nand[x1_,x1_],nand[x2_,nand[x2_,x2_]]]→x1
```

contains a subpattern of the form:

```
nand[x1_,x1_]
```

which can be unified with the input for the rule:

```
nand[nand[x1_,nand[x2_,x3_]],nand[x1_,nand[x2_,x3_]]]↔nand[nand[nand[x2_,x2_],x1_],na
```

where these rules follow from Critical Pair Lemma 1 and Axiom 3 respectively.

Substitution Lemma 2

It can be shown that:

```
nand[nand[x1,x1],x2]==nand[x2,nand[x1,x1]]
```

Proof

We start by taking Critical Pair Lemma 5, and apply the substitution:

```
nand[nand[x1_,x1_],nand[x2_,nand[x2_,x2_]]]→x1
```

which follows from Critical Pair Lemma 1.

Critical Pair Lemma 6

The following expressions are equivalent:

```
nand[x1,nand[nand[x2,x2],nand[x2,x2]]]==nand[x2,x1]
```

Proof

Note that the input for the rule:

```
nand[nand[x1_,x1_],x2_]↔nand[x2_,nand[x1_,x1_]]
```

contains a subpattern of the form:

```
nand[x1_,x1_]
```

which can be unified with the input for the rule:

```
nand[nand[x1_,x1_],nand[x1_,x1_]]→x1
```

where these rules follow from Substitution Lemma 2 and Axiom 1 respectively.

Substitution Lemma 3

It can be shown that:

```
nand[x1,x2]==nand[x2,x1]
```

Proof

We start by taking Critical Pair Lemma 6, and apply the substitution:

```
nand[nand[x1_,x1_],nand[x1_,x1_]]→x1
```

which follows from Axiom 1.

Critical Pair Lemma 7

The following expressions are equivalent:

`nand[nand[x1,nand[x1,x1]],nand[x2,x2]]==x2`

Proof

Note that the input for the rule:

`nand[x1_,x2_]↔nand[x2_,x1_]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[nand[x1_,x1_],nand[x2_,nand[x2_,x2_]]]→x1`

where these rules follow from Substitution Lemma 3 and Critical Pair Lemma 1 respectively.

Critical Pair Lemma 8

The following expressions are equivalent:

`nand[x1,x1]==nand[nand[x2,nand[x2,x2]],x1]`

Proof

Note that the input for the rule:

`nand[nand[x1_,nand[x1_,x1_]],nand[x2_,x2_]]→x2`

contains a subpattern of the form:

`nand[x2_,x2_]`

which can be unified with the input for the rule:

`nand[nand[x1_,x1_],nand[x1_,x1_]]→x1`

where these rules follow from Critical Pair Lemma 7 and Axiom 1 respectively.

Substitution Lemma 4

It can be shown that:

`nand[x1,x1]==nand[x1,nand[x2,nand[x2,x2]]]`

Proof

We start by taking Critical Pair Lemma 2, and apply the substitution:

`nand[x1_,x2_]→nand[x2,x1]`

which follows from Substitution Lemma 3.

Critical Pair Lemma 9

The following expressions are equivalent:

`x1==nand[nand[x1,nand[x2,nand[x2,x2]]],nand[x1,x1]]`

Proof

Note that the input for the rule:

`nand[nand[x1_,x1_],nand[x1_,x1_]]→x1`

contains a subpattern of the form:

`nand[x1_,x1_]`

which can be unified with the input for the rule:

`nand[x1_,x1_]↔nand[x1_,nand[x2_,nand[x2_,x2_]]]`

where these rules follow from Axiom 1 and Substitution Lemma 4 respectively.

Critical Pair Lemma 10

The following expressions are equivalent:

`x1==nand[nand[x1,nand[x2,nand[x2,x2]]],nand[x3,nand[x3,x3]]]`

Proof

Note that the input for the rule:

`nand[nand[x1_,x1_],nand[x2_,nand[x2_,x2_]]]→x1`

contains a subpattern of the form:

`nand[x1_,x1_]`

which can be unified with the input for the rule:

`nand[x1_,x1_]↔nand[x1_,nand[x2_,nand[x2_,x2_]]]`

where these rules follow from Critical Pair Lemma 1 and Substitution Lemma 4 respectively.

Critical Pair Lemma 11

The following expressions are equivalent:

`nand[nand[x1,x1],nand[nand[x2,x2],nand[x2,x2]]]==nand[nand[nand[nand[nand[x2,x2],nand[`

Proof

Note that the input for the rule:

`nand[nand[x1_,nand[x2_,nand[x2_,x2_]]],nand[x3_,nand[x3_,x3_]]]→x1`

contains a subpattern of the form:

`nand[x1_,nand[x2_,nand[x2_,x2_]]]`

which can be unified with the input for the rule:

`nand[nand[x1_,nand[x2_,x3_]],nand[x1_,nand[x2_,x3_]]]↔nand[nand[nand[x2_,x2_],x1_],na`

where these rules follow from Critical Pair Lemma 10 and Axiom 3 respectively.

Substitution Lemma 5

It can be shown that:

`nand[nand[x1,x1],x2]==nand[nand[nand[nand[x2,x2],nand[x2,x2]],nand[x1,x2]],nand[n`

Proof

We start by taking Critical Pair Lemma 11, and apply the substitution:

`nand[nand[x1_,x1_],nand[x1_,x1_]]→x1`

which follows from Axiom 1.

Substitution Lemma 6

It can be shown that:

`nand[nand[x1,x1],x2]==nand[nand[nand[x2,x2],nand[x2,x2]],nand[x1,x2]]`

Proof

We start by taking Substitution Lemma 5, and apply the substitution:

`nand[nand[x1_,x1_],nand[x2_,nand[x2_,x2_]]]→x1` `nand[nand[x1_,x1_],nand[x2_,nand[x2_,x2_]]]→x1`

which follows from Critical Pair Lemma 1.

Substitution Lemma 7

It can be shown that:

`nand[nand[x1,x1],x2]==nand[x2,nand[x1,x2]]`

Proof

We start by taking Substitution Lemma 6, and apply the substitution:

`nand[nand[x1_,x1_],nand[x1_,x1_]]→x1`

which follows from Axiom 1.

Substitution Lemma 8

It can be shown that:

`nand[a,nand[a,b]]==nand[nand[b,b],a]`

Proof

We start by taking Hypothesis 2, and apply the substitution:

`nand[x1_,nand[x2_,x2_]]→nand[nand[x2,x2],x1]`

which follows from Substitution Lemma 2.

Substitution Lemma 9

It can be shown that:

`nand[a,nand[b,a]]==nand[nand[b,b],a]`

Proof

We start by taking Substitution Lemma 8, and apply the substitution:

`nand[x1_,x2_]→nand[x2,x1]`

which follows from Substitution Lemma 3.

Conclusion 1

We obtain the conclusion:

`True`

Proof

Take Substitution Lemma 9, and apply the substitution:

`nand[nand[x1_,x1_],x2_]→nand[x2,nand[x1,x2]]`

which follows from Substitution Lemma 7.

Critical Pair Lemma 12

The following expressions are equivalent:

`nand[x1,nand[x2,x1]]==nand[x1,nand[x2,x2]]`

Proof

Note that the input for the rule:

`nand[nand[x1_,x1_],x2_]↔nand[x2_,nand[x1_,x2_]]`

contains a subpattern of the form:

```
nand[nand[x1_,x1_],x2_]
```

which can be unified with the input for the rule:

```
nand[x1_,x2_]↔nand[x2_,x1_]
```

where these rules follow from Substitution Lemma 7 and Substitution Lemma 3 respectively.

Critical Pair Lemma 13

The following expressions are equivalent:

```
nand[x1,nand[nand[x2,x2],x1]]==nand[x2,x1]
```

Proof

Note that the input for the rule:

```
nand[nand[x1_,x1_],x2_]↔nand[x2_,nand[x1_,x2_]]
```

contains a subpattern of the form:

```
nand[x1_,x1_]
```

which can be unified with the input for the rule:

```
nand[nand[x1_,x1_],nand[x1_,x1_]]→x1
```

where these rules follow from Substitution Lemma 7 and Axiom 1 respectively.

Critical Pair Lemma 14

The following expressions are equivalent:

```
nand[x1,nand[x2,x1]]==nand[nand[nand[x3,nand[x3,x3]],x2],x1]
```

Proof

Note that the input for the rule:

```
nand[nand[x1_,x1_],x2_]↔nand[x2_,nand[x1_,x2_]]
```

contains a subpattern of the form:

```
nand[x1_,x1_]
```

which can be unified with the input for the rule:

```
nand[x1_,x1_]↔nand[nand[x2_,nand[x2_,x2_]],x1_]
```

where these rules follow from Substitution Lemma 7 and Critical Pair Lemma 8 respectively.

Critical Pair Lemma 15

The following expressions are equivalent:

```
nand[x1,nand[x2,x1]]==nand[nand[x2,nand[x3,nand[x3,x3]]],x1]
```

Proof

Note that the input for the rule:

```
nand[nand[x1_,x1_],x2_]↔nand[x2_,nand[x1_,x2_]]
```

contains a subpattern of the form:

```
nand[x1_,x1_]
```

which can be unified with the input for the rule:

```
nand[x1_,x1_]↔nand[x1_,nand[x2_,nand[x2_,x2_]]]
```

where these rules follow from Substitution Lemma 7 and Substitution Lemma 4 respectively.

Critical Pair Lemma 16

The following expressions are equivalent:

nand[nand[nand[x1,x1],nand[x1,x1]],x2]==nand[x2,nand[x2,nand[x1,x2]]]

Proof

Note that the input for the rule:

nand[nand[x1_,x1_],x2_]↔nand[x2_,nand[x1_,x2_]]

contains a subpattern of the form:

nand[x2_,x1_]

which can be unified with the input for the rule:

nand[nand[x1_,x1_],x2_]↔nand[x2_,nand[x1_,x2_]]

where these rules follow from Substitution Lemma 7 and Substitution Lemma 7 respectively.

Substitution Lemma 10

It can be shown that:

nand[x1,x2]==nand[x2,nand[x2,nand[x1,x2]]]

Proof

We start by taking Critical Pair Lemma 16, and apply the substitution:

nand[nand[x1_,x1_],nand[x1_,x1_]]→x1

which follows from Axiom 1.

Critical Pair Lemma 17

The following expressions are equivalent:

nand[nand[x1,x1],x2]==nand[x2,nand[x2,x1]]

Proof

Note that the input for the rule:

nand[nand[x1_,x1_],x2_]↔nand[x2_,nand[x1_,x2_]]

contains a subpattern of the form:

nand[x2_,x1_]

which can be unified with the input for the rule:

nand[x1_,x2_]↔nand[x2_,x1_]

where these rules follow from Substitution Lemma 7 and Substitution Lemma 3 respectively.

Critical Pair Lemma 18

The following expressions are equivalent:

nand[x1,x2]==nand[x2,nand[x2,nand[x1,x1]]]

Proof

Note that the input for the rule:

nand[x1_,nand[nand[x2_,x2_],x1_]]→nand[x2,x1]

contains a subpattern of the form:

nand[nand[x2_,x2_],x1_]

which can be unified with the input for the rule:

nand[x1_,x2_]↔nand[x2_,x1_]

where these rules follow from Critical Pair Lemma 12 and Substitution Lemma 3 respectively.

where these rules follow from Critical Pair Lemma 13 and Substitution Lemma 3 respectively.

Critical Pair Lemma 19

The following expressions are equivalent:

`nand[x1,x2]==nand[x2,nand[nand[x1,nand[x3,nand[x3,x3]]],x2]]`

PROOF

Note that the input for the rule:

`nand[x1_,nand[nand[x2_,x2_],x1_]]→nand[x2,x1]`

contains a subpattern of the form:

`nand[x2_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,x1_]↔nand[x1_,nand[x2_,nand[x2_,x2_]]]`

where these rules follow from Critical Pair Lemma 13 and Substitution Lemma 4 respectively.

Critical Pair Lemma 20

The following expressions are equivalent:

`nand[x1,x2]==nand[x2,nand[x2,nand[x2,x1]]]`

PROOF

Note that the input for the rule:

`nand[x1_,nand[x1_,nand[x2_,x1_]]]→nand[x2,x1]`

contains a subpattern of the form:

`nand[x2_,x1_]`

which can be unified with the input for the rule:

`nand[x1_,x2_]↔nand[x2_,x1_]`

where these rules follow from Substitution Lemma 10 and Substitution Lemma 3 respectively.

Critical Pair Lemma 21

The following expressions are equivalent:

`nand[x1,nand[x2,x2]]==nand[x1,nand[x1,x2]]`

PROOF

Note that the input for the rule:

`nand[x1_,nand[x2_,x1_]]↔nand[x1_,nand[x2_,x2_]]`

contains a subpattern of the form:

`nand[x2_,x1_]`

which can be unified with the input for the rule:

`nand[x1_,x2_]↔nand[x2_,x1_]`

where these rules follow from Critical Pair Lemma 12 and Substitution Lemma 3 respectively.

Critical Pair Lemma 22

The following expressions are equivalent:

`nand[x1,nand[x1,x2]]==nand[nand[x2,nand[x3,nand[x3,x3]]],x1]`

PROOF

Note that the input for the rule:

nand[nand[x1_,x1_],x2_]↔nand[x2_,nand[x2_,x1_]]

contains a subpattern of the form:

nand[x1_,x1_]

which can be unified with the input for the rule:

nand[x1_,x1_]↔nand[x1_,nand[x2_,nand[x2_,x2_]]]

where these rules follow from Critical Pair Lemma 17 and Substitution Lemma 4 respectively.

Critical Pair Lemma 23

The following expressions are equivalent:

nand[x1,nand[x1,x2]]==nand[x1,nand[x2,nand[x3,nand[x3,x3]]]]

Proof

Note that the input for the rule:

nand[x1_,nand[x2_,x2_]]↔nand[x1_,nand[x1_,x2_]]

contains a subpattern of the form:

nand[x2_,x2_]

which can be unified with the input for the rule:

nand[x1_,x1_]↔nand[x1_,nand[x2_,nand[x2_,x2_]]]

where these rules follow from Critical Pair Lemma 21 and Substitution Lemma 4 respectively.

Critical Pair Lemma 24

The following expressions are equivalent:

nand[nand[nand[x1,nand[x1,x1]],x2],x3]==nand[x3,nand[x2,x2]]

Proof

Note that the input for the rule:

nand[x1_,nand[x2_,x1_]]↔nand[nand[nand[x3_,nand[x3_,x3_]],x2_],x1_]

contains a subpattern of the form:

nand[x1_,nand[x2_,x1_]]

which can be unified with the input for the rule:

nand[x1_,nand[x2_,x1_]]↔nand[x1_,nand[x2_,x2_]]

where these rules follow from Critical Pair Lemma 14 and Critical Pair Lemma 12 respectively.

Critical Pair Lemma 25

The following expressions are equivalent:

nand[nand[x1,nand[x2,nand[x2,x2]]],x3]==nand[x3,nand[x1,x1]]

Proof

Note that the input for the rule:

nand[x1_,nand[x2_,x1_]]↔nand[nand[x2_,nand[x3_,nand[x3_,x3_]]],x1_]

contains a subpattern of the form:

nand[x1_,nand[x2_,x1_]]

which can be unified with the input for the rule:

nand[x1_,nand[x2_,x1_]]↔nand[x1_,nand[x2_,x2_]] nand[x1_,nand[x2_,x1_]]↔nand[x1_,nand[x2_,x2_]]

where these rules follow from Critical Pair Lemma 15 and Critical Pair Lemma 12 respectively.

Critical Pair Lemma 26

The following expressions are equivalent:

```
nand[nand[x1,nand[x2,nand[x2,x2]]],nand[x1,x1]]==nand[nand[nand[x2,x2],x1],nand[nand[n
```

Proof

Note that the input for the rule:

```
nand[nand[x1_,nand[x2_,nand[x2_,x2_]]],x3_]↔nand[x3_,nand[x1_,x1_]]
```

contains a subpattern of the form:

```
nand[nand[x1_,nand[x2_,nand[x2_,x2_]]],x3_]
```

which can be unified with the input for the rule:

```
nand[nand[x1_,nand[x2_,x3_]],nand[x1_,nand[x2_,x3_]]]↔nand[nand[nand[x2_,x2_],x1_],na
```

where these rules follow from Critical Pair Lemma 25 and Axiom 3 respectively.

Substitution Lemma 11

It can be shown that:

```
x1==nand[nand[nand[x2,x2],x1],nand[nand[nand[x2,x2],nand[x2,x2]],x1]]
```

Proof

We start by taking Critical Pair Lemma 26, and apply the substitution:

```
nand[nand[x1_,nand[x2_,nand[x2_,x2_]]],nand[x1_,x1_]]→x1
```

which follows from Critical Pair Lemma 9.

Substitution Lemma 12

It can be shown that:

```
x1==nand[nand[nand[x2,x2],x1],nand[x2,x1]]
```

Proof

We start by taking Substitution Lemma 11, and apply the substitution:

```
nand[nand[x1_,x1_],nand[x1_,x1_]]→x1
```

which follows from Axiom 1.

Substitution Lemma 13

It can be shown that:

```
x1==nand[nand[x2,x1],nand[nand[x2,x2],x1]]
```

Proof

We start by taking Substitution Lemma 12, and apply the substitution:

```
nand[x1_,x2_]→nand[x2,x1]
```

which follows from Substitution Lemma 3.

Critical Pair Lemma 27

The following expressions are equivalent:

```
x1==nand[nand[x1,nand[x2,x1]],nand[nand[nand[x2,x2],nand[x2,x2]],x1]]
```

Proof

PROOF

Note that the input for the rule:

`nand[nand[x1_,x2_],nand[nand[x1_,x1_],x2_]]→x2`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[nand[x1_,x1_],x2_]↔nand[x2_,nand[x1_,x2_]]`

where these rules follow from Substitution Lemma 13 and Substitution Lemma 7 respectively.

Substitution Lemma 14

It can be shown that:

`x1==nand[nand[x1,nand[x2,x1]],nand[x2,x1]]`

PROOF

We start by taking Critical Pair Lemma 27, and apply the substitution:

`nand[nand[x1_,x1_],nand[x1_,x1_]]→x1`

which follows from Axiom 1.

Substitution Lemma 15

It can be shown that:

`x1==nand[nand[x2,x1],nand[x1,nand[x2,x1]]]`

PROOF

We start by taking Substitution Lemma 14, and apply the substitution:

`nand[x1_,x2_]→nand[x2,x1]`

which follows from Substitution Lemma 3.

Substitution Lemma 16

It can be shown that:

`x1==nand[nand[x2,x1],nand[x1,x1]]`

PROOF

We start by taking Substitution Lemma 15, and apply the substitution:

`nand[x1_,nand[x2_,x1_]]→nand[x1,nand[x2,x2]]`

which follows from Critical Pair Lemma 12.

Critical Pair Lemma 28

The following expressions are equivalent:

`x1==nand[nand[x1,x1],nand[x2,x1]]`

PROOF

Note that the input for the rule:

`nand[nand[x1_,x2_],nand[x2_,x2_]]→x2`

contains a subpattern of the form:

`nand[nand[x1_,x2_],nand[x2_,x2_]]`

which can be unified with the input for the rule:

```
nand[x1_,x2_]↔nand[x2_,x1_]
```

where these rules follow from Substitution Lemma 16 and Substitution Lemma 3 respectively.

Critical Pair Lemma 29

The following expressions are equivalent:

```
x1==nand[nand[x1,x2],nand[x1,x1]]
```

Proof

Note that the input for the rule:

```
nand[nand[x1_,x2_],nand[x2_,x2_]]→x2
```

contains a subpattern of the form:

```
nand[x1_,x2_]
```

which can be unified with the input for the rule:

```
nand[x1_,x2_]↔nand[x2_,x1_]
```

where these rules follow from Substitution Lemma 16 and Substitution Lemma 3 respectively.

Critical Pair Lemma 30

The following expressions are equivalent:

```
nand[x1,x1]==nand[nand[x2,nand[x2,x1]],nand[nand[x1,x1],nand[x1,x1]]]
```

Proof

Note that the input for the rule:

```
nand[nand[x1_,x2_],nand[x2_,x2_]]→x2
```

contains a subpattern of the form:

```
nand[x1_,x2_]
```

which can be unified with the input for the rule:

```
nand[x1_,nand[x2_,x2_]]↔nand[x1_,nand[x1_,x2_]]
```

where these rules follow from Substitution Lemma 16 and Critical Pair Lemma 21 respectively.

Substitution Lemma 17

It can be shown that:

```
nand[x1,x1]==nand[nand[x2,nand[x2,x1]],x1]
```

Proof

We start by taking Critical Pair Lemma 30, and apply the substitution:

```
nand[nand[x1_,x2_],nand[x2_,x2_]]→x2
```

which follows from Substitution Lemma 16.

Critical Pair Lemma 31

The following expressions are equivalent:

```
nand[x1,x1]==nand[nand[x2,nand[x1,x2]],nand[nand[x1,x1],nand[x1,x1]]]
```

Proof

Note that the input for the rule:

`nand[nand[x1_,x2_],nand[x2_,x2_]]→x2`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,nand[x2_,x1_]]↔nand[x1_,nand[x2_,x2_]]`

where these rules follow from Substitution Lemma 16 and Critical Pair Lemma 12 respectively.

Substitution Lemma 18

It can be shown that:

`nand[x1,x1]⩵nand[nand[x2,nand[x1,x2]],x1]`

Proof

We start by taking Critical Pair Lemma 31, and apply the substitution:

`nand[nand[x1_,x2_],nand[x2_,x2_]]→x2`

which follows from Substitution Lemma 16.

Critical Pair Lemma 32

The following expressions are equivalent:

`nand[nand[x1,x1],nand[nand[x2,x1],nand[x2,x1]]]⩵nand[nand[x1,x1],x1]`

Proof

Note that the input for the rule:

`nand[x1_,nand[x2_,x1_]]↔nand[x1_,nand[x2_,x2_]]`

contains a subpattern of the form:

`nand[x2_,x1_]`

which can be unified with the input for the rule:

`nand[nand[x1_,x2_],nand[x2_,x2_]]→x2`

where these rules follow from Critical Pair Lemma 12 and Substitution Lemma 16 respectively.

Critical Pair Lemma 33

The following expressions are equivalent:

`x1⩵nand[nand[x1,x1],nand[x1,x2]]`

Proof

Note that the input for the rule:

`nand[nand[x1_,x1_],nand[x2_,x1_]]→x1`

contains a subpattern of the form:

`nand[x2_,x1_]`

which can be unified with the input for the rule:

`nand[x1_,x2_]↔nand[x2_,x1_]`

where these rules follow from Critical Pair Lemma 28 and Substitution Lemma 3 respectively.

Substitution Lemma 19

It can be shown that:

It can be shown that:

`nand[nand[a,b],nand[a,nand[a,b]]]==a`

PROOF

We start by taking Hypothesis 1, and apply the substitution:

`nand[nand[x1_,x1_],x2_]→nand[x2,nand[x1,x2]]`

which follows from Substitution Lemma 7.

Substitution Lemma 20

It can be shown that:

`nand[nand[a,b],nand[a,a]]==a`

PROOF

We start by taking Substitution Lemma 19, and apply the substitution:

`nand[x1_,nand[x2_,x1_]]→nand[x1,nand[x2,x2]]`

which follows from Critical Pair Lemma 12.

Conclusion 2

We obtain the conclusion:

`True`

PROOF

Take Substitution Lemma 20, and apply the substitution:

`nand[nand[x1_,x2_],nand[x1_,x1_]]→x1`

which follows from Critical Pair Lemma 29.

Critical Pair Lemma 34

The following expressions are equivalent:

`x1==nand[nand[x1,nand[x2,nand[x2,x2]]],nand[x1,x3]]`

PROOF

Note that the input for the rule:

`nand[nand[x1_,x2_],nand[x1_,x1_]]→x1`

contains a subpattern of the form:

`nand[nand[x1_,x2_],nand[x1_,x1_]]`

which can be unified with the input for the rule:

`nand[nand[x1_,nand[x2_,nand[x2_,x2_]]],x3_]↔nand[x3_,nand[x1_,x1_]]`

where these rules follow from Critical Pair Lemma 29 and Critical Pair Lemma 25 respectively.

Critical Pair Lemma 35

The following expressions are equivalent:

`x1==nand[nand[nand[x2,nand[x2,x2]],x1],nand[x1,x3]]`

PROOF

Note that the input for the rule:

`nand[nand[x1_,x2_],nand[x1_,x1_]]→x1`

contains a subpattern of the form:

contains a subpattern of the form:

`nand[nand[x1_,x2_],nand[x1_,x1_]]`

which can be unified with the input for the rule:

`nand[nand[nand[x1_,nand[x1_,x1_]],x2_],x3_]↔nand[x3_,nand[x2_,x2_]]`

where these rules follow from Critical Pair Lemma 29 and Critical Pair Lemma 24 respectively.

Substitution Lemma 21

It can be shown that:

`nand[x1,x1]==nand[x1,nand[x2,nand[x2,x1]]]`

Proof

We start by taking Substitution Lemma 17, and apply the substitution:

`nand[x1_,x2_]→nand[x2,x1]`

which follows from Substitution Lemma 3.

Substitution Lemma 22

It can be shown that:

`nand[x1,x1]==nand[x1,nand[x2,nand[x1,x2]]]`

Proof

We start by taking Substitution Lemma 18, and apply the substitution:

`nand[x1_,x2_]→nand[x2,x1]`

which follows from Substitution Lemma 3.

Substitution Lemma 23

It can be shown that:

`nand[nand[x1,x1],nand[nand[x2,x1],nand[x2,x1]]]==nand[x1,nand[x1,x1]]`

Proof

We start by taking Critical Pair Lemma 32, and apply the substitution:

`nand[nand[x1_,x1_],x2_]→nand[x2,nand[x2,x1]]`

which follows from Critical Pair Lemma 17.

Critical Pair Lemma 36

The following expressions are equivalent:

`nand[nand[nand[nand[x1,x1],nand[x1,x1]],x2],nand[nand[nand[nand[x3,x1],nand[x3,x1]],na`

Proof

Note that the input for the rule:

`nand[nand[x1_,nand[x2_,x3_]],nand[x1_,nand[x2_,x3_]]]↔nand[nand[nand[x2_,x2_],x1_],na`

contains a subpattern of the form:

`nand[x2_,x3_]`

which can be unified with the input for the rule:

`nand[nand[x1_,x1_],nand[nand[x2_,x1_],nand[x2_,x1_]]]→nand[x1,nand[x1,x1]]`

where these rules follow from Axiom 3 and Substitution Lemma 23 respectively.

Substitution Lemma 24

It can be shown that:

```
nand[nand[x1,x2],nand[nand[nand[nand[x3,x1],nand[x3,x1]],nand[nand[x3,x1],nand[x3,x1]]
```

PROOF

We start by taking Critical Pair Lemma 36, and apply the substitution:

```
nand[nand[x1_,x2_],nand[x2_,x2_]]→x2
```

which follows from Substitution Lemma 16.

Substitution Lemma 25

It can be shown that:

```
nand[nand[x1,x2],nand[nand[x3,x1],x2]]==nand[nand[x2,nand[x1,nand[x1,x1]]],nand[x2,nan
```

PROOF

We start by taking Substitution Lemma 24, and apply the substitution:

```
nand[nand[x1_,x2_],nand[x2_,x2_]]→x2
```

which follows from Substitution Lemma 16.

Substitution Lemma 26

It can be shown that:

```
nand[nand[x1,x2],nand[nand[x3,x1],x2]]==x2
```

PROOF

We start by taking Substitution Lemma 25, and apply the substitution:

```
nand[nand[x1_,nand[x2_,nand[x2_,x2_]]],nand[x1_,x3_]]→x1
```

which follows from Critical Pair Lemma 34.

Critical Pair Lemma 37

The following expressions are equivalent:

```
nand[x1,x2]==nand[x2,nand[nand[x3,nand[x2,x2]],nand[x1,x2]]]
```

PROOF

Note that the input for the rule:

```
nand[nand[x1_,x2_],nand[nand[x3_,x1_],x2_]]→x2
```

contains a subpattern of the form:

```
nand[x1_,x2_]
```

which can be unified with the input for the rule:

```
nand[nand[x1_,x1_],nand[x2_,x1_]]→x1
```

where these rules follow from Substitution Lemma 26 and Critical Pair Lemma 28 respectively.

Critical Pair Lemma 38

The following expressions are equivalent:

```
x1==nand[nand[x1,x2],nand[nand[x3,x2],x1]]
```

PROOF

Note that the input for the rule:

nand[nand[x1_,x2_],nand[nand[x3_,x1_],x2_]]→x2

contains a subpattern of the form:

nand[x1_,x2_]

which can be unified with the input for the rule:

nand[x1_,x2_]↔nand[x2_,x1_]

where these rules follow from Substitution Lemma 26 and Substitution Lemma 3 respectively.

Critical Pair Lemma 39

The following expressions are equivalent:

x1==nand[nand[x1,nand[x1,x2]],nand[nand[x3,nand[x2,x2]],x1]]

Proof

Note that the input for the rule:

nand[nand[x1_,x2_],nand[nand[x3_,x1_],x2_]]→x2

contains a subpattern of the form:

nand[x1_,x2_]

which can be unified with the input for the rule:

nand[nand[x1_,x1_],x2_]↔nand[x2_,nand[x2_,x1_]]

where these rules follow from Substitution Lemma 26 and Critical Pair Lemma 17 respectively.

Critical Pair Lemma 40

The following expressions are equivalent:

x1==nand[nand[x2,x1],nand[x1,nand[x3,x2]]]

Proof

Note that the input for the rule:

nand[nand[x1_,x2_],nand[nand[x3_,x1_],x2_]]→x2

contains a subpattern of the form:

nand[nand[x3_,x1_],x2_]

which can be unified with the input for the rule:

nand[x1_,x2_]↔nand[x2_,x1_]

where these rules follow from Substitution Lemma 26 and Substitution Lemma 3 respectively.

Critical Pair Lemma 41

The following expressions are equivalent:

x1==nand[nand[nand[x2,nand[x2,x3]],x1],nand[nand[x3,x3],x1]]

Proof

Note that the input for the rule:

nand[nand[x1_,x2_],nand[nand[x3_,x1_],x2_]]→x2

contains a subpattern of the form:

nand[x3_,x1_]

which can be unified with the input for the rule:

nand[x1_,nand[x2_,nand[x2_,x1_]]]→nand[x1,x1]

`nand[x1_,nand[x1_,nand[x1_,x1_]]]→nand[x1,x1]`

where these rules follow from Substitution Lemma 26 and Substitution Lemma 21 respectively.

Critical Pair Lemma 42

The following expressions are equivalent:

`x1==nand[nand[nand[x2,x3],x1],nand[x2,x1]]`

Proof

Note that the input for the rule:

`nand[nand[x1_,x2_],nand[nand[x3_,x1_],x2_]]→x2`

contains a subpattern of the form:

`nand[x3_,x1_]`

which can be unified with the input for the rule:

`nand[nand[nand[x1_,nand[x1_,x1_]],x2_],nand[x2_,x3_]]→x2`

where these rules follow from Substitution Lemma 26 and Critical Pair Lemma 35 respectively.

Critical Pair Lemma 43

The following expressions are equivalent:

`nand[nand[x1,x2],nand[x1,x2]]==nand[nand[x1,x2],nand[nand[nand[x3,x1],x2],x2]]`

Proof

Note that the input for the rule:

`nand[x1_,nand[x2_,nand[x1_,x2_]]]→nand[x1,x1]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[nand[x1_,x2_],nand[nand[x3_,x1_],x2_]]→x2`

where these rules follow from Substitution Lemma 22 and Substitution Lemma 26 respectively.

Critical Pair Lemma 44

The following expressions are equivalent:

`x1==nand[nand[x1,x2],nand[x1,nand[x3,x2]]]`

Proof

Note that the input for the rule:

`nand[nand[x1_,x2_],nand[nand[x3_,x2_],x1_]]→x1`

contains a subpattern of the form:

`nand[nand[x3_,x2_],x1_]`

which can be unified with the input for the rule:

`nand[x1_,x2_]↔nand[x2_,x1_]`

where these rules follow from Critical Pair Lemma 38 and Substitution Lemma 3 respectively.

Critical Pair Lemma 45

The following expressions are equivalent:

`x1==nand[nand[x1,nand[x2,x3]],nand[x2,x1]]`

PROOF

Note that the input for the rule:

`nand[nand[x1_,x2_],nand[nand[x3_,x2_],x1_]]→x1`

contains a subpattern of the form:

`nand[x3_,x2_]`

which can be unified with the input for the rule:

`nand[nand[nand[x1_,nand[x1_,x1_]],x2_],nand[x2_,x3_]]→x2`

where these rules follow from Critical Pair Lemma 38 and Critical Pair Lemma 35 respectively.

Critical Pair Lemma 46

The following expressions are equivalent:

`x1==nand[nand[x1,x2],nand[nand[x2,x3],x1]]`

Out[ ]=

PROOF

Note that the input for the rule:

`nand[nand[x1_,x2_],nand[nand[x3_,x2_],x1_]]→x1`

contains a subpattern of the form:

`nand[x3_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,x2_]↔nand[x2_,x1_]`

where these rules follow from Critical Pair Lemma 38 and Substitution Lemma 3 respectively.

Critical Pair Lemma 47

The following expressions are equivalent:

`nand[nand[x1,x2],x3]==nand[nand[nand[nand[x1,x2],x3],x2],x3]`

PROOF

Note that the input for the rule:

`nand[nand[x1_,x2_],nand[nand[x3_,x2_],x1_]]→x1`

contains a subpattern of the form:

`nand[nand[x3_,x2_],x1_]`

which can be unified with the input for the rule:

`nand[nand[x1_,x2_],nand[nand[x3_,x2_],x1_]]→x1`

where these rules follow from Critical Pair Lemma 38 and Critical Pair Lemma 38 respectively.

Critical Pair Lemma 48

The following expressions are equivalent:

`nand[nand[x1,x2],nand[x1,x2]]==nand[nand[x1,x2],nand[nand[nand[x3,x2],x1],x1]]`

PROOF

Note that the input for the rule:

`nand[x1_,nand[x2_,nand[x1_,x2_]]]→nand[x1,x1]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[nand[x1_,x2_],nand[nand[x3_,x2_],x1_]]→x1`

where these rules follow from Substitution Lemma 22 and Critical Pair Lemma 38 respectively.

Substitution Lemma 27

It can be shown that:

`x1==nand[nand[x2,x1],nand[nand[x2,x3],x1]]`

Proof

We start by taking Critical Pair Lemma 42, and apply the substitution:

`nand[x1_,x2_]→nand[x2,x1]`

which follows from Substitution Lemma 3.

Critical Pair Lemma 49

The following expressions are equivalent:

`nand[x1,nand[x2,x3]]==nand[nand[x3,nand[x1,nand[x2,x3]]],x1]`

Proof

Note that the input for the rule:

`nand[nand[x1_,x2_],nand[nand[x1_,x3_],x2_]]→x2`

contains a subpattern of the form:

`nand[nand[x1_,x3_],x2_]`

which can be unified with the input for the rule:

`nand[nand[x1_,x2_],nand[x2_,nand[x3_,x1_]]]→x2`

where these rules follow from Substitution Lemma 27 and Critical Pair Lemma 40 respectively.

Critical Pair Lemma 50

The following expressions are equivalent:

`x1==nand[nand[x1,nand[x2,x3]],nand[x1,x2]]`

Proof

Note that the input for the rule:

`nand[nand[x1_,x2_],nand[x1_,nand[x3_,x2_]]]→x1`

contains a subpattern of the form:

`nand[x3_,x2_]`

which can be unified with the input for the rule:

`nand[nand[nand[x1_,nand[x1_,x1_]],x2_],nand[x2_,x3_]]→x2`

where these rules follow from Critical Pair Lemma 44 and Critical Pair Lemma 35 respectively.

Critical Pair Lemma 51

The following expressions are equivalent:

`nand[x1,x2]==nand[nand[nand[x3,x2],nand[x1,x2]],x1]`

PROOF

Note that the input for the rule:

`nand[nand[x1_,x2_],nand[x2_,nand[x3_,x1_]]]→x2`

contains a subpattern of the form:

`nand[x2_,nand[x3_,x1_]]`

which can be unified with the input for the rule:

`nand[nand[x1_,x2_],nand[x1_,nand[x3_,x2_]]]→x1`

where these rules follow from Critical Pair Lemma 40 and Critical Pair Lemma 44 respectively.

Substitution Lemma 28

It can be shown that:

`x1==nand[nand[x2,x1],nand[x1,nand[x2,x3]]]`

PROOF

We start by taking Critical Pair Lemma 45, and apply the substitution:

`nand[x1_,x2_]→nand[x2,x1]`

which follows from Substitution Lemma 3.

Critical Pair Lemma 52

The following expressions are equivalent:

`nand[x1,nand[x2,x3]]==nand[nand[x2,nand[x1,nand[x2,x3]]],x1]`

PROOF

Note that the input for the rule:

`nand[nand[x1_,x2_],nand[nand[x1_,x3_],x2_]]→x2`

contains a subpattern of the form:

`nand[nand[x1_,x3_],x2_]`

which can be unified with the input for the rule:

`nand[nand[x1_,x2_],nand[x2_,nand[x1_,x3_]]]→x2`

where these rules follow from Substitution Lemma 27 and Substitution Lemma 28 respectively.

Substitution Lemma 29

It can be shown that:

`x1==nand[nand[x1,x2],nand[x1,nand[x2,x3]]]`

PROOF

We start by taking Critical Pair Lemma 50, and apply the substitution:

`nand[x1_,x2_]→nand[x2,x1]`

which follows from Substitution Lemma 3.

Critical Pair Lemma 53

The following expressions are equivalent:

`nand[x1,x2]==nand[nand[nand[x1,x2],nand[x2,x3]],x1]`

```
nand[x1,x2]==nand[nand[nand[x1,x2],nand[x2,x3]],x1]
```

Proof

Note that the input for the rule:

```
nand[nand[x1_,x2_],nand[x1_,nand[x2_,x3_]]]→x1
```

contains a subpattern of the form:

```
nand[x1_,nand[x2_,x3_]]
```

which can be unified with the input for the rule:

```
nand[nand[x1_,x2_],nand[nand[x2_,x3_],x1_]]→x1
```

where these rules follow from Substitution Lemma 29 and Critical Pair Lemma 46 respectively.

Substitution Lemma 30

It can be shown that:

```
nand[x1,x2]==nand[x1,nand[nand[x3,x2],nand[x1,x2]]]
```

Proof

We start by taking Critical Pair Lemma 51, and apply the substitution:

```
nand[x1_,x2_]→nand[x2,x1]
```

which follows from Substitution Lemma 3.

Substitution Lemma 31

It can be shown that:

```
nand[x1,x2]==nand[x1,nand[nand[x1,x2],nand[x2,x3]]]
```

Proof

We start by taking Critical Pair Lemma 53, and apply the substitution:

```
nand[x1_,x2_]→nand[x2,x1]
```

which follows from Substitution Lemma 3.

Substitution Lemma 32

It can be shown that:

```
nand[nand[x1,x2],x3]==nand[x3,nand[nand[nand[x1,x2],x3],x2]]
```

Proof

We start by taking Critical Pair Lemma 47, and apply the substitution:

```
nand[x1_,x2_]→nand[x2,x1]
```

which follows from Substitution Lemma 3.

Substitution Lemma 33

It can be shown that:

```
nand[nand[x1,x2],x3]==nand[x3,nand[x2,nand[nand[x1,x2],x3]]]
```

Proof

We start by taking Substitution Lemma 32, and apply the substitution:

```
nand[x1_,x2_]→nand[x2,x1]
```

which follows from Substitution Lemma 3.

Substitution Lemma 34

It can be shown that:

`nand[x1,nand[x2,x3]]==nand[x1,nand[x3,nand[x1,nand[x2,x3]]]]`

Proof

We start by taking Critical Pair Lemma 49, and apply the substitution:

`nand[x1_,x2_]→nand[x2,x1]`

which follows from Substitution Lemma 3.

Substitution Lemma 35

It can be shown that:

`nand[x1,nand[x2,x3]]==nand[x1,nand[x2,nand[x1,nand[x2,x3]]]]`

Proof

We start by taking Critical Pair Lemma 52, and apply the substitution:

`nand[x1_,x2_]→nand[x2,x1]`

which follows from Substitution Lemma 3.

Critical Pair Lemma 54

The following expressions are equivalent:

`nand[x1,x2]==nand[x2,nand[nand[x3,nand[x2,x3]],nand[x1,x2]]]`

Proof

Note that the input for the rule:

`nand[x1_,nand[nand[x2_,nand[x1_,x1_]],nand[x3_,x1_]]]→nand[x3,x1]`

contains a subpattern of the form:

`nand[x2_,nand[x1_,x1_]]`

which can be unified with the input for the rule:

`nand[x1_,nand[x2_,x1_]]↔nand[x1_,nand[x2_,x2_]]`

where these rules follow from Critical Pair Lemma 37 and Critical Pair Lemma 12 respectively.

Critical Pair Lemma 55

The following expressions are equivalent:

`x1==nand[nand[x1,nand[x1,x2]],nand[nand[x3,nand[x3,x2]],x1]]`

Proof

Note that the input for the rule:

`nand[nand[x1_,nand[x1_,x2_]],nand[nand[x3_,nand[x2_,x2_]],x1_]]→x1`

contains a subpattern of the form:

`nand[x3_,nand[x2_,x2_]]`

which can be unified with the input for the rule:

`nand[x1_,nand[x2_,x2_]]↔nand[x1_,nand[x1_,x2_]]`

where these rules follow from Critical Pair Lemma 39 and Critical Pair Lemma 21 respectively.

Critical Pair Lemma 56

The following expressions are equivalent:

`nand[nand[nand[nand[x1,x1],nand[x1,x1]],nand[nand[x2,nand[x2,x1]],x3]],nand[nand[x3,x3`

PROOF

Note that the input for the rule:

`nand[nand[x1_,nand[x2_,x3_]],nand[x1_,nand[x2_,x3_]]]↔nand[nand[nand[x2_,x2_],x1_],na`

contains a subpattern of the form:

`nand[x1_,nand[x2_,x3_]]`

which can be unified with the input for the rule:

`nand[nand[nand[x1_,nand[x1_,x2_]],x3_],nand[nand[x2_,x2_],x3_]]→x3`

where these rules follow from Axiom 3 and Critical Pair Lemma 41 respectively.

Substitution Lemma 36

It can be shown that:

`nand[nand[x1,nand[nand[x2,nand[x2,x1]],x3]],nand[nand[x3,x3],nand[nand[x2,nand[x2,x1]]`

PROOF

We start by taking Critical Pair Lemma 56, and apply the substitution:

`nand[nand[x1_,x2_],nand[x2_,x2_]]→x2`

which follows from Substitution Lemma 16.

Substitution Lemma 37

It can be shown that:

`nand[nand[x1,nand[nand[x2,nand[x2,x1]],x3]],x3]==nand[x3,nand[nand[nand[x2,nand[x2,x1]]`

PROOF

We start by taking Substitution Lemma 36, and apply the substitution:

`nand[nand[x1_,x1_],nand[x2_,x1_]]→x1`

which follows from Critical Pair Lemma 28.

Substitution Lemma 38

It can be shown that:

`nand[nand[x1,nand[nand[x2,nand[x2,x1]],x3]],x3]==nand[x3,x3]`

PROOF

We start by taking Substitution Lemma 37, and apply the substitution:

`nand[nand[nand[x1_,nand[x1_,x2_]],x3_],nand[nand[x2_,x2_],x3_]]→x3`

which follows from Critical Pair Lemma 41.

Substitution Lemma 39

It can be shown that:

`nand[x1,nand[x2,nand[nand[x3,nand[x3,x2]],x1]]]==nand[x1,x1]`

PROOF

We start by taking Substitution Lemma 38, and apply the substitution:

`nand[x1_,x2_]→nand[x2,x1]`

which follows from Substitution Lemma 3.

Critical Pair Lemma 57

The following expressions are equivalent:

```
nand[x1,x1]==nand[nand[x2,nand[x2,nand[x3,nand[x1,x3]]]],x1]
```

Proof

Note that the input for the rule:

```
nand[x1_,nand[x2_,nand[nand[x3_,nand[x3_,x2_]],x1_]]]→nand[x1,x1]
```

contains a subpattern of the form:

```
nand[x1_,nand[x2_,nand[nand[x3_,nand[x3_,x2_]],x1_]]]
```

which can be unified with the input for the rule:

```
nand[x1_,nand[nand[x2_,nand[x1_,x2_]],nand[x3_,x1_]]]→nand[x3,x1]
```

where these rules follow from Substitution Lemma 39 and Critical Pair Lemma 54 respectively.

Substitution Lemma 40

It can be shown that:

```
nand[x1,x1]==nand[x1,nand[x2,nand[x2,nand[x3,nand[x1,x3]]]]]
```

Proof

We start by taking Critical Pair Lemma 57, and apply the substitution:

```
nand[x1_,x2_]→nand[x2,x1]
```

which follows from Substitution Lemma 3.

Critical Pair Lemma 58

The following expressions are equivalent:

```
nand[nand[x1,nand[x2,nand[x2,x2]]],nand[x1,nand[x2,nand[x2,x2]]]]==nand[nand[x3,nand[x
```

Proof

Note that the input for the rule:

```
nand[x1_,nand[x2_,nand[x2_,nand[x3_,nand[x1_,x3_]]]]]→nand[x1,x1]
```

contains a subpattern of the form:

```
nand[x1_,nand[x2_,nand[x2_,nand[x3_,nand[x1_,x3_]]]]]
```

which can be unified with the input for the rule:

```
nand[nand[x1_,nand[x2_,nand[x2_,x2_]]],x3_]↔nand[x3_,nand[x1_,x1_]]
```

where these rules follow from Substitution Lemma 40 and Critical Pair Lemma 25 respectively.

Substitution Lemma 41

It can be shown that:

```
x1==nand[nand[x2,nand[x2,nand[x3,nand[nand[x1,nand[x4,nand[x4,x4]]],x3]]]],nand[x1,x1]]
```

Proof

We start by taking Critical Pair Lemma 58, and apply the substitution:

```
nand[nand[x1_,nand[x2_,nand[x2_,x2_]]],nand[x1_,x3_]]→x1
```

which follows from Critical Pair Lemma 34.

Substitution Lemma 42

It can be shown that:

`x1==nand[nand[x2,nand[x2,nand[x1,x3]]],nand[x1,x1]]`

Proof

We start by taking Substitution Lemma 41, and apply the substitution:

`nand[x1_,nand[nand[x2_,nand[x3_,nand[x3_,x3_]]],x1_]]→nand[x2,x1]`

which follows from Critical Pair Lemma 19.

Critical Pair Lemma 59

The following expressions are equivalent:

`nand[x1,x1]==nand[x1,nand[x2,nand[nand[x3,nand[x1,x3]],x2]]]`

Proof

Note that the input for the rule:

`nand[x1_,nand[x2_,nand[x2_,nand[x3_,nand[x1_,x3_]]]]]→nand[x1,x1]`

contains a subpattern of the form:

`nand[x2_,nand[x3_,nand[x1_,x3_]]]`

which can be unified with the input for the rule:

`nand[x1_,x2_]↔nand[x2_,x1_]`

where these rules follow from Substitution Lemma 40 and Substitution Lemma 3 respectively.

Critical Pair Lemma 60

The following expressions are equivalent:

`x1==nand[nand[x2,nand[x2,nand[x3,x1]]],nand[x1,x1]]`

Proof

Note that the input for the rule:

`nand[nand[x1_,nand[x1_,nand[x2_,x3_]]],nand[x2_,x2_]]→x2`

contains a subpattern of the form:

`nand[x1_,nand[x2_,x3_]]`

which can be unified with the input for the rule:

`nand[x1_,nand[x2_,nand[x1_,nand[x3_,x2_]]]]→nand[x1,nand[x3,x2]]`

where these rules follow from Substitution Lemma 42 and Substitution Lemma 34 respectively.

Critical Pair Lemma 61

The following expressions are equivalent:

`x1==nand[nand[nand[x2,nand[x2,nand[x3,x4]]],x1],nand[x1,x4]]`

Proof

Note that the input for the rule:

`nand[nand[x1_,x2_],nand[x2_,nand[x1_,x3_]]]→x2`

contains a subpattern of the form:

contains a subpattern of the form:

`nand[x1_,x3_]`

which can be unified with the input for the rule:

`nand[nand[x1_,nand[x1_,nand[x2_,x3_]]],nand[x3_,x3_]]→x3`

where these rules follow from Substitution Lemma 28 and Critical Pair Lemma 60 respectively.

Critical Pair Lemma 62

The following expressions are equivalent:

`nand[x1,x1]==nand[x1,nand[nand[x2,nand[nand[x1,x2],x3]],x2]]`

Proof

Note that the input for the rule:

`nand[x1_,nand[x2_,nand[nand[x3_,nand[x1_,x3_]],x2_]]]→nand[x1,x1]`

contains a subpattern of the form:

`nand[nand[x3_,nand[x1_,x3_]],x2_]`

which can be unified with the input for the rule:

`nand[nand[x1_,x2_],nand[x1_,nand[x2_,x3_]]]→x1`

where these rules follow from Critical Pair Lemma 59 and Substitution Lemma 29 respectively.

Critical Pair Lemma 63

The following expressions are equivalent:

`nand[x1,x1]==nand[x1,nand[nand[nand[x2,nand[x1,x3]],x3],x3]]`

Proof

Note that the input for the rule:

`nand[x1_,nand[x2_,nand[nand[x3_,nand[x1_,x3_]],x2_]]]→nand[x1,x1]`

contains a subpattern of the form:

`nand[nand[x3_,nand[x1_,x3_]],x2_]`

which can be unified with the input for the rule:

`nand[nand[x1_,x2_],nand[nand[x3_,x2_],x1_]]→x1`

where these rules follow from Critical Pair Lemma 59 and Critical Pair Lemma 38 respectively.

Substitution Lemma 43

It can be shown that:

`nand[x1,x1]==nand[x1,nand[x2,nand[x2,nand[nand[x1,x2],x3]]]]`

Proof

We start by taking Critical Pair Lemma 62, and apply the substitution:

`nand[x1_,x2_]→nand[x2,x1]`

which follows from Substitution Lemma 3.

Substitution Lemma 44

It can be shown that:

`nand[x1,x1]==nand[x1,nand[x2,nand[nand[x3,nand[x1,x2]],x2]]]` nand[x1,x1]==nand[x1,nand[x2,nand[nand[x3,nand[x1,x2]],x2]]]

PROOF

We start by taking Critical Pair Lemma 63, and apply the substitution:

`nand[x1_,x2_]→nand[x2,x1]`

which follows from Substitution Lemma 3.

Substitution Lemma 45

It can be shown that:

`nand[x1,x1]==nand[x1,nand[x2,nand[x2,nand[x3,nand[x1,x2]]]]]`

PROOF

We start by taking Substitution Lemma 44, and apply the substitution:

`nand[x1_,x2_]→nand[x2,x1]`

which follows from Substitution Lemma 3.

Critical Pair Lemma 64

The following expressions are equivalent:

`nand[x1,x1]==nand[x1,nand[nand[x1,nand[nand[x2,x1],x3]],nand[nand[x1,nand[nand[x2,x1],x`

PROOF

Note that the input for the rule:

`nand[x1_,nand[x2_,nand[x2_,nand[x3_,nand[x1_,x2_]]]]]→nand[x1,x1]`

contains a subpattern of the form:

`nand[x3_,nand[x1_,x2_]]`

which can be unified with the input for the rule:

`nand[x1_,nand[x2_,nand[x2_,nand[nand[x1_,x2_],x3_]]]]→nand[x1,x1]`

where these rules follow from Substitution Lemma 45 and Substitution Lemma 43 respectively.

Substitution Lemma 46

It can be shown that:

`nand[x1,x1]==nand[x1,nand[x2,nand[x1,nand[nand[x2,x1],x3]]]]`

PROOF

We start by taking Critical Pair Lemma 64, and apply the substitution:

`nand[x1_,nand[x1_,nand[x2_,x2_]]]→nand[x2,x1]`

which follows from Critical Pair Lemma 18.

Critical Pair Lemma 65

The following expressions are equivalent:

`nand[x1,nand[x2,nand[nand[x1,x2],x3]]]==nand[x1,nand[x2,x2]]`

PROOF

Note that the input for the rule:

`nand[x1_,nand[x2_,nand[x1_,nand[x2_,x3_]]]]→nand[x1,nand[x2,x3]]`

contains a subpattern of the form:

nand[x2_,nand[x1_,nand[x2_,x3_]]]

which can be unified with the input for the rule:

nand[x1_,nand[x2_,nand[x1_,nand[nand[x2_,x1_],x3_]]]]→nand[x1,x1]

where these rules follow from Substitution Lemma 35 and Substitution Lemma 46 respectively.

Critical Pair Lemma 66

The following expressions are equivalent:

nand[x1,nand[x2,x2]]==nand[x1,nand[nand[x3,nand[x1,x2]],x2]]

Proof

Note that the input for the rule:

nand[x1_,nand[x2_,nand[nand[x1_,x2_],x3_]]]→nand[x1,nand[x2,x2]]

contains a subpattern of the form:

nand[x2_,nand[nand[x1_,x2_],x3_]]

which can be unified with the input for the rule:

nand[x1_,nand[x2_,nand[nand[x3_,x2_],x1_]]]→nand[nand[x3,x2],x1]

where these rules follow from Critical Pair Lemma 65 and Substitution Lemma 33 respectively.

Substitution Lemma 47

It can be shown that:

nand[x1,nand[x2,x2]]==nand[x1,nand[x2,nand[x3,nand[x1,x2]]]]

Proof

We start by taking Critical Pair Lemma 66, and apply the substitution:

nand[x1_,x2_]→nand[x2,x1]

which follows from Substitution Lemma 3.

Critical Pair Lemma 67

The following expressions are equivalent:

x1==nand[nand[x1,nand[x1,nand[x2,x3]]],nand[nand[x3,x2],x1]]

Proof

Note that the input for the rule:

nand[nand[x1_,nand[x1_,x2_]],nand[nand[x3_,nand[x3_,x2_]],x1_]]→x1

contains a subpattern of the form:

nand[x3_,nand[x3_,x2_]]

which can be unified with the input for the rule:

nand[x1_,nand[x1_,nand[x1_,x2_]]]→nand[x2,x1]

where these rules follow from Critical Pair Lemma 55 and Critical Pair Lemma 20 respectively.

Substitution Lemma 48

It can be shown that:

x1==nand[nand[x1,x2],nand[nand[x3,nand[x3,nand[x4,x2]]],x1]]

Proof

We start by taking Critical Pair Lemma 61, and apply the substitution:

`nand[x1_,x2_]→nand[x2,x1]`

which follows from Substitution Lemma 3.

Critical Pair Lemma 68

The following expressions are equivalent:

`x1==nand[nand[x1,nand[x2,nand[x3,x4]]],nand[nand[x3,nand[x3,nand[x4,x4]]],x1]]`

Proof

Note that the input for the rule:

`nand[nand[x1_,x2_],nand[nand[x3_,nand[x3_,nand[x4_,x2_]]],x1_]]→x1`

contains a subpattern of the form:

`nand[x3_,nand[x4_,x2_]]`

which can be unified with the input for the rule:

`nand[x1_,nand[x2_,nand[x3_,nand[x1_,x2_]]]]→nand[x1,nand[x2,x2]]`

where these rules follow from Substitution Lemma 48 and Substitution Lemma 47 respectively.

Substitution Lemma 49

It can be shown that:

`x1==nand[nand[x1,nand[x2,nand[x3,x4]]],nand[nand[x4,x3],x1]]`

Proof

We start by taking Critical Pair Lemma 68, and apply the substitution:

`nand[x1_,nand[x1_,nand[x2_,x2_]]]→nand[x2,x1]`

which follows from Critical Pair Lemma 18.

Substitution Lemma 50

It can be shown that:

`nand[x1,nand[nand[nand[x2,x3],nand[x2,x3]],nand[x1,nand[x2,x3]]]]==nand[nand[nand[x1,n`

Proof

We start by taking Critical Pair Lemma 3, and apply the substitution:

`nand[nand[x1_,x1_],nand[x1_,x2_]]→x1`

which follows from Critical Pair Lemma 33.

Substitution Lemma 51

It can be shown that:

`nand[x1,nand[x2,x3]]==nand[nand[nand[x1,nand[x2,x3]],nand[x1,nand[x2,x3]]],nand[nand[n`

Proof

We start by taking Substitution Lemma 50, and apply the substitution:

`nand[x1_,nand[nand[x2_,x3_],nand[x1_,x3_]]]→nand[x1,x3]`

which follows from Substitution Lemma 30.

Substitution Lemma 52

It can be shown that:

`x1==nand[nand[nand[x2,x3],x1],nand[x1,nand[x1,nand[x3,x2]]]]`

Proof

We start by taking Critical Pair Lemma 67, and apply the substitution:

`nand[x1_,x2_]→nand[x2,x1]`

which follows from Substitution Lemma 3.

Critical Pair Lemma 69

The following expressions are equivalent:

`nand[x1,nand[x1,nand[nand[x2,x3],nand[x3,x2]]]]==nand[x1,nand[x3,x2]]`

Proof

Note that the input for the rule:

`nand[x1_,nand[x1_,x2_]]↔nand[x1_,nand[x2_,nand[x3_,nand[x3_,x3_]]]]`

contains a subpattern of the form:

`nand[x2_,nand[x3_,nand[x3_,x3_]]]`

which can be unified with the input for the rule:

`nand[nand[nand[x1_,x2_],x3_],nand[x3_,nand[x3_,nand[x2_,x1_]]]]→x3`

where these rules follow from Critical Pair Lemma 23 and Substitution Lemma 52 respectively.

Critical Pair Lemma 70

The following expressions are equivalent:

`nand[x1,nand[x2,x3]]==nand[x1,nand[x1,nand[nand[x3,x2],nand[x3,x2]]]]`

Proof

Note that the input for the rule:

`nand[x1_,nand[x1_,nand[nand[x2_,x3_],nand[x3_,x2_]]]]→nand[x1,nand[x3,x2]]`

contains a subpattern of the form:

`nand[x3_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,x2_]↔nand[x2_,x1_]`

where these rules follow from Critical Pair Lemma 69 and Substitution Lemma 3 respectively.

Substitution Lemma 53

It can be shown that:

`nand[x1,nand[x2,x3]]==nand[nand[x3,x2],x1]`

Proof

We start by taking Critical Pair Lemma 70, and apply the substitution:

`nand[x1_,nand[x1_,nand[x2_,x2_]]]→nand[x2,x1]`

which follows from Critical Pair Lemma 18.

Critical Pair Lemma 71

Critical Pair Lemma 71

The following expressions are equivalent:

`nand[nand[x1,nand[x2,x2]],x3]==nand[x3,nand[x1,nand[x1,x2]]]`

PROOF

Note that the input for the rule:

`nand[x1_,nand[x2_,x3_]]↔nand[nand[x3_,x2_],x1_]`

contains a subpattern of the form:

`nand[x2_,x3_]`

which can be unified with the input for the rule:

`nand[nand[x1_,x1_],x2_]↔nand[x2_,nand[x2_,x1_]]`

where these rules follow from Substitution Lemma 53 and Critical Pair Lemma 17 respectively.

Critical Pair Lemma 72

The following expressions are equivalent:

`nand[x1,nand[nand[x2,x3],x4]]==nand[nand[nand[x3,x2],x4],x1]`

PROOF

Note that the input for the rule:

`nand[x1_,nand[x2_,x3_]]↔nand[nand[x3_,x2_],x1_]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,nand[x2_,x3_]]↔nand[nand[x3_,x2_],x1_]`

where these rules follow from Substitution Lemma 53 and Substitution Lemma 53 respectively.

Substitution Lemma 54

It can be shown that:

`x1==nand[nand[nand[x2,x3],x1],nand[nand[x4,nand[x3,x2]],x1]]`

PROOF

We start by taking Substitution Lemma 49, and apply the substitution:

`nand[nand[x1_,x2_],x3_]→nand[x3,nand[x2,x1]]`

which follows from Substitution Lemma 53.

Critical Pair Lemma 73

The following expressions are equivalent:

`nand[nand[nand[x1,x2],x3],x4]==nand[x4,nand[x3,nand[x2,x1]]]`

PROOF

Note that the input for the rule:

`nand[x1_,nand[nand[x2_,x3_],x4_]]↔nand[nand[nand[x3_,x2_],x4_],x1_]`

contains a subpattern of the form:

`nand[nand[x2_,x3_],x4_]`

nand[nand[x2_,x3_],x4_]

which can be unified with the input for the rule:

`nand[x1_,x2_]↔nand[x2_,x1_]`

where these rules follow from Critical Pair Lemma 72 and Substitution Lemma 3 respectively.

Critical Pair Lemma 74

The following expressions are equivalent:

`nand[x1,nand[nand[x2,x3],x4]]==nand[nand[x4,nand[x3,x2]],x1]`

Proof

Note that the input for the rule:

`nand[x1_,nand[nand[x2_,x3_],x4_]]↔nand[nand[nand[x3_,x2_],x4_],x1_]`

contains a subpattern of the form:

`nand[nand[x1_,x2_],x3_]`

which can be unified with the input for the rule:

`nand[x1_,x2_]↔nand[x2_,x1_]`

where these rules follow from Critical Pair Lemma 72 and Substitution Lemma 3 respectively.

Substitution Lemma 55

It can be shown that:

`nand[nand[x1,x2],nand[x1,x2]]==nand[nand[x1,x2],nand[x2,nand[x2,nand[x1,x3]]]]`

Proof

We start by taking Critical Pair Lemma 43, and apply the substitution:

`nand[nand[nand[x1_,x2_],x3_],x4_]→nand[x4,nand[x3,nand[x2,x1]]]`

which follows from Critical Pair Lemma 73.

Substitution Lemma 56

It can be shown that:

`nand[nand[x1,x2],nand[x1,x2]]==nand[nand[x1,x2],nand[x1,nand[x1,nand[x2,x3]]]]`

Proof

We start by taking Critical Pair Lemma 48, and apply the substitution:

`nand[nand[nand[x1_,x2_],x3_],x4_]→nand[x4,nand[x3,nand[x2,x1]]]`

which follows from Critical Pair Lemma 73.

Substitution Lemma 57

It can be shown that:

`nand[nand[x1,nand[nand[x2,x2],x3]],x3]==nand[nand[nand[x3,nand[x2,x1]],nand[x3,nand[x2`

Proof

We start by taking Substitution Lemma 1, and apply the substitution:

`nand[nand[x1_,x1_],nand[x2_,x1_]]→x1`

which follows from Critical Pair Lemma 28.

Substitution Lemma 58

It can be shown that:

It can be shown that:

```
nand[x1,nand[nand[x1,nand[x2,x2]],x3]]==nand[nand[nand[x1,nand[x2,x3]],nand[x1,nand[x2
```

PROOF

We start by taking Substitution Lemma 57, and apply the substitution:

```
nand[nand[x1_,nand[x2_,x3_]],x4_]→nand[x4,nand[nand[x3,x2],x1]]
```

which follows from Critical Pair Lemma 74.

Substitution Lemma 59

It can be shown that:

```
nand[x1,nand[nand[x1,nand[x2,x2]],x3]]==nand[x1,nand[x2,x3]]
```

PROOF

We start by taking Substitution Lemma 58, and apply the substitution:

```
nand[nand[nand[x1_,nand[x2_,x3_]],nand[x1_,nand[x2_,x3_]]],nand[nand[nand[x2_,x2_],x1
```

which follows from Substitution Lemma 51.

Critical Pair Lemma 75

The following expressions are equivalent:

```
nand[x1,nand[x2,x3]]==nand[x1,nand[x3,nand[x1,nand[x1,x2]]]]
```

PROOF

Note that the input for the rule:

```
nand[x1_,nand[nand[x1_,nand[x2_,x2_]],x3_]]→nand[x1,nand[x2,x3]]
```

contains a subpattern of the form:

```
nand[nand[x1_,nand[x2_,x2_]],x3_]
```

which can be unified with the input for the rule:

```
nand[nand[x1_,nand[x2_,x2_]],x3_]↔nand[x3_,nand[x1_,nand[x1_,x2_]]]
```

where these rules follow from Substitution Lemma 59 and Critical Pair Lemma 71 respectively.

Critical Pair Lemma 76

The following expressions are equivalent:

```
nand[x1,nand[nand[nand[x1,x2],nand[x2,x3]],x4]]==nand[x1,nand[x4,nand[x1,nand[x1,x2]]]]
```

PROOF

Note that the input for the rule:

```
nand[x1_,nand[x2_,nand[x1_,nand[x1_,x3_]]]]→nand[x1,nand[x3,x2]]
```

contains a subpattern of the form:

```
nand[x1_,x3_]
```

which can be unified with the input for the rule:

```
nand[x1_,nand[nand[x1_,x2_],nand[x2_,x3_]]]→nand[x1,x2]
```

where these rules follow from Critical Pair Lemma 75 and Substitution Lemma 31 respectively.

Substitution Lemma 60

It can be shown that:

`nand[x1,nand[nand[nand[x1,x2],nand[x2,x3]],x4]]==nand[x1,nand[x2,x4]]`

PROOF

We start by taking Critical Pair Lemma 76, and apply the substitution:

`nand[x1_,nand[x2_,nand[x1_,nand[x1_,x3_]]]]→nand[x1,nand[x3,x2]]`

which follows from Critical Pair Lemma 75.

Critical Pair Lemma 77

The following expressions are equivalent:

`nand[x1,nand[x2,x3]]==nand[x1,nand[x3,nand[x3,nand[x1,x2]]]]`

PROOF

Note that the input for the rule:

`nand[x1_,nand[nand[nand[x1_,x2_],nand[x2_,x3_]],x4_]]→nand[x1,nand[x2,x4]]`

contains a subpattern of the form:

`nand[nand[nand[x1_,x2_],nand[x2_,x3_]],x4_]`

which can be unified with the input for the rule:

`nand[x1_,nand[x1_,x2_]]↔nand[nand[x2_,nand[x3_,nand[x3_,x3_]]],x1_]`

where these rules follow from Substitution Lemma 60 and Critical Pair Lemma 22 respectively.

Critical Pair Lemma 78

The following expressions are equivalent:

`nand[nand[x1,nand[x1,nand[x2,x3]]],nand[x2,x2]]==nand[nand[x1,nand[x1,nand[x2,x3]]],na`

PROOF

Note that the input for the rule:

`nand[x1_,nand[x2_,x1_]]↔nand[x1_,nand[x2_,x2_]]`

contains a subpattern of the form:

`nand[x2_,x1_]`

which can be unified with the input for the rule:

`nand[x1_,nand[x2_,nand[x2_,nand[x1_,x3_]]]]→nand[x1,nand[x3,x2]]`

where these rules follow from Critical Pair Lemma 12 and Critical Pair Lemma 77 respectively.

Substitution Lemma 61

It can be shown that:

`x1==nand[nand[x2,nand[x2,nand[x1,x3]]],nand[x1,nand[x3,x2]]]`

PROOF

We start by taking Critical Pair Lemma 78, and apply the substitution:

`nand[nand[x1_,nand[x1_,nand[x2_,x3_]]],nand[x2_,x2_]]→x2`

which follows from Substitution Lemma 42.

Substitution Lemma 62

It can be shown that:

```
x1==nand[nand[x1,nand[x2,x3]],nand[nand[nand[x1,x2],x3],x3]]
```

Proof

We start by taking Substitution Lemma 61, and apply the substitution:

```
nand[nand[x1_,nand[x2_,x3_]],x4_]→nand[x4,nand[nand[x3,x2],x1]]
```

which follows from Critical Pair Lemma 74.

Substitution Lemma 63

It can be shown that:

```
x1==nand[nand[x1,nand[x2,x3]],nand[x3,nand[x3,nand[x2,x1]]]]
```

Proof

We start by taking Substitution Lemma 62, and apply the substitution:

```
nand[nand[nand[x1_,x2_],x3_],x4_]→nand[x4,nand[x3,nand[x2,x1]]]
```

which follows from Critical Pair Lemma 73.

Critical Pair Lemma 79

The following expressions are equivalent:

```
nand[x1,nand[x1,nand[x2,x3]]]==nand[nand[nand[x2,x1],nand[x1,nand[x1,nand[x2,x3]]]],x3
```

Proof

Note that the input for the rule:

```
nand[nand[x1_,x2_],nand[nand[x3_,x1_],x2_]]→x2
```

contains a subpattern of the form:

```
nand[nand[x3_,x1_],x2_]
```

which can be unified with the input for the rule:

```
nand[nand[x1_,nand[x2_,x3_]],nand[x3_,nand[x3_,nand[x2_,x1_]]]]→x1
```

where these rules follow from Substitution Lemma 26 and Substitution Lemma 63 respectively.

Substitution Lemma 64

It can be shown that:

```
nand[x1,nand[x1,nand[x2,x3]]]==nand[nand[nand[x2,x1],nand[x2,x1]],x3]
```

Proof

We start by taking Critical Pair Lemma 79, and apply the substitution:

```
nand[nand[x1_,x2_],nand[x2_,nand[x2_,nand[x1_,x3_]]]]→nand[nand[x1,x2],nand[x1,x2]]
```

which follows from Substitution Lemma 55.

Critical Pair Lemma 80

The following expressions are equivalent:

```
nand[x1,nand[x1,nand[x2,x3]]]==nand[nand[nand[x1,x2],nand[x1,nand[x1,nand[x2,x3]]]],x3
```

Proof

Note that the input for the rule:

```
nand[nand[nand[x1_,x2_],x3_],nand[nand[x4_,nand[x2_,x1_]],x3_]]→x3
```

contains a subpattern of the form:

`nand[nand[x4_,nand[x2_,x1_]],x3_]`

which can be unified with the input for the rule:

`nand[nand[x1_,nand[x2_,x3_]],nand[x3_,nand[x3_,nand[x2_,x1_]]]]→x1`

where these rules follow from Substitution Lemma 54 and Substitution Lemma 63 respectively.

Substitution Lemma 65

It can be shown that:

`nand[x1,nand[x1,nand[x2,x3]]]==nand[nand[nand[x1,x2],nand[x1,x2]],x3]`

Proof

We start by taking Critical Pair Lemma 80, and apply the substitution:

`nand[nand[x1_,x2_],nand[x1_,nand[x1_,nand[x2_,x3_]]]]→nand[nand[x1,x2],nand[x1,x2]]`

which follows from Substitution Lemma 56.

Substitution Lemma 66

It can be shown that:

`nand[x1,nand[x1,nand[x2,x3]]]==nand[x2,nand[x2,nand[x1,x3]]]`

Proof

We start by taking Substitution Lemma 65, and apply the substitution:

`nand[nand[nand[x1_,x2_],nand[x1_,x2_]],x3_]→nand[x2,nand[x2,nand[x1,x3]]]`

which follows from Substitution Lemma 64.

Conclusion 3

We obtain the conclusion:

`True`

Proof

Take Hypothesis 3, and apply the substitution:

`nand[x1_,nand[x1_,nand[x2_,x3_]]]→nand[x2,nand[x2,nand[x1,x3]]]`

which follows from Substitution Lemma 66.

Substitution Lemma 67

It can be shown that:

`nand[b,nand[b,nand[c,a]]]==nand[b,nand[b,nand[a,c]]]`

Proof

We start by taking Conclusion 3, and apply the substitution:

`nand[x1_,x2_]→nand[x2,x1]`

which follows from Substitution Lemma 3.

Substitution Lemma 68

It can be shown that:

`nand[c,nand[c,nand[b,a]]]==nand[b,nand[b,nand[a,c]]]`

PROOF

We start by taking Substitution Lemma 67, and apply the substitution:

`nand[x1_,nand[x1_,nand[x2_,x3_]]]→nand[x2,nand[x2,nand[x1,x3]]]`

which follows from Substitution Lemma 66.

Substitution Lemma 69

It can be shown that:

`nand[c,nand[c,nand[b,a]]]==nand[b,nand[b,nand[c,a]]]`

PROOF

We start by taking Substitution Lemma 68, and apply the substitution:

`nand[x1_,x2_]→nand[x2,x1]`

which follows from Substitution Lemma 3.

Conclusion 4

We obtain the conclusion:

`True`

PROOF

Take Substitution Lemma 69, and apply the substitution:

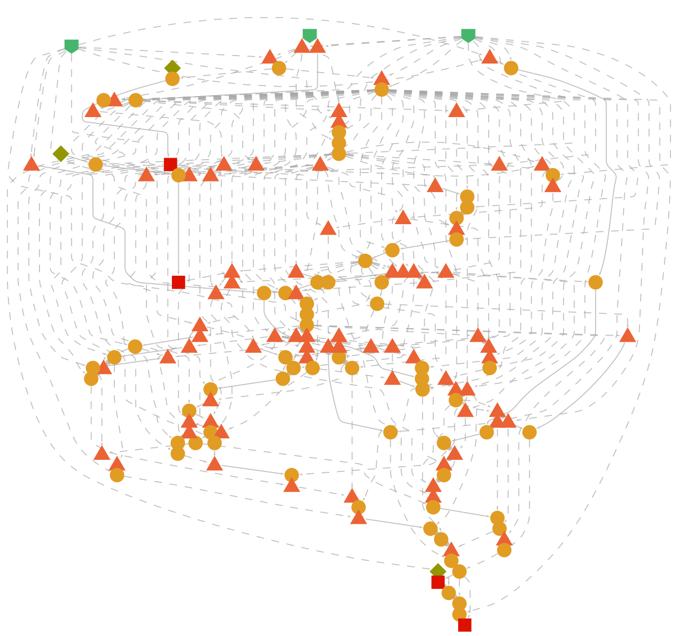
`nand[x1_,nand[x1_,nand[x2_,x3_]]]→nand[x2,nand[x2,nand[x1,x3]]]`

which follows from Substitution Lemma 66.

large output | show less | show more | show all | set size limit...

In[ ]:= `proofShortfromSheffer["ProofGraph"]`

Out[ ]=



In[ ]:=
```
Clear[proofShortfromSheffer]
```

Appendix 10. Derivation of Sheffer logic from equational Boolean logic

In[◦]:= proofShefferfromBoolean["ProofNotebook"]

Axiom 1

We are given that:

`x1==and[x1,or[x2,not[x2]]]`

Axiom 2

We are given that:

`x1==or[x1,and[x2,not[x2]]]`

Axiom 3

We are given that:

`and[x1,x2]==and[x2,x1]`

Axiom 4

We are given that:

`and[x1,or[x2,x3]]==or[and[x1,x2],and[x1,x3]]`

Axiom 5

We are given that:

`and[x1,not[x1]]==0`

Axiom 6

We are given that:

`and[or[x1,x2],or[x1,x3]]==or[x1,and[x2,x3]]`

Axiom 7

We are given that:

`or[x1,x2]==or[x2,x1]`

Axiom 8

We are given that:

`or[x1,not[x1]]==1`

Axiom 9

We are given that:

`not[and[x1,x2]]==nand[x1,x2]`

Hypothesis 1

We would like to show that:

`nand[nand[nand[b,b],a],nand[nand[c,c],a]]==nand[nand[a,nand[b,c]],nand[a,nand[b,c]]]`

Hypothesis 2

We would like to show that:

`nand[nand[a,a],nand[a,a]]==a`

Hypothesis 3

We would like to show that:

`nand[a,nand[b,nand[b,b]]]==nand[a,a]`

Critical Pair Lemma 1

The following expressions are equivalent:

`and[x1,or[x2,x3]]==or[and[x2,x1],and[x1,x3]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],and[x1_,x3_]]→and[x1,or[x2,x3]]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Axiom 4 and Axiom 3 respectively.

Critical Pair Lemma 2

The following expressions are equivalent:

`and[x1,or[x2,x3]]==or[and[x1,x2],and[x3,x1]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],and[x1_,x3_]]→and[x1,or[x2,x3]]`

contains a subpattern of the form:

`and[x1_,x3_]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Axiom 4 and Axiom 3 respectively.

Substitution Lemma 1

It can be shown that:

`or[x1,0]==x1`

Proof

We start by taking Axiom 2, and apply the substitution:

`and[x1_,not[x1_]]→0`

which follows from Axiom 5.

Critical Pair Lemma 3

The following expressions are equivalent:

`or[x1,and[x2,x3]]==and[or[x2,x1],or[x1,x3]]`

Proof

PROOF

Note that the input for the rule:

`and[or[x1_,x2_],or[x1_,x3_]]→or[x1,and[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Axiom 6 and Axiom 7 respectively.

Critical Pair Lemma 4

The following expressions are equivalent:

`or[x1,and[not[x1],x2]]==and[1,or[x1,x2]]`

Proof

Note that the input for the rule:

`and[or[x1_,x2_],or[x1_,x3_]]→or[x1,and[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,not[x1_]]→1`

where these rules follow from Axiom 6 and Axiom 8 respectively.

Substitution Lemma 2

It can be shown that:

`and[x1,1]==x1`

Proof

We start by taking Axiom 1, and apply the substitution:

`or[x1_,not[x1_]]→1`

which follows from Axiom 8.

Critical Pair Lemma 5

The following expressions are equivalent:

`nand[x1,not[x1]]==not[0]`

Proof

Note that the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,not[x1_]]→0`

where these rules follow from Axiom 9 and Axiom 5 respectively.

Critical Pair Lemma 6

The following expressions are equivalent:

`nand[x1,x2]==not[and[x2,x1]]`

Proof

Note that the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Axiom 9 and Axiom 3 respectively.

Substitution Lemma 3

It can be shown that:

`nand[x1,x2]==nand[x2,x1]`

Proof

We start by taking Critical Pair Lemma 6, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Axiom 9.

Critical Pair Lemma 7

The following expressions are equivalent:

`1==or[and[x1,x2],nand[x1,x2]]`

Proof

Note that the input for the rule:

`or[x1_,not[x1_]]→1`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

where these rules follow from Axiom 8 and Axiom 9 respectively.

Critical Pair Lemma 8

The following expressions are equivalent:

`x1==or[0,x1]`

Proof

Note that the input for the rule:

`or[x1_,0]→x1`

contains a subpattern of the form:

`or[x1_,0]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Substitution Lemma 1 and Axiom 7 respectively.

where these rules follow from Substitution Lemma 1 and Axiom 7 respectively.

Critical Pair Lemma 9

The following expressions are equivalent:

`or[x1,and[0,x2]]==and[x1,or[x1,x2]]`

Proof

Note that the input for the rule:

`and[or[x1_,x2_],or[x1_,x3_]]→or[x1,and[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,0]→x1`

where these rules follow from Axiom 6 and Substitution Lemma 1 respectively.

Critical Pair Lemma 10

The following expressions are equivalent:

`x1==and[1,x1]`

Proof

Note that the input for the rule:

`and[x1_,1]→x1`

contains a subpattern of the form:

`and[x1_,1]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Substitution Lemma 2 and Axiom 3 respectively.

Critical Pair Lemma 11

The following expressions are equivalent:

`nand[x1,1]==not[x1]`

Proof

Note that the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,1]→x1`

where these rules follow from Axiom 9 and Substitution Lemma 2 respectively.

Critical Pair Lemma 12

The following expressions are equivalent:

`not[0]==1`

Proof

Note that the input for the rule:

`or[0,x1_]→x1`

contains a subpattern of the form:

`or[0,x1_]`

which can be unified with the input for the rule:

`or[x1_,not[x1_]]→1`

where these rules follow from Critical Pair Lemma 8 and Axiom 8 respectively.

Critical Pair Lemma 13

The following expressions are equivalent:

`not[1]==0`

Proof

Note that the input for the rule:

`and[1,x1_]→x1`

contains a subpattern of the form:

`and[1,x1_]`

which can be unified with the input for the rule:

`and[x1_,not[x1_]]→0`

where these rules follow from Critical Pair Lemma 10 and Axiom 5 respectively.

Critical Pair Lemma 14

The following expressions are equivalent:

`nand[1,x1]==not[x1]`

Proof

Note that the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[1,x1_]→x1`

where these rules follow from Axiom 9 and Critical Pair Lemma 10 respectively.

Substitution Lemma 4

It can be shown that:

`nand[x1,not[x1]]==1`

Proof

We start by taking Critical Pair Lemma 5, and apply the substitution:

`not[0]→1`

which follows from Critical Pair Lemma 12.

Critical Pair Lemma 15

The following expressions are equivalent:

`or[x1,and[0,0]]==and[x1,x1]`

Proof

Note that the input for the rule:

`and[x1_,or[x1_,x2_]]→or[x1,and[0,x2]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,0]→x1`

where these rules follow from Critical Pair Lemma 9 and Substitution Lemma 1 respectively.

Critical Pair Lemma 16

The following expressions are equivalent:

`or[x1,and[0,x2]]==and[x1,or[x2,x1]]`

Proof

Note that the input for the rule:

`and[x1_,or[x1_,x2_]]→or[x1,and[0,x2]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Critical Pair Lemma 9 and Axiom 7 respectively.

Critical Pair Lemma 17

The following expressions are equivalent:

`or[not[x1],and[0,x1]]==and[not[x1],1]`

Proof

Note that the input for the rule:

`and[x1_,or[x2_,x1_]]→or[x1,and[0,x2]]`

contains a subpattern of the form:

`or[x2_,x1_]`

which can be unified with the input for the rule:

`or[x1_,not[x1_]]→1`

where these rules follow from Critical Pair Lemma 16 and Axiom 8 respectively.

Substitution Lemma 5

It can be shown that:

`or[not[x1],and[0,x1]]==not[x1]`

Proof

We start by taking Critical Pair Lemma 17, and apply the substitution:

`and[x1_,1]→x1`

which follows from Substitution Lemma 2.

Critical Pair Lemma 18

The following expressions are equivalent:

`or[and[0,x1],and[0,not[x1]]]==and[and[0,x1],not[x1]]`

Proof

Note that the input for the rule:

`and[x1_,or[x2_,x1_]]→or[x1,and[0,x2]]`

contains a subpattern of the form:

`or[x2_,x1_]`

which can be unified with the input for the rule:

`or[not[x1_],and[0,x1_]]→not[x1]`

where these rules follow from Critical Pair Lemma 16 and Substitution Lemma 5 respectively.

Substitution Lemma 6

It can be shown that:

`and[0,or[x1,not[x1]]]==and[and[0,x1],not[x1]]`

Proof

We start by taking Critical Pair Lemma 18, and apply the substitution:

`or[and[x1_,x2_],and[x1_,x3_]]→and[x1,or[x2,x3]]`

which follows from Axiom 4.

Substitution Lemma 7

It can be shown that:

`and[0,1]==and[and[0,x1],not[x1]]`

Proof

We start by taking Substitution Lemma 6, and apply the substitution:

`or[x1_,not[x1_]]→1`

which follows from Axiom 8.

Substitution Lemma 8

It can be shown that:

`0==and[and[0,x1],not[x1]]`

Proof

We start by taking Substitution Lemma 7, and apply the substitution:

`and[x1_,1]→x1`

which follows from Substitution Lemma 2.

Substitution Lemma 9

It can be shown that:

`0==and[not[x1],and[0,x1]]`

Proof

We start by taking Substitution Lemma 8, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Axiom 3.

Critical Pair Lemma 19

The following expressions are equivalent:

`0⩵and[0,and[0,1]]`

Proof

Note that the input for the rule:

`and[not[x1_],and[0,x1_]]→0`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[1]→0`

where these rules follow from Substitution Lemma 9 and Critical Pair Lemma 13 respectively.

Substitution Lemma 10

It can be shown that:

`0⩵and[0,0]`

Proof

We start by taking Critical Pair Lemma 19, and apply the substitution:

`and[x1_,1]→x1`

which follows from Substitution Lemma 2.

Substitution Lemma 11

It can be shown that:

`1⩵or[nand[x1,x2],and[x1,x2]]`

Proof

We start by taking Critical Pair Lemma 7, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 7.

Critical Pair Lemma 20

The following expressions are equivalent:

`1⩵or[nand[x1,x2],and[x2,x1]]`

Proof

Note that the input for the rule:

`or[nand[x1_,x2_],and[x1_,x2_]]→1`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,x2_]↔nand[x2_,x1_]`

where these rules follow from Substitution Lemma 11 and Substitution Lemma 3 respectively.

where these rules follow from Substitution Lemma 11 and Substitution Lemma 3 respectively.

Substitution Lemma 12

It can be shown that:

`or[x1,0]==and[x1,x1]`

Proof

We start by taking Critical Pair Lemma 15, and apply the substitution:

`and[0,0]→0`

which follows from Substitution Lemma 10.

Substitution Lemma 13

It can be shown that:

`x1==and[x1,x1]`

Proof

We start by taking Substitution Lemma 12, and apply the substitution:

`or[x1_,0]→x1`

which follows from Substitution Lemma 1.

Critical Pair Lemma 21

The following expressions are equivalent:

`and[x1,or[x2,x1]]==or[and[x1,x2],x1]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],and[x1_,x3_]]→and[x1,or[x2,x3]]`

contains a subpattern of the form:

`and[x1_,x3_]`

which can be unified with the input for the rule:

`and[x1_,x1_]→x1`

where these rules follow from Axiom 4 and Substitution Lemma 13 respectively.

Substitution Lemma 14

It can be shown that:

`or[x1,and[0,x2]]==or[and[x1,x2],x1]`

Proof

We start by taking Critical Pair Lemma 21, and apply the substitution:

`and[x1_,or[x2_,x1_]]→or[x1,and[0,x2]]`

which follows from Critical Pair Lemma 16.

Critical Pair Lemma 22

The following expressions are equivalent:

`nand[x1,x1]==not[x1]`

Proof

Note that the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,x1_]→x1`

where these rules follow from Axiom 9 and Substitution Lemma 13 respectively.

Substitution Lemma 15

It can be shown that:

`nand[nand[b,nand[b,b]],a]==nand[a,a]`

Proof

We start by taking Hypothesis 3, and apply the substitution:

`nand[x1_,x2_]→nand[x2,x1]`

which follows from Substitution Lemma 3.

Substitution Lemma 16

It can be shown that:

`nand[nand[b,nand[b,b]],a]==not[a]`

Proof

We start by taking Substitution Lemma 15, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Critical Pair Lemma 22.

Substitution Lemma 17

It can be shown that:

`nand[nand[b,not[b]],a]==not[a]`

Proof

We start by taking Substitution Lemma 16, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Critical Pair Lemma 22.

Substitution Lemma 18

It can be shown that:

`nand[1,a]==not[a]`

Proof

We start by taking Substitution Lemma 17, and apply the substitution:

`nand[x1_,not[x1_]]→1`

which follows from Substitution Lemma 4.

Conclusion 1

We obtain the conclusion:

`True`

PROOF

Take Substitution Lemma 18, and apply the substitution:

`nand[1,x1_]→not[x1]`

which follows from Critical Pair Lemma 14.

Substitution Lemma 19

It can be shown that:

`or[x1,and[0,x2]]==or[x1,and[x1,x2]]`

PROOF

We start by taking Substitution Lemma 14, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 7.

Critical Pair Lemma 23

The following expressions are equivalent:

`or[x1,and[x1,not[0]]]==or[x1,0]`

PROOF

Note that the input for the rule:

`or[x1_,and[0,x2_]]↔or[x1_,and[x1_,x2_]]`

contains a subpattern of the form:

`and[0,x2_]`

which can be unified with the input for the rule:

`and[x1_,not[x1_]]→0`

where these rules follow from Substitution Lemma 19 and Axiom 5 respectively.

Substitution Lemma 20

It can be shown that:

`or[x1,and[x1,1]]==or[x1,0]`

PROOF

We start by taking Critical Pair Lemma 23, and apply the substitution:

`not[0]→1`

which follows from Critical Pair Lemma 12.

Substitution Lemma 21

It can be shown that:

`or[x1,x1]==or[x1,0]`

PROOF

We start by taking Substitution Lemma 20, and apply the substitution:

`and[x1_,1]→x1`

which follows from Substitution Lemma 2.

Substitution Lemma 22

It can be shown that:

`or[x1,x1]==x1`

Proof

We start by taking Substitution Lemma 21, and apply the substitution:

`or[x1_,0]→x1`

which follows from Substitution Lemma 1.

Critical Pair Lemma 24

The following expressions are equivalent:

`or[x1,and[x2,x1]]==and[or[x1,x2],x1]`

Proof

Note that the input for the rule:

`and[or[x1_,x2_],or[x1_,x3_]]→or[x1,and[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x3_]`

which can be unified with the input for the rule:

`or[x1_,x1_]→x1`

where these rules follow from Axiom 6 and Substitution Lemma 22 respectively.

Substitution Lemma 23

It can be shown that:

`or[x1,and[x2,x1]]==and[x1,or[x1,x2]]`

Proof

We start by taking Critical Pair Lemma 24, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Axiom 3.

Substitution Lemma 24

It can be shown that:

`or[x1,and[x2,x1]]==or[x1,and[0,x2]]`

Proof

We start by taking Substitution Lemma 23, and apply the substitution:

`and[x1_,or[x1_,x2_]]→or[x1,and[0,x2]]`

which follows from Critical Pair Lemma 9.

Critical Pair Lemma 25

The following expressions are equivalent:

`or[and[0,x1],and[0,not[x1]]]==or[and[0,x1],0]`

Proof

Note that the input for the rule:

`or[x1_,and[x2_,x1_]]↔or[x1_,and[0,x2_]]`

contains a subpattern of the form:

`and[x2_,x1_]`

which can be unified with the input for the rule:

`and[not[x1_],and[0,x1_]]→0`

where these rules follow from Substitution Lemma 24 and Substitution Lemma 9 respectively.

Substitution Lemma 25

It can be shown that:

`and[0,or[x1,not[x1]]]==or[and[0,x1],0]`

Proof

We start by taking Critical Pair Lemma 25, and apply the substitution:

`or[and[x1_,x2_],and[x1_,x3_]]→and[x1,or[x2,x3]]`

which follows from Axiom 4.

Substitution Lemma 26

It can be shown that:

`and[0,1]==or[and[0,x1],0]`

Proof

We start by taking Substitution Lemma 25, and apply the substitution:

`or[x1_,not[x1_]]→1`

which follows from Axiom 8.

Substitution Lemma 27

It can be shown that:

`0==or[and[0,x1],0]`

Proof

We start by taking Substitution Lemma 26, and apply the substitution:

`and[x1_,1]→x1`

which follows from Substitution Lemma 2.

Substitution Lemma 28

It can be shown that:

`0==and[0,x1]`

Proof

We start by taking Substitution Lemma 27, and apply the substitution:

`or[x1_,0]→x1`

which follows from Substitution Lemma 1.

Substitution Lemma 29

It can be shown that:

`and[x1_,or[x2_,x1_]]→or[x1,0]`

Proof

We start by taking Critical Pair Lemma 16, and apply the substitution:

```
and[0,x1_]→0
```

which follows from Substitution Lemma 28.

Substitution Lemma 30

It can be shown that:

```
and[x1_,or[x2_,x1_]]→x1
```

Proof

We start by taking Substitution Lemma 29, and apply the substitution:

```
or[x1_,0]→x1
```

which follows from Substitution Lemma 1.

Substitution Lemma 31

It can be shown that:

```
and[x1_,or[x1_,x2_]]→or[x1,0]
```

Proof

We start by taking Critical Pair Lemma 9, and apply the substitution:

```
and[0,x1_]→0
```

which follows from Substitution Lemma 28.

Substitution Lemma 32

It can be shown that:

```
and[x1_,or[x1_,x2_]]→x1
```

Proof

We start by taking Substitution Lemma 31, and apply the substitution:

```
or[x1_,0]→x1
```

which follows from Substitution Lemma 1.

Substitution Lemma 33

It can be shown that:

```
or[x1,and[x2,x1]]==or[x1,0]
```

Proof

We start by taking Substitution Lemma 24, and apply the substitution:

```
and[0,x1_]→0
```

which follows from Substitution Lemma 28.

Substitution Lemma 34

It can be shown that:

```
or[x1,and[x2,x1]]==x1
```

Proof

We start by taking Substitution Lemma 33, and apply the substitution:

We start by taking Substitution Lemma 33, and apply the substitution:

`or[x1_,0]→x1`

which follows from Substitution Lemma 1.

Substitution Lemma 35

It can be shown that:

`or[x1,0]==or[x1,and[x1,x2]]`

Proof

We start by taking Substitution Lemma 19, and apply the substitution:

`and[0,x1_]→0`

which follows from Substitution Lemma 28.

Substitution Lemma 36

It can be shown that:

`x1==or[x1,and[x1,x2]]`

Proof

We start by taking Substitution Lemma 35, and apply the substitution:

`or[x1_,0]→x1`

which follows from Substitution Lemma 1.

Critical Pair Lemma 26

The following expressions are equivalent:

`and[not[x1],or[x1,x2]]==or[0,and[not[x1],x2]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],and[x2_,x3_]]→and[x2,or[x1,x3]]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,not[x1_]]→0`

where these rules follow from Critical Pair Lemma 1 and Axiom 5 respectively.

Substitution Lemma 37

It can be shown that:

`and[not[x1],or[x1,x2]]==and[not[x1],x2]`

Proof

We start by taking Critical Pair Lemma 26, and apply the substitution:

`or[0,x1_]→x1`

which follows from Critical Pair Lemma 8.

Critical Pair Lemma 27

The following expressions are equivalent:

`and[x1,or[x2,not[x1]]]==or[and[x2,x1],0]` and[x1,or[x1,not[x1]]]==or[and[x1,x1],0]

PROOF

Note that the input for the rule:

`or[and[x1_,x2_],and[x2_,x3_]]→and[x2,or[x1,x3]]`

contains a subpattern of the form:

`and[x2_,x3_]`

which can be unified with the input for the rule:

`and[x1_,not[x1_]]→0`

where these rules follow from Critical Pair Lemma 1 and Axiom 5 respectively.

Substitution Lemma 38

It can be shown that:

`and[x1,or[x2,not[x1]]]==and[x2,x1]`

PROOF

We start by taking Critical Pair Lemma 27, and apply the substitution:

`or[x1_,0]→x1`

which follows from Substitution Lemma 1.

Critical Pair Lemma 28

The following expressions are equivalent:

`or[x1,x2]==or[or[x1,x2],x2]`

PROOF

Note that the input for the rule:

`or[x1_,and[x2_,x1_]]→x1`

contains a subpattern of the form:

`and[x2_,x1_]`

which can be unified with the input for the rule:

`Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[541,or[x1_,0]→x`

where these rules follow from Substitution Lemma 34 and Substitution Lemma 30 respectively.

Critical Pair Lemma 29

The following expressions are equivalent:

`and[x1,x2]==and[and[x1,x2],x2]`

PROOF

Note that the input for the rule:

`Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[541,or[x1_,0]→x`

contains a subpattern of the form:

`or[x2_,x1_]`

which can be unified with the input for the rule:

`or[x1_,and[x2_,x1_]]→x1`

where these rules follow from Substitution Lemma 30 and Substitution Lemma 34 respectively.

Critical Pair Lemma 30

Critical Pair Lemma 30

The following expressions are equivalent:

`or[x1,and[and[x2,x1],x3]]==and[x1,or[x1,x3]]`

Proof

Note that the input for the rule:

`and[or[x1_,x2_],or[x1_,x3_]]→or[x1,and[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,and[x2_,x1_]]→x1`

where these rules follow from Axiom 6 and Substitution Lemma 34 respectively.

Substitution Lemma 39

It can be shown that:

`or[x1,and[and[x2,x1],x3]]==x1`

Proof

We start by taking Critical Pair Lemma 30, and apply the substitution:

`and[x1_,or[x1_,x2_]]→x1`

which follows from Substitution Lemma 32.

Critical Pair Lemma 31

The following expressions are equivalent:

`and[x1,x2]==and[and[x1,x2],x1]`

Proof

Note that the input for the rule:

`` Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[541,or[x1_,0]→x ``

contains a subpattern of the form:

`or[x2_,x1_]`

which can be unified with the input for the rule:

`or[x1_,and[x1_,x2_]]→x1`

where these rules follow from Substitution Lemma 30 and Substitution Lemma 36 respectively.

Substitution Lemma 40

It can be shown that:

`or[x1,x2]==or[x2,or[x1,x2]]`

Proof

We start by taking Critical Pair Lemma 28, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 7.

Substitution Lemma 41

It can be shown that:

`and[x1,x2]==and[x2,and[x1,x2]]`

Proof

We start by taking Critical Pair Lemma 29, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Axiom 3.

Critical Pair Lemma 32

The following expressions are equivalent:

`and[x1,or[and[x2,x1],x3]]==or[and[x2,x1],and[x1,x3]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],and[x1_,x3_]]→and[x1,or[x2,x3]]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,and[x2_,x1_]]→and[x2,x1]`

where these rules follow from Axiom 4 and Substitution Lemma 41 respectively.

Substitution Lemma 42

It can be shown that:

`and[x1,or[and[x2,x1],x3]]==and[x1,or[x2,x3]]`

Proof

We start by taking Critical Pair Lemma 32, and apply the substitution:

`or[and[x1_,x2_],and[x2_,x3_]]→and[x2,or[x1,x3]]`

which follows from Critical Pair Lemma 1.

Critical Pair Lemma 33

The following expressions are equivalent:

`nand[x1,and[x2,x1]]==not[and[x2,x1]]`

Proof

Note that the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,and[x2_,x1_]]→and[x2,x1]`

where these rules follow from Axiom 9 and Substitution Lemma 41 respectively.

Substitution Lemma 43

It can be shown that:

`nand[x1,and[x2,x1]]==nand[x2,x1]`

Proof

We start by taking Critical Pair Lemma 33, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Axiom 9.

Substitution Lemma 44

It can be shown that:

`and[x1,x2]==and[x1,and[x1,x2]]`

Proof

We start by taking Critical Pair Lemma 31, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Axiom 3.

Critical Pair Lemma 34

The following expressions are equivalent:

`nand[x1,and[x1,x2]]==not[and[x1,x2]]`

Proof

Note that the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,and[x1_,x2_]]→and[x1,x2]`

where these rules follow from Axiom 9 and Substitution Lemma 44 respectively.

Substitution Lemma 45

It can be shown that:

`nand[x1,and[x1,x2]]==nand[x1,x2]`

Proof

We start by taking Critical Pair Lemma 34, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Axiom 9.

Critical Pair Lemma 35

The following expressions are equivalent:

`and[x1,x2]==and[x2,or[not[x2],x1]]`

Proof

Note that the input for the rule:

`and[x1_,or[x2_,not[x1_]]]→and[x2,x1]`

contains a subpattern of the form:

`or[x2_,not[x1_]]`

which can be unified with the input for the rule:

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Substitution Lemma 38 and Axiom 7 respectively.

Critical Pair Lemma 36

The following expressions are equivalent:

`nand[x1,or[x2,not[x1]]]==nand[x1,and[x2,x1]]`

Proof

Note that the input for the rule:

`nand[x1_,and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,or[x2_,not[x1_]]]→and[x2,x1]`

where these rules follow from Substitution Lemma 45 and Substitution Lemma 38 respectively.

Substitution Lemma 46

It can be shown that:

`nand[x1,or[x2,not[x1]]]==nand[x2,x1]`

Proof

We start by taking Critical Pair Lemma 36, and apply the substitution:

`nand[x1_,and[x2_,x1_]]→nand[x2,x1]`

which follows from Substitution Lemma 43.

Critical Pair Lemma 37

The following expressions are equivalent:

`and[not[not[x1]],x1]==and[x1,1]`

Proof

Note that the input for the rule:

`and[x1_,or[not[x1_],x2_]]→and[x2,x1]`

contains a subpattern of the form:

`or[not[x1_],x2_]`

which can be unified with the input for the rule:

`or[x1_,not[x1_]]→1`

where these rules follow from Critical Pair Lemma 35 and Axiom 8 respectively.

Substitution Lemma 47

It can be shown that:

`and[not[not[x1]],x1]==x1`

Proof

We start by taking Critical Pair Lemma 37, and apply the substitution:

`and[x1_,1]→x1`

which follows from Substitution Lemma 2.

Substitution Lemma 48

It can be shown that:

`and[x1,not[not[x1]]]==x1`

Proof

We start by taking Substitution Lemma 47, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Axiom 3.

Critical Pair Lemma 38

The following expressions are equivalent:

`and[not[x1],x2]==and[not[x1],or[x2,x1]]`

Proof

Note that the input for the rule:

`and[not[x1_],or[x1_,x2_]]→and[not[x1],x2]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Substitution Lemma 37 and Axiom 7 respectively.

Out[∘]=

Critical Pair Lemma 39

The following expressions are equivalent:

`nand[not[x1],or[x1,x2]]==nand[not[x1],and[not[x1],x2]]`

Proof

Note that the input for the rule:

`nand[x1_,and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[not[x1_],or[x1_,x2_]]→and[not[x1],x2]`

where these rules follow from Substitution Lemma 45 and Substitution Lemma 37 respectively.

Substitution Lemma 49

It can be shown that:

`nand[not[x1],or[x1,x2]]==nand[not[x1],x2]`

Proof

We start by taking Critical Pair Lemma 39, and apply the substitution:

`nand[x1_,and[x1_,x2_]]→nand[x1,x2]`

which follows from Substitution Lemma 45.

Critical Pair Lemma 40

Critical Pair Lemma 40

The following expressions are equivalent:

`and[not[not[x1]],x1]==and[not[not[x1]],1]`

PROOF

Note that the input for the rule:

`and[not[x1_],or[x2_,x1_]]→and[not[x1],x2]`

contains a subpattern of the form:

`or[x2_,x1_]`

which can be unified with the input for the rule:

`or[x1_,not[x1_]]→1`

where these rules follow from Critical Pair Lemma 38 and Axiom 8 respectively.

Substitution Lemma 50

It can be shown that:

`and[not[not[x1]],x1]==not[not[x1]]`

PROOF

We start by taking Critical Pair Lemma 40, and apply the substitution:

`and[x1_,1]→x1`

which follows from Substitution Lemma 2.

Critical Pair Lemma 41

The following expressions are equivalent:

`nand[not[x1],or[x2,x1]]==nand[not[x1],and[not[x1],x2]]`

PROOF

Note that the input for the rule:

`nand[x1_,and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[not[x1_],or[x2_,x1_]]→and[not[x1],x2]`

where these rules follow from Substitution Lemma 45 and Critical Pair Lemma 38 respectively.

Substitution Lemma 51

It can be shown that:

`nand[not[x1],or[x2,x1]]==nand[not[x1],x2]`

PROOF

We start by taking Critical Pair Lemma 41, and apply the substitution:

`nand[x1_,and[x1_,x2_]]→nand[x1,x2]`

which follows from Substitution Lemma 45.

Substitution Lemma 52

It can be shown that:

```
and[x1,not[not[x1]]]==not[not[x1]]
```

PROOF

We start by taking Substitution Lemma 50, and apply the substitution:

```
and[x1_,x2_]→and[x2,x1]
```

which follows from Axiom 3.

Substitution Lemma 53

It can be shown that:

```
x1==not[not[x1]]
```

PROOF

We start by taking Substitution Lemma 52, and apply the substitution:

```
and[x1_,not[not[x1_]]]→x1
```

which follows from Substitution Lemma 48.

Substitution Lemma 54

It can be shown that:

```
not[nand[a,a]]==a
```

PROOF

We start by taking Hypothesis 2, and apply the substitution:

```
nand[x1_,x1_]→not[x1]
```

which follows from Critical Pair Lemma 22.

Substitution Lemma 55

It can be shown that:

```
not[not[a]]==a
```

PROOF

We start by taking Substitution Lemma 54, and apply the substitution:

```
nand[x1_,x1_]→not[x1]
```

which follows from Critical Pair Lemma 22.

Conclusion 2

We obtain the conclusion:

```
True
```

PROOF

Take Substitution Lemma 55, and apply the substitution:

```
not[not[x1_]]→x1
```

which follows from Substitution Lemma 53.

Critical Pair Lemma 42

The following expressions are equivalent:

```
and[x1,x2]==not[nand[x1,x2]]
```

PROOF

PROOF

Note that the input for the rule:

`not[not[x1_]]→x1`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

where these rules follow from Substitution Lemma 53 and Axiom 9 respectively.

Critical Pair Lemma 43

The following expressions are equivalent:

`nand[not[and[x1,x2]],nand[x2,x1]]==nand[not[and[x1,x2]],1]`

PROOF

Note that the input for the rule:

`nand[not[x1_],or[x2_,x1_]]→nand[not[x1],x2]`

contains a subpattern of the form:

`or[x2_,x1_]`

which can be unified with the input for the rule:

`or[nand[x1_,x2_],and[x2_,x1_]]→1`

where these rules follow from Substitution Lemma 51 and Critical Pair Lemma 20 respectively.

Substitution Lemma 56

It can be shown that:

`nand[nand[x1,x2],nand[x2,x1]]==nand[not[and[x1,x2]],1]`

PROOF

We start by taking Critical Pair Lemma 43, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Axiom 9.

Substitution Lemma 57

It can be shown that:

`nand[nand[x1,x2],nand[x2,x1]]==not[not[and[x1,x2]]]`

PROOF

We start by taking Substitution Lemma 56, and apply the substitution:

`nand[x1_,1]→not[x1]`

which follows from Critical Pair Lemma 11.

Substitution Lemma 58

It can be shown that:

`nand[nand[x1,x2],nand[x2,x1]]==and[x1,x2]`

PROOF

We start by taking Substitution Lemma 57, and apply the substitution:

```
not[not[x1_]]→x1
```

which follows from Substitution Lemma 53.

Substitution Lemma 59

It can be shown that:

```
or[x1,and[not[x1],x2]]==or[x1,x2]
```

Proof

We start by taking Critical Pair Lemma 4, and apply the substitution:

```
and[1,x1_]→x1
```

which follows from Critical Pair Lemma 10.

Critical Pair Lemma 44

The following expressions are equivalent:

```
or[x1,or[not[x1],x2]]==or[x1,not[x1]]
```

Proof

Note that the input for the rule:

```
or[x1_,and[not[x1_],x2_]]→or[x1,x2]
```

contains a subpattern of the form:

```
and[not[x1_],x2_]
```

which can be unified with the input for the rule:

```
Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[543,or[x1_,0]→x
```

where these rules follow from Substitution Lemma 59 and Substitution Lemma 32 respectively.

Substitution Lemma 60

It can be shown that:

```
or[x1,or[not[x1],x2]]==1
```

Proof

We start by taking Critical Pair Lemma 44, and apply the substitution:

```
or[x1_,not[x1_]]→1
```

which follows from Axiom 8.

Critical Pair Lemma 45

The following expressions are equivalent:

```
or[x1,or[x2,not[x1]]]==or[x1,not[x1]]
```

Proof

Note that the input for the rule:

```
or[x1_,and[not[x1_],x2_]]→or[x1,x2]
```

contains a subpattern of the form:

```
and[not[x1_],x2_]
```

which can be unified with the input for the rule:

```
Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[541,or[x1_,0]→x
```

where these rules follow from Substitution Lemma 59 and Substitution Lemma 30 respectively.

Substitution Lemma 61

It can be shown that:

`or[x1,or[x2,not[x1]]]==1`

Proof

We start by taking Critical Pair Lemma 45, and apply the substitution:

`or[x1_,not[x1_]]→1`

which follows from Axiom 8.

Critical Pair Lemma 46

The following expressions are equivalent:

`1==or[not[x1],or[x1,x2]]`

Proof

Note that the input for the rule:

`or[x1_,or[not[x1_],x2_]]→1`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Substitution Lemma 60 and Substitution Lemma 53 respectively.

Critical Pair Lemma 47

The following expressions are equivalent:

`1==or[not[x1],or[x2,x1]]`

Proof

Note that the input for the rule:

`or[x1_,or[x2_,not[x1_]]]→1`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Substitution Lemma 61 and Substitution Lemma 53 respectively.

Critical Pair Lemma 48

The following expressions are equivalent:

`nand[not[or[x1,not[x2]]],x2]==nand[not[or[x1,not[x2]]],1]`

Proof

Note that the input for the rule:

`nand[not[x1_],or[x2_,x1_]]→nand[not[x1],x2]`

contains a subpattern of the form:

`or[x2_,x1_]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,not[x1_]]]→1`

where these rules follow from Substitution Lemma 51 and Substitution Lemma 61 respectively.

Substitution Lemma 62

It can be shown that:

`nand[not[or[x1,not[x2]]],x2]⩵not[not[or[x1,not[x2]]]]`

Proof

We start by taking Critical Pair Lemma 48, and apply the substitution:

`nand[x1_,1]→not[x1]`

which follows from Critical Pair Lemma 11.

Substitution Lemma 63

It can be shown that:

`nand[not[or[x1,not[x2]]],x2]⩵or[x1,not[x2]]`

Proof

We start by taking Substitution Lemma 62, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 53.

Critical Pair Lemma 49

The following expressions are equivalent:

`nand[not[or[x1,x2]],not[x1]]⩵nand[not[or[x1,x2]],1]`

Proof

Note that the input for the rule:

`nand[not[x1_],or[x2_,x1_]]→nand[not[x1],x2]`

contains a subpattern of the form:

`or[x2_,x1_]`

which can be unified with the input for the rule:

`or[not[x1_],or[x1_,x2_]]→1`

where these rules follow from Substitution Lemma 51 and Critical Pair Lemma 46 respectively.

Substitution Lemma 64

It can be shown that:

`nand[not[or[x1,x2]],not[x1]]⩵not[not[or[x1,x2]]]`

Proof

We start by taking Critical Pair Lemma 49, and apply the substitution:

`nand[x1_,1]→not[x1]`

which follows from Critical Pair Lemma 11.

Substitution Lemma 65

It can be shown that:

`nand[not[or[x1,x2]],not[x1]]==or[x1,x2]`

Proof

We start by taking Substitution Lemma 64, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 53.

Critical Pair Lemma 50

The following expressions are equivalent:

`1==or[not[and[and[x1,x2],x3]],x2]`

Proof

Note that the input for the rule:

`or[not[x1_],or[x2_,x1_]]→1`

contains a subpattern of the form:

`or[x2_,x1_]`

which can be unified with the input for the rule:

`or[x1_,and[and[x2_,x1_],x3_]]→x1`

where these rules follow from Critical Pair Lemma 47 and Substitution Lemma 39 respectively.

Substitution Lemma 66

It can be shown that:

`1==or[nand[and[x1,x2],x3],x2]`

Proof

We start by taking Critical Pair Lemma 50, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Axiom 9.

Critical Pair Lemma 51

The following expressions are equivalent:

`1==or[not[and[x1,x2]],x1]`

Proof

Note that the input for the rule:

`or[not[x1_],or[x2_,x1_]]→1`

contains a subpattern of the form:

`or[x2_,x1_]`

which can be unified with the input for the rule:

`or[x1_,and[x1_,x2_]]→x1`

where these rules follow from Critical Pair Lemma 47 and Substitution Lemma 36 respectively.

Substitution Lemma 67

It can be shown that:

`1==or[nand[x1,x2],x1]`

Proof

PROOF

We start by taking Critical Pair Lemma 51, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Axiom 9.

Substitution Lemma 68

It can be shown that:

`1==or[x1,nand[x1,x2]]`

PROOF

We start by taking Substitution Lemma 67, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 7.

Critical Pair Lemma 52

The following expressions are equivalent:

`and[nand[not[x1],x2],x1]==and[x1,1]`

PROOF

Note that the input for the rule:

`and[x1_,or[not[x1_],x2_]]→and[x2,x1]`

contains a subpattern of the form:

`or[not[x1_],x2_]`

which can be unified with the input for the rule:

`or[x1_,nand[x1_,x2_]]→1`

where these rules follow from Critical Pair Lemma 35 and Substitution Lemma 68 respectively.

Substitution Lemma 69

It can be shown that:

`and[nand[not[x1],x2],x1]==x1`

PROOF

We start by taking Critical Pair Lemma 52, and apply the substitution:

`and[x1_,1]→x1`

which follows from Substitution Lemma 2.

Substitution Lemma 70

It can be shown that:

`and[x1,nand[not[x1],x2]]==x1`

PROOF

We start by taking Substitution Lemma 69, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Axiom 3.

Critical Pair Lemma 53

The following expressions are equivalent:

The following expressions are equivalent:

`nand[not[x1],x2]==or[nand[not[x1],x2],x1]`

Proof

Note that the input for the rule:

`or[x1_,and[x2_,x1_]]→x1`

contains a subpattern of the form:

`and[x2_,x1_]`

which can be unified with the input for the rule:

`and[x1_,nand[not[x1_],x2_]]→x1`

where these rules follow from Substitution Lemma 34 and Substitution Lemma 70 respectively.

Substitution Lemma 71

It can be shown that:

`1==or[x1,nand[and[x2,x1],x3]]`

Proof

We start by taking Substitution Lemma 66, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 7.

Critical Pair Lemma 54

The following expressions are equivalent:

`1==or[or[x1,x2],nand[x2,x3]]`

Proof

Note that the input for the rule:

`or[x1_,nand[and[x2_,x1_],x3_]]→1`

contains a subpattern of the form:

`and[x2_,x1_]`

which can be unified with the input for the rule:

`Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[541,or[x1_,0]→x`

where these rules follow from Substitution Lemma 71 and Substitution Lemma 30 respectively.

Critical Pair Lemma 55

The following expressions are equivalent:

`nand[not[nand[x1,x2]],or[x3,x1]]==nand[not[nand[x1,x2]],1]`

Proof

Note that the input for the rule:

`nand[not[x1_],or[x2_,x1_]]→nand[not[x1],x2]`

contains a subpattern of the form:

`or[x2_,x1_]`

which can be unified with the input for the rule:

`or[or[x1_,x2_],nand[x2_,x3_]]→1`

where these rules follow from Substitution Lemma 51 and Critical Pair Lemma 54 respectively.

where these rules follow from Substitution Lemma 51 and Critical Pair Lemma 54 respectively.

Substitution Lemma 72

It can be shown that:

`nand[and[x1,x2],or[x3,x1]]==nand[not[nand[x1,x2]],1]`

Proof

We start by taking Critical Pair Lemma 55, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Critical Pair Lemma 42.

Substitution Lemma 73

It can be shown that:

`nand[and[x1,x2],or[x3,x1]]==not[not[nand[x1,x2]]]`

Proof

We start by taking Substitution Lemma 72, and apply the substitution:

`nand[x1_,1]→not[x1]`

which follows from Critical Pair Lemma 11.

Substitution Lemma 74

It can be shown that:

`nand[and[x1,x2],or[x3,x1]]==nand[x1,x2]`

Proof

We start by taking Substitution Lemma 73, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 53.

Substitution Lemma 75

It can be shown that:

`nand[not[x1],x2]==or[x1,nand[not[x1],x2]]`

Proof

We start by taking Critical Pair Lemma 53, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 7.

Substitution Lemma 76

It can be shown that:

`nand[not[x1],not[or[x1,x2]]]==or[x1,x2]`

Proof

We start by taking Substitution Lemma 65, and apply the substitution:

`nand[x1_,x2_]→nand[x2,x1]`

which follows from Substitution Lemma 3.

Critical Pair Lemma 56

The following expressions are equivalent:

`nand[x1,x2]==nand[or[x3,x1],and[x1,x2]]`

PROOF

Note that the input for the rule:

`nand[and[x1_,x2_],or[x3_,x1_]]→nand[x1,x2]`

contains a subpattern of the form:

`nand[and[x1_,x2_],or[x3_,x1_]]`

which can be unified with the input for the rule:

`nand[x1_,x2_]↔nand[x2_,x1_]`

where these rules follow from Substitution Lemma 74 and Substitution Lemma 3 respectively.

Substitution Lemma 77

It can be shown that:

`nand[x1,not[or[x2,not[x1]]]]==or[x2,not[x1]]`

PROOF

We start by taking Substitution Lemma 63, and apply the substitution:

`nand[x1_,x2_]→nand[x2,x1]`

which follows from Substitution Lemma 3.

Critical Pair Lemma 57

The following expressions are equivalent:

`and[x1,or[x2,and[x3,x2]]]==and[x1,and[x2,or[x1,x3]]]`

PROOF

Note that the input for the rule:

`and[x1_,or[and[x2_,x1_],x3_]]→and[x1,or[x2,x3]]`

contains a subpattern of the form:

`or[and[x2_,x1_],x3_]`

which can be unified with the input for the rule:

`or[and[x1_,x2_],and[x3_,x1_]]→and[x1,or[x2,x3]]`

where these rules follow from Substitution Lemma 42 and Critical Pair Lemma 2 respectively.

Substitution Lemma 78

It can be shown that:

`and[x1,x2]==and[x1,and[x2,or[x1,x3]]]`

PROOF

We start by taking Critical Pair Lemma 57, and apply the substitution:

`or[x1_,and[x2_,x1_]]→x1`

which follows from Substitution Lemma 34.

Critical Pair Lemma 58

The following expressions are equivalent:

`and[x1,x2]==and[x1,and[or[x1,x3],x2]]`

PROOF

Note that the input for the rule:

`and[x1_,and[x2_,or[x1_,x3_]]]→and[x1,x2]`

contains a subpattern of the form:

`and[x2_,or[x1_,x3_]]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Substitution Lemma 78 and Axiom 3 respectively.

Critical Pair Lemma 59

The following expressions are equivalent:

`and[x1,x2]==and[x1,and[x2,or[x3,x1]]]`

PROOF

Note that the input for the rule:

`and[x1_,and[x2_,or[x1_,x3_]]]→and[x1,x2]`

contains a subpattern of the form:

`or[x1_,x3_]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,x1_]]→or[x2,x1]`

where these rules follow from Substitution Lemma 78 and Substitution Lemma 40 respectively.

Critical Pair Lemma 60

The following expressions are equivalent:

`and[x1,x2]==and[x1,and[or[x3,x1],x2]]`

PROOF

Note that the input for the rule:

`and[x1_,and[or[x1_,x2_],x3_]]→and[x1,x3]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,x1_]]→or[x2,x1]`

where these rules follow from Critical Pair Lemma 58 and Substitution Lemma 40 respectively.

Critical Pair Lemma 61

The following expressions are equivalent:

`nand[x1,and[or[x1,x2],x3]]==nand[or[x4,x1],and[x1,x3]]`

PROOF

Note that the input for the rule:

`nand[or[x1_,x2_],and[x2_,x3_]]→nand[x2,x3]`

contains a subpattern of the form:

`and[x2_,x3_]`

```
and[x2_,x3_]
```

which can be unified with the input for the rule:

```
and[x1_,and[or[x1_,x2_],x3_]]→and[x1,x3]
```

where these rules follow from Critical Pair Lemma 56 and Critical Pair Lemma 58 respectively.

Substitution Lemma 79

It can be shown that:

```
nand[x1,and[or[x1,x2],x3]]==nand[x1,x3]
```

Proof

We start by taking Critical Pair Lemma 61, and apply the substitution:

```
nand[or[x1_,x2_],and[x2_,x3_]]→nand[x2,x3]
```

which follows from Critical Pair Lemma 56.

Critical Pair Lemma 62

The following expressions are equivalent:

```
and[and[x1,x2],x3]==and[and[x1,x2],and[x3,x1]]
```

Proof

Note that the input for the rule:

```
and[x1_,and[x2_,or[x3_,x1_]]]→and[x1,x2]
```

contains a subpattern of the form:

```
or[x3_,x1_]
```

which can be unified with the input for the rule:

```
or[x1_,and[x1_,x2_]]→x1
```

where these rules follow from Critical Pair Lemma 59 and Substitution Lemma 36 respectively.

Critical Pair Lemma 63

The following expressions are equivalent:

```
and[and[x1,x2],x3]==and[and[x1,x2],and[x2,x3]]
```

Proof

Note that the input for the rule:

```
and[x1_,and[or[x2_,x1_],x3_]]→and[x1,x3]
```

contains a subpattern of the form:

```
or[x2_,x1_]
```

which can be unified with the input for the rule:

```
or[x1_,and[x2_,x1_]]→x1
```

where these rules follow from Critical Pair Lemma 60 and Substitution Lemma 34 respectively.

Critical Pair Lemma 64

The following expressions are equivalent:

```
nand[x1,or[x2,x3]]==nand[x1,or[x2,and[x1,x3]]]
```

Proof

Note that the input for the rule:

`nand[x1_,and[or[x1_,x2_],x3_]]→nand[x1,x3]`

contains a subpattern of the form:

`and[or[x1_,x2_],x3_]`

which can be unified with the input for the rule:

`and[or[x1_,x2_],or[x2_,x3_]]→or[x2,and[x1,x3]]`

where these rules follow from Substitution Lemma 79 and Critical Pair Lemma 3 respectively.

Critical Pair Lemma 65

The following expressions are equivalent:

`and[and[x1,x2],x3]==and[and[x3,x1],and[x1,x2]]`

Proof

Note that the input for the rule:

`and[and[x1_,x2_],and[x3_,x1_]]→and[and[x1,x2],x3]`

contains a subpattern of the form:

`and[and[x1_,x2_],and[x3_,x1_]]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Critical Pair Lemma 62 and Axiom 3 respectively.

Substitution Lemma 80

It can be shown that:

`and[and[x1,x2],x3]==and[and[x3,x1],x2]`

Proof

We start by taking Critical Pair Lemma 65, and apply the substitution:

`and[and[x1_,x2_],and[x2_,x3_]]→and[and[x1,x2],x3]`

which follows from Critical Pair Lemma 63.

Critical Pair Lemma 66

The following expressions are equivalent:

`and[and[x1,x2],x3]==and[x1,and[x2,x3]]`

Proof

Note that the input for the rule:

`and[and[x1_,x2_],x3_]↔and[and[x3_,x1_],x2_]`

contains a subpattern of the form:

`and[and[x1_,x2_],x3_]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Substitution Lemma 80 and Axiom 3 respectively.

Critical Pair Lemma 67

The following expressions are equivalent:

`and[x1,and[x2,not[and[x1,x2]]]]==0`

Proof

Note that the input for the rule:

`and[and[x1_,x2_],x3_]→and[x1,and[x2,x3]]`

contains a subpattern of the form:

`and[and[x1_,x2_],x3_]`

which can be unified with the input for the rule:

`and[x1_,not[x1_]]→0`

where these rules follow from Critical Pair Lemma 66 and Axiom 5 respectively.

Substitution Lemma 81

It can be shown that:

`and[x1,and[x2,nand[x1,x2]]]==0`

Proof

We start by taking Critical Pair Lemma 67, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Axiom 9.

Critical Pair Lemma 68

The following expressions are equivalent:

`or[x1,and[x2,nand[not[x1],x2]]]==or[x1,0]`

Proof

Note that the input for the rule:

`or[x1_,and[not[x1_],x2_]]→or[x1,x2]`

contains a subpattern of the form:

`and[not[x1_],x2_]`

which can be unified with the input for the rule:

`and[x1_,and[x2_,nand[x1_,x2_]]]→0`

where these rules follow from Substitution Lemma 59 and Substitution Lemma 81 respectively.

Substitution Lemma 82

It can be shown that:

`or[x1,and[x2,nand[not[x1],x2]]]==x1`

Proof

We start by taking Critical Pair Lemma 68, and apply the substitution:

`or[x1_,0]→x1`

which follows from Substitution Lemma 1.

Critical Pair Lemma 69

The following expressions are equivalent:

`nand[x1,or[x2,nand[not[x2],x1]]]==nand[x1,x2]`

Proof

Note that the input for the rule:

`nand[x1_,or[x2_,and[x1_,x3_]]]→nand[x1,or[x2,x3]]`

contains a subpattern of the form:

`or[x2_,and[x1_,x3_]]`

which can be unified with the input for the rule:

`or[x1_,and[x2_,nand[not[x1_],x2_]]]→x1`

where these rules follow from Critical Pair Lemma 64 and Substitution Lemma 82 respectively.

Substitution Lemma 83

It can be shown that:

`nand[x1,nand[not[x2],x1]]==nand[x1,x2]`

Proof

We start by taking Critical Pair Lemma 69, and apply the substitution:

`or[x1_,nand[not[x1_],x2_]]→nand[not[x1],x2]`

which follows from Substitution Lemma 75.

Critical Pair Lemma 70

The following expressions are equivalent:

`nand[x1,not[x2]]==nand[x1,nand[x2,x1]]`

Proof

Note that the input for the rule:

`nand[x1_,nand[not[x2_],x1_]]→nand[x1,x2]`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Substitution Lemma 83 and Substitution Lemma 53 respectively.

Critical Pair Lemma 71

The following expressions are equivalent:

`nand[x1,not[x2]]==nand[x1,nand[x1,x2]]`

Proof

Note that the input for the rule:

`nand[x1_,nand[x2_,x1_]]→nand[x1,not[x2]]`

contains a subpattern of the form:

`nand[x2_,x1_]`

which can be unified with the input for the rule:

`nand[x1_,x2_]↔nand[x2_,x1_]`

where these rules follow from Critical Pair Lemma 70 and Substitution Lemma 3 respectively.

Critical Pair Lemma 72

The following expressions are equivalent:

`nand[x1,not[or[x2,not[x1]]]]==nand[x1,nand[x2,x1]]`

Proof

Note that the input for the rule:

`nand[x1_,nand[x1_,x2_]]→nand[x1,not[x2]]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,or[x2_,not[x1_]]]→nand[x2,x1]`

where these rules follow from Critical Pair Lemma 71 and Substitution Lemma 46 respectively.

Substitution Lemma 84

It can be shown that:

`or[x1,not[x2]]==nand[x2,nand[x1,x2]]`

Proof

We start by taking Critical Pair Lemma 72, and apply the substitution:

`nand[x1_,not[or[x2_,not[x1_]]]]→or[x2,not[x1]]`

which follows from Substitution Lemma 77.

Substitution Lemma 85

It can be shown that:

`or[x1,not[x2]]==nand[x2,not[x1]]`

Proof

We start by taking Substitution Lemma 84, and apply the substitution:

`nand[x1_,nand[x2_,x1_]]→nand[x1,not[x2]]`

which follows from Critical Pair Lemma 70.

Critical Pair Lemma 73

The following expressions are equivalent:

`nand[not[x1],not[or[x1,x2]]]==nand[not[x1],nand[not[x1],x2]]`

Proof

Note that the input for the rule:

`nand[x1_,nand[x1_,x2_]]→nand[x1,not[x2]]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[not[x1_],or[x1_,x2_]]→nand[not[x1],x2]`

where these rules follow from Critical Pair Lemma 71 and Substitution Lemma 49 respectively.

Substitution Lemma 86

It can be shown that:

`or[x1,x2]==nand[not[x1],nand[not[x1],x2]]`

Proof

We start by taking Critical Pair Lemma 73, and apply the substitution:

`nand[not[x1_],not[or[x1_,x2_]]]→or[x1,x2]`

which follows from Substitution Lemma 76.

Substitution Lemma 87

It can be shown that:

`or[x1,x2]⩵nand[not[x1],not[x2]]`

Proof

We start by taking Substitution Lemma 86, and apply the substitution:

`nand[x1_,nand[x1_,x2_]]→nand[x1,not[x2]]`

which follows from Critical Pair Lemma 71.

Critical Pair Lemma 74

The following expressions are equivalent:

`and[not[x1],not[x2]]⩵not[or[x1,x2]]`

Proof

Note that the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[not[x1_],not[x2_]]→or[x1,x2]`

where these rules follow from Critical Pair Lemma 42 and Substitution Lemma 87 respectively.

Critical Pair Lemma 75

The following expressions are equivalent:

`not[or[and[x1,x2],x3]]⩵and[nand[x1,x2],not[x3]]`

Proof

Note that the input for the rule:

`and[not[x1_],not[x2_]]→not[or[x1,x2]]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

where these rules follow from Critical Pair Lemma 74 and Axiom 9 respectively.

Critical Pair Lemma 76

The following expressions are equivalent:

`and[nand[x1,x2],not[and[x3,x1]]]⩵not[and[x1,or[x2,x3]]]`

Proof

Note that the input for the rule:

`not[or[and[x1_,x2_],x3_]]→and[nand[x1,x2],not[x3]]`

contains a subpattern of the form:

`or[and[x1_,x2_],x3_]`

which can be unified with the input for the rule:

`or[and[x1_,x2_],and[x3_,x1_]]→and[x1,or[x2,x3]]`

where these rules follow from Critical Pair Lemma 75 and Critical Pair Lemma 2 respectively.

Substitution Lemma 88

It can be shown that:

`and[nand[x1,x2],nand[x3,x1]]==not[and[x1,or[x2,x3]]]`

Proof

We start by taking Critical Pair Lemma 76, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Axiom 9.

Substitution Lemma 89

It can be shown that:

`and[nand[x1,x2],nand[x3,x1]]==nand[x1,or[x2,x3]]`

Proof

We start by taking Substitution Lemma 88, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Axiom 9.

Critical Pair Lemma 77

The following expressions are equivalent:

`nand[nand[x1,x2],nand[x3,x1]]==not[nand[x1,or[x2,x3]]]`

Proof

Note that the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[nand[x1_,x2_],nand[x3_,x1_]]→nand[x1,or[x2,x3]]`

where these rules follow from Axiom 9 and Substitution Lemma 89 respectively.

Substitution Lemma 90

It can be shown that:

`nand[nand[x1,x2],nand[x3,x1]]==and[x1,or[x2,x3]]`

Proof

We start by taking Critical Pair Lemma 77, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Critical Pair Lemma 42.

which follows from Critical Pair Lemma 42.

Substitution Lemma 91

It can be shown that:

`nand[nand[nand[b,b],a],nand[a,nand[c,c]]]==nand[nand[a,nand[b,c]],nand[a,nand[b,c]]]`

Proof

We start by taking Hypothesis 1, and apply the substitution:

`nand[x1_,x2_]→nand[x2,x1]`

which follows from Substitution Lemma 3.

Substitution Lemma 92

It can be shown that:

`nand[nand[nand[b,b],a],nand[a,nand[c,c]]]==nand[nand[nand[b,c],a],nand[a,nand[b,c]]]`

Proof

We start by taking Substitution Lemma 91, and apply the substitution:

`nand[x1_,x2_]→nand[x2,x1]`

which follows from Substitution Lemma 3.

Substitution Lemma 93

It can be shown that:

`nand[nand[a,nand[c,c]],nand[nand[b,b],a]]==nand[nand[nand[b,c],a],nand[a,nand[b,c]]]`

Proof

We start by taking Substitution Lemma 92, and apply the substitution:

`nand[x1_,x2_]→nand[x2,x1]`

which follows from Substitution Lemma 3.

Substitution Lemma 94

It can be shown that:

`nand[nand[a,nand[c,c]],nand[not[b],a]]==nand[nand[nand[b,c],a],nand[a,nand[b,c]]]`

Proof

We start by taking Substitution Lemma 93, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Critical Pair Lemma 22.

Substitution Lemma 95

It can be shown that:

`nand[nand[a,not[c]],nand[not[b],a]]==nand[nand[nand[b,c],a],nand[a,nand[b,c]]]`

Proof

We start by taking Substitution Lemma 94, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Critical Pair Lemma 22.

Substitution Lemma 96

It can be shown that:

`nand[nand[a,not[c]],nand[not[b],a]]==and[nand[b,c],a]`

PROOF

We start by taking Substitution Lemma 95, and apply the substitution:

`nand[nand[x1_,x2_],nand[x2_,x1_]]→and[x1,x2]`

which follows from Substitution Lemma 58.

Substitution Lemma 97

It can be shown that:

`and[a,or[not[c],not[b]]]==and[nand[b,c],a]`

PROOF

We start by taking Substitution Lemma 96, and apply the substitution:

`nand[nand[x1_,x2_],nand[x3_,x1_]]→and[x1,or[x2,x3]]`

which follows from Substitution Lemma 90.

Substitution Lemma 98

It can be shown that:

`and[or[not[c],not[b]],a]==and[nand[b,c],a]`

PROOF

We start by taking Substitution Lemma 97, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Axiom 3.

Substitution Lemma 99

It can be shown that:

`and[nand[b,not[not[c]]],a]==and[nand[b,c],a]`

PROOF

We start by taking Substitution Lemma 98, and apply the substitution:

`or[x1_,not[x2_]]→nand[x2,not[x1]]`

which follows from Substitution Lemma 85.

Conclusion 3

We obtain the conclusion:

`True`

PROOF

Take Substitution Lemma 99, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 53.

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Axiom 1

We are given that:

`x1==and[x1,or[x2,not[x2]]]`

Axiom 2

We are given that:

`x1==or[x1,and[x2,not[x2]]]`

Axiom 3

We are given that:

`and[x1,x2]==and[x2,x1]`

Axiom 4

We are given that:

`and[x1,or[x2,x3]]==or[and[x1,x2],and[x1,x3]]`

Axiom 5

We are given that:

`and[x1,not[x1]]==0`

Axiom 6

We are given that:

`and[or[x1,x2],or[x1,x3]]==or[x1,and[x2,x3]]`

Axiom 7

We are given that:

`or[x1,x2]==or[x2,x1]`

Axiom 8

We are given that:

`or[x1,not[x1]]==1`

Axiom 9

We are given that:

`not[and[x1,x2]]==nand[x1,x2]`

Hypothesis 1

We would like to show that:

`nand[nand[nand[b,b],a],nand[nand[c,c],a]]==nand[nand[a,nand[b,c]],nand[a,nand[b,c]]]`

Hypothesis 2

We would like to show that:

`nand[nand[a,a],nand[a,a]]==a`

Hypothesis 3

We would like to show that:

`nand[a,nand[b,nand[b,b]]]==nand[a,a]`

Critical Pair Lemma 1

The following expressions are equivalent:

`and[x1,or[x2,x3]]==or[and[x2,x1],and[x1,x3]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],and[x1_,x3_]]→and[x1,or[x2,x3]]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Axiom 4 and Axiom 3 respectively.

Critical Pair Lemma 2

The following expressions are equivalent:

`and[x1,or[x2,x3]]==or[and[x1,x2],and[x3,x1]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],and[x1_,x3_]]→and[x1,or[x2,x3]]`

contains a subpattern of the form:

`and[x1_,x3_]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Axiom 4 and Axiom 3 respectively.

Substitution Lemma 1

It can be shown that:

`or[x1,0]==x1`

Proof

We start by taking Axiom 2, and apply the substitution:

`and[x1_,not[x1_]]→0`

which follows from Axiom 5.

Critical Pair Lemma 3

The following expressions are equivalent:

`or[x1,and[x2,x3]]==and[or[x2,x1],or[x1,x3]]`

Proof

Note that the input for the rule:

`and[or[x1_,x2_],or[x1_,x3_]]→or[x1,and[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Axiom 6 and Axiom 7 respectively.

Critical Pair Lemma 4

The following expressions are equivalent:

`or[x1,and[not[x1],x2]]==and[1,or[x1,x2]]`

Proof

Note that the input for the rule:

`and[or[x1_,x2_],or[x1_,x3_]]→or[x1,and[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,not[x1_]]→1`

where these rules follow from Axiom 6 and Axiom 8 respectively.

Substitution Lemma 2

It can be shown that:

`and[x1,1]==x1`

Proof

We start by taking Axiom 1, and apply the substitution:

`or[x1_,not[x1_]]→1`

which follows from Axiom 8.

Critical Pair Lemma 5

The following expressions are equivalent:

`nand[x1,not[x1]]==not[0]`

Proof

Note that the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,not[x1_]]→0`

where these rules follow from Axiom 9 and Axiom 5 respectively.

Critical Pair Lemma 6

The following expressions are equivalent:

`nand[x1,x2]==not[and[x2,x1]]`

Proof

Note that the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Axiom 9 and Axiom 3 respectively.

Substitution Lemma 3

It can be shown that:

`nand[x1,x2]==nand[x2,x1]`

Proof

We start by taking Critical Pair Lemma 6, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Axiom 9.

Critical Pair Lemma 7

The following expressions are equivalent:

`1==or[and[x1,x2],nand[x1,x2]]`

Proof

Note that the input for the rule:

`or[x1_,not[x1_]]→1`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

where these rules follow from Axiom 8 and Axiom 9 respectively.

Critical Pair Lemma 8

The following expressions are equivalent:

`x1==or[0,x1]`

Proof

Note that the input for the rule:

`or[x1_,0]→x1`

contains a subpattern of the form:

`or[x1_,0]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Substitution Lemma 1 and Axiom 7 respectively.

Critical Pair Lemma 9

The following expressions are equivalent:

`or[x1,and[0,x2]]==and[x1,or[x1,x2]]`

Proof

Note that the input for the rule:

`and[or[x1_,x2_],or[x1_,x3_]]→or[x1,and[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,0]→x1`

where these rules follow from Axiom 6 and Substitution Lemma 1 respectively.

Critical Pair Lemma 10

The following expressions are equivalent:

`x1==and[1,x1]`

Proof

Note that the input for the rule:

`and[x1_,1]→x1`

contains a subpattern of the form:

`and[x1_,1]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Substitution Lemma 2 and Axiom 3 respectively.

Critical Pair Lemma 11

The following expressions are equivalent:

`nand[x1,1]==not[x1]`

Proof

Note that the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,1]→x1`

where these rules follow from Axiom 9 and Substitution Lemma 2 respectively.

Critical Pair Lemma 12

The following expressions are equivalent:

`not[0]==1`

Proof

Note that the input for the rule:

`or[0,x1_]→x1`

contains a subpattern of the form:

`or[0,x1_]`

which can be unified with the input for the rule:

`or[x1_,not[x1_]]→1`

where these rules follow from Critical Pair Lemma 8 and Axiom 8 respectively.

Critical Pair Lemma 13

The following expressions are equivalent:

`not[1]==0`

Proof

Note that the input for the rule:

`and[1,x1_]→x1`

contains a subpattern of the form:

`and[1,x1_]`

which can be unified with the input for the rule:

`and[x1_,not[x1_]]→0`

where these rules follow from Critical Pair Lemma 10 and Axiom 5 respectively.

Critical Pair Lemma 14

The following expressions are equivalent:

`nand[1,x1]==not[x1]`

Proof

Note that the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[1,x1_]→x1`

where these rules follow from Axiom 9 and Critical Pair Lemma 10 respectively.

Substitution Lemma 4

It can be shown that:

`nand[x1,not[x1]]==1`

Proof

We start by taking Critical Pair Lemma 5, and apply the substitution:

`not[0]→1`

which follows from Critical Pair Lemma 12.

Critical Pair Lemma 15

The following expressions are equivalent:

`or[x1,and[0,0]]==and[x1,x1]`

Proof

Note that the input for the rule:

`and[x1_,or[x1_,x2_]]→or[x1,and[0,x2]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,0]→x1`

where these rules follow from Critical Pair Lemma 9 and Substitution Lemma 1 respectively.

Critical Pair Lemma 16

The following expressions are equivalent:

`or[x1,and[0,x2]]==and[x1,or[x2,x1]]`

Proof

Note that the input for the rule:

`and[x1_,or[x1_,x2_]]→or[x1,and[0,x2]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Critical Pair Lemma 9 and Axiom 7 respectively.

Critical Pair Lemma 17

The following expressions are equivalent:

`or[not[x1],and[0,x1]]==and[not[x1],1]`

Proof

Note that the input for the rule:

`and[x1_,or[x2_,x1_]]→or[x1,and[0,x2]]`

contains a subpattern of the form:

`or[x2_,x1_]`

which can be unified with the input for the rule:

`or[x1_,not[x1_]]→1`

where these rules follow from Critical Pair Lemma 16 and Axiom 8 respectively.

Substitution Lemma 5

It can be shown that:

`or[not[x1],and[0,x1]]==not[x1]`

Proof

We start by taking Critical Pair Lemma 17, and apply the substitution:

`and[x1_,1]→x1`

which follows from Substitution Lemma 2.

Critical Pair Lemma 18

The following expressions are equivalent:

`or[and[0,x1],and[0,not[x1]]]==and[and[0,x1],not[x1]]`

Proof

Note that the input for the rule:

Note that the input for the rule:

`and[x1_,or[x2_,x1_]]→or[x1,and[0,x2]]`

contains a subpattern of the form:

`or[x2_,x1_]`

which can be unified with the input for the rule:

`or[not[x1_],and[0,x1_]]→not[x1]`

where these rules follow from Critical Pair Lemma 16 and Substitution Lemma 5 respectively.

Substitution Lemma 6

It can be shown that:

`and[0,or[x1,not[x1]]]==and[and[0,x1],not[x1]]`

Proof

We start by taking Critical Pair Lemma 18, and apply the substitution:

`or[and[x1_,x2_],and[x1_,x3_]]→and[x1,or[x2,x3]]`

which follows from Axiom 4.

Substitution Lemma 7

It can be shown that:

`and[0,1]==and[and[0,x1],not[x1]]`

Proof

We start by taking Substitution Lemma 6, and apply the substitution:

`or[x1_,not[x1_]]→1`

which follows from Axiom 8.

Substitution Lemma 8

It can be shown that:

`0==and[and[0,x1],not[x1]]`

Proof

We start by taking Substitution Lemma 7, and apply the substitution:

`and[x1_,1]→x1`

which follows from Substitution Lemma 2.

Substitution Lemma 9

It can be shown that:

`0==and[not[x1],and[0,x1]]`

Proof

We start by taking Substitution Lemma 8, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Axiom 3.

Critical Pair Lemma 19

The following expressions are equivalent:

`0==and[0,and[0,1]]`

Proof

Note that the input for the rule:

`and[not[x1_],and[0,x1_]]→0`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[1]→0`

where these rules follow from Substitution Lemma 9 and Critical Pair Lemma 13 respectively.

Substitution Lemma 10

It can be shown that:

`0==and[0,0]`

Proof

We start by taking Critical Pair Lemma 19, and apply the substitution:

`and[x1_,1]→x1`

which follows from Substitution Lemma 2.

Substitution Lemma 11

It can be shown that:

`1==or[nand[x1,x2],and[x1,x2]]`

Proof

We start by taking Critical Pair Lemma 7, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 7.

Critical Pair Lemma 20

The following expressions are equivalent:

`1==or[nand[x1,x2],and[x2,x1]]`

Proof

Note that the input for the rule:

`or[nand[x1_,x2_],and[x1_,x2_]]→1`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,x2_]↔nand[x2_,x1_]`

where these rules follow from Substitution Lemma 11 and Substitution Lemma 3 respectively.

Substitution Lemma 12

It can be shown that:

`or[x1,0]==and[x1,x1]`

Proof

We start by taking Critical Pair Lemma 15, and apply the substitution:

`and[0,0]→0`

which follows from Substitution Lemma 10.

Substitution Lemma 13

It can be shown that:

`x1==and[x1,x1]`

Proof

We start by taking Substitution Lemma 12, and apply the substitution:

`or[x1_,0]→x1`

which follows from Substitution Lemma 1.

Critical Pair Lemma 21

The following expressions are equivalent:

`and[x1,or[x2,x1]]==or[and[x1,x2],x1]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],and[x1_,x3_]]→and[x1,or[x2,x3]]`

contains a subpattern of the form:

`and[x1_,x3_]`

which can be unified with the input for the rule:

`and[x1_,x1_]→x1`

where these rules follow from Axiom 4 and Substitution Lemma 13 respectively.

Substitution Lemma 14

It can be shown that:

`or[x1,and[0,x2]]==or[and[x1,x2],x1]`

Proof

We start by taking Critical Pair Lemma 21, and apply the substitution:

`and[x1_,or[x2_,x1_]]→or[x1,and[0,x2]]`

which follows from Critical Pair Lemma 16.

Critical Pair Lemma 22

The following expressions are equivalent:

`nand[x1,x1]==not[x1]`

Proof

Note that the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,x1_]→x1`

`and[x1_,x1_]→x1`

where these rules follow from Axiom 9 and Substitution Lemma 13 respectively.

Substitution Lemma 15

It can be shown that:

`nand[nand[b,nand[b,b]],a]==nand[a,a]`

Proof

We start by taking Hypothesis 3, and apply the substitution:

`nand[x1_,x2_]→nand[x2,x1]`

which follows from Substitution Lemma 3.

Substitution Lemma 16

It can be shown that:

`nand[nand[b,nand[b,b]],a]==not[a]`

Proof

We start by taking Substitution Lemma 15, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Critical Pair Lemma 22.

Substitution Lemma 17

It can be shown that:

`nand[nand[b,not[b]],a]==not[a]`

Proof

We start by taking Substitution Lemma 16, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Critical Pair Lemma 22.

Substitution Lemma 18

It can be shown that:

`nand[1,a]==not[a]`

Proof

We start by taking Substitution Lemma 17, and apply the substitution:

`nand[x1_,not[x1_]]→1`

which follows from Substitution Lemma 4.

Conclusion 1

We obtain the conclusion:

`True`

Proof

Take Substitution Lemma 18, and apply the substitution:

`nand[1,x1_]→not[x1]`

which follows from Critical Pair Lemma 14.

Substitution Lemma 19

It can be shown that:

`or[x1,and[0,x2]]==or[x1,and[x1,x2]]`

Proof

We start by taking Substitution Lemma 14, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 7.

Critical Pair Lemma 23

The following expressions are equivalent:

`or[x1,and[x1,not[0]]]==or[x1,0]`

Proof

Note that the input for the rule:

`or[x1_,and[0,x2_]]↔or[x1_,and[x1_,x2_]]`

contains a subpattern of the form:

`and[0,x2_]`

which can be unified with the input for the rule:

`and[x1_,not[x1_]]→0`

where these rules follow from Substitution Lemma 19 and Axiom 5 respectively.

Substitution Lemma 20

It can be shown that:

`or[x1,and[x1,1]]==or[x1,0]`

Proof

We start by taking Critical Pair Lemma 23, and apply the substitution:

`not[0]→1`

which follows from Critical Pair Lemma 12.

Substitution Lemma 21

It can be shown that:

`or[x1,x1]==or[x1,0]`

Proof

We start by taking Substitution Lemma 20, and apply the substitution:

`and[x1_,1]→x1`

which follows from Substitution Lemma 2.

Substitution Lemma 22

It can be shown that:

`or[x1,x1]==x1`

Proof

We start by taking Substitution Lemma 21, and apply the substitution:

`or[x1_,0]→x1`

which follows from Substitution Lemma 1.

Critical Pair Lemma 24

The following expressions are equivalent:

`or[x1,and[x2,x1]]==and[or[x1,x2],x1]`

Proof

Note that the input for the rule:

`and[or[x1_,x2_],or[x1_,x3_]]→or[x1,and[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x3_]`

which can be unified with the input for the rule:

`or[x1_,x1_]→x1`

where these rules follow from Axiom 6 and Substitution Lemma 22 respectively.

Substitution Lemma 23

It can be shown that:

`or[x1,and[x2,x1]]==and[x1,or[x1,x2]]`

Proof

We start by taking Critical Pair Lemma 24, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Axiom 3.

Substitution Lemma 24

It can be shown that:

`or[x1,and[x2,x1]]==or[x1,and[0,x2]]`

Proof

We start by taking Substitution Lemma 23, and apply the substitution:

`and[x1_,or[x1_,x2_]]→or[x1,and[0,x2]]`

which follows from Critical Pair Lemma 9.

Critical Pair Lemma 25

The following expressions are equivalent:

`or[and[0,x1],and[0,not[x1]]]==or[and[0,x1],0]`

Proof

Note that the input for the rule:

`or[x1_,and[x2_,x1_]]↔or[x1_,and[0,x2_]]`

contains a subpattern of the form:

`and[x2_,x1_]`

which can be unified with the input for the rule:

`and[not[x1_],and[0,x1_]]→0`

where these rules follow from Substitution Lemma 24 and Substitution Lemma 9 respectively.

where these rules follow from Substitution Lemma 24 and Substitution Lemma 9 respectively.

Substitution Lemma 25

It can be shown that:

`and[0,or[x1,not[x1]]]==or[and[0,x1],0]`

Proof

We start by taking Critical Pair Lemma 25, and apply the substitution:

`or[and[x1_,x2_],and[x1_,x3_]]→and[x1,or[x2,x3]]`

which follows from Axiom 4.

Substitution Lemma 26

It can be shown that:

`and[0,1]==or[and[0,x1],0]`

Proof

We start by taking Substitution Lemma 25, and apply the substitution:

`or[x1_,not[x1_]]→1`

which follows from Axiom 8.

Substitution Lemma 27

It can be shown that:

`0==or[and[0,x1],0]`

Proof

We start by taking Substitution Lemma 26, and apply the substitution:

`and[x1_,1]→x1`

which follows from Substitution Lemma 2.

Substitution Lemma 28

It can be shown that:

`0==and[0,x1]`

Proof

We start by taking Substitution Lemma 27, and apply the substitution:

`or[x1_,0]→x1`

which follows from Substitution Lemma 1.

Substitution Lemma 29

It can be shown that:

`and[x1_,or[x2_,x1_]]→or[x1,0]`

Proof

We start by taking Critical Pair Lemma 16, and apply the substitution:

`and[0,x1_]→0`

which follows from Substitution Lemma 28.

Substitution Lemma 30

It can be shown that:

`and[x1_,or[x2_,x1_]]→x1`

PROOF

We start by taking Substitution Lemma 29, and apply the substitution:

`or[x1_,0]→x1`

which follows from Substitution Lemma 1.

Substitution Lemma 31

It can be shown that:

`and[x1_,or[x1_,x2_]]→or[x1,0]`

PROOF

We start by taking Critical Pair Lemma 9, and apply the substitution:

`and[0,x1_]→0`

which follows from Substitution Lemma 28.

Substitution Lemma 32

It can be shown that:

`and[x1_,or[x1_,x2_]]→x1`

PROOF

We start by taking Substitution Lemma 31, and apply the substitution:

`or[x1_,0]→x1`

which follows from Substitution Lemma 1.

Substitution Lemma 33

It can be shown that:

`or[x1,and[x2,x1]]==or[x1,0]`

PROOF

We start by taking Substitution Lemma 24, and apply the substitution:

`and[0,x1_]→0`

which follows from Substitution Lemma 28.

Substitution Lemma 34

It can be shown that:

`or[x1,and[x2,x1]]==x1`

PROOF

We start by taking Substitution Lemma 33, and apply the substitution:

`or[x1_,0]→x1`

which follows from Substitution Lemma 1.

Substitution Lemma 35

It can be shown that:

`or[x1,0]==or[x1,and[x1,x2]]`

`or[x1,0]==or[x1,and[x1,x2]]`

PROOF

We start by taking Substitution Lemma 19, and apply the substitution:

`and[0,x1_]→0`

which follows from Substitution Lemma 28.

Substitution Lemma 36

It can be shown that:

`x1==or[x1,and[x1,x2]]`

PROOF

We start by taking Substitution Lemma 35, and apply the substitution:

`or[x1_,0]→x1`

which follows from Substitution Lemma 1.

Critical Pair Lemma 26

The following expressions are equivalent:

`and[not[x1],or[x1,x2]]==or[0,and[not[x1],x2]]`

PROOF

Note that the input for the rule:

`or[and[x1_,x2_],and[x2_,x3_]]→and[x2,or[x1,x3]]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,not[x1_]]→0`

where these rules follow from Critical Pair Lemma 1 and Axiom 5 respectively.

Substitution Lemma 37

It can be shown that:

`and[not[x1],or[x1,x2]]==and[not[x1],x2]`

PROOF

We start by taking Critical Pair Lemma 26, and apply the substitution:

`or[0,x1_]→x1`

which follows from Critical Pair Lemma 8.

Critical Pair Lemma 27

The following expressions are equivalent:

`and[x1,or[x2,not[x1]]]==or[and[x2,x1],0]`

PROOF

Note that the input for the rule:

`or[and[x1_,x2_],and[x2_,x3_]]→and[x2,or[x1,x3]]`

contains a subpattern of the form:

`and[x2_,x3_]`

`and[x2_,x3_]`

which can be unified with the input for the rule:

`and[x1_,not[x1_]]→0`

where these rules follow from Critical Pair Lemma 1 and Axiom 5 respectively.

Substitution Lemma 38

It can be shown that:

`and[x1,or[x2,not[x1]]]==and[x2,x1]`

Proof

We start by taking Critical Pair Lemma 27, and apply the substitution:

`or[x1_,0]→x1`

which follows from Substitution Lemma 1.

Critical Pair Lemma 28

The following expressions are equivalent:

`or[x1,x2]==or[or[x1,x2],x2]`

Proof

Note that the input for the rule:

`or[x1_,and[x2_,x1_]]→x1`

contains a subpattern of the form:

`and[x2_,x1_]`

which can be unified with the input for the rule:

`Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[541,or[x1_,0]→x`

where these rules follow from Substitution Lemma 34 and Substitution Lemma 30 respectively.

Critical Pair Lemma 29

The following expressions are equivalent:

`and[x1,x2]==and[and[x1,x2],x2]`

Proof

Note that the input for the rule:

`Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[541,or[x1_,0]→x`

contains a subpattern of the form:

`or[x2_,x1_]`

which can be unified with the input for the rule:

`or[x1_,and[x2_,x1_]]→x1`

where these rules follow from Substitution Lemma 30 and Substitution Lemma 34 respectively.

Critical Pair Lemma 30

The following expressions are equivalent:

`or[x1,and[and[x2,x1],x3]]==and[x1,or[x1,x3]]`

Proof

Note that the input for the rule:

`and[or[x1_,x2_],or[x1_,x3_]]→or[x1,and[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,and[x2_,x1_]]→x1`

where these rules follow from Axiom 6 and Substitution Lemma 34 respectively.

Substitution Lemma 39

It can be shown that:

`or[x1,and[and[x2,x1],x3]]==x1`

Proof

We start by taking Critical Pair Lemma 30, and apply the substitution:

`and[x1_,or[x1_,x2_]]→x1`

which follows from Substitution Lemma 32.

Critical Pair Lemma 31

The following expressions are equivalent:

`and[x1,x2]==and[and[x1,x2],x1]`

Proof

Note that the input for the rule:

`Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[541,or[x1_,0]→x`

contains a subpattern of the form:

`or[x2_,x1_]`

which can be unified with the input for the rule:

`or[x1_,and[x1_,x2_]]→x1`

where these rules follow from Substitution Lemma 30 and Substitution Lemma 36 respectively.

Substitution Lemma 40

It can be shown that:

`or[x1,x2]==or[x2,or[x1,x2]]`

Proof

We start by taking Critical Pair Lemma 28, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 7.

Substitution Lemma 41

It can be shown that:

`and[x1,x2]==and[x2,and[x1,x2]]`

Proof

We start by taking Critical Pair Lemma 29, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Axiom 3.

Critical Pair Lemma 32

The following expressions are equivalent:

`and[x1,or[and[x2,x1],x3]]==or[and[x2,x1],and[x1,x3]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],and[x1_,x3_]]→and[x1,or[x2,x3]]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,and[x2_,x1_]]→and[x2,x1]`

where these rules follow from Axiom 4 and Substitution Lemma 41 respectively.

Substitution Lemma 42

It can be shown that:

`and[x1,or[and[x2,x1],x3]]==and[x1,or[x2,x3]]`

Proof

We start by taking Critical Pair Lemma 32, and apply the substitution:

`or[and[x1_,x2_],and[x2_,x3_]]→and[x2,or[x1,x3]]`

which follows from Critical Pair Lemma 1.

Critical Pair Lemma 33

The following expressions are equivalent:

`nand[x1,and[x2,x1]]==not[and[x2,x1]]`

Proof

Note that the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,and[x2_,x1_]]→and[x2,x1]`

where these rules follow from Axiom 9 and Substitution Lemma 41 respectively.

Substitution Lemma 43

It can be shown that:

`nand[x1,and[x2,x1]]==nand[x2,x1]`

Proof

We start by taking Critical Pair Lemma 33, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Axiom 9.

Substitution Lemma 44

It can be shown that:

`and[x1,x2]==and[x1,and[x1,x2]]`

Proof

We start by taking Critical Pair Lemma 31, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Axiom 3.

Critical Pair Lemma 34

The following expressions are equivalent:

`nand[x1,and[x1,x2]]==not[and[x1,x2]]`

Proof

Note that the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,and[x1_,x2_]]→and[x1,x2]`

where these rules follow from Axiom 9 and Substitution Lemma 44 respectively.

Substitution Lemma 45

It can be shown that:

`nand[x1,and[x1,x2]]==nand[x1,x2]`

Proof

We start by taking Critical Pair Lemma 34, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Axiom 9.

Critical Pair Lemma 35

The following expressions are equivalent:

`and[x1,x2]==and[x2,or[not[x2],x1]]`

Proof

Note that the input for the rule:

`and[x1_,or[x2_,not[x1_]]]→and[x2,x1]`

contains a subpattern of the form:

`or[x2_,not[x1_]]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Substitution Lemma 38 and Axiom 7 respectively.

Critical Pair Lemma 36

The following expressions are equivalent:

`nand[x1,or[x2,not[x1]]]==nand[x1,and[x2,x1]]` `nand[x1,or[x2,not[x1]]]==nand[x1,and[x2,x1]]`

Proof

Note that the input for the rule:

`nand[x1_,and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,or[x2_,not[x1_]]]→and[x2,x1]`

where these rules follow from Substitution Lemma 45 and Substitution Lemma 38 respectively.

Substitution Lemma 46

It can be shown that:

`nand[x1,or[x2,not[x1]]]==nand[x2,x1]`

Proof

We start by taking Critical Pair Lemma 36, and apply the substitution:

`nand[x1_,and[x2_,x1_]]→nand[x2,x1]`

which follows from Substitution Lemma 43.

Critical Pair Lemma 37

The following expressions are equivalent:

`and[not[not[x1]],x1]==and[x1,1]`

Proof

Note that the input for the rule:

`and[x1_,or[not[x1_],x2_]]→and[x2,x1]`

contains a subpattern of the form:

`or[not[x1_],x2_]`

which can be unified with the input for the rule:

`or[x1_,not[x1_]]→1`

where these rules follow from Critical Pair Lemma 35 and Axiom 8 respectively.

Substitution Lemma 47

It can be shown that:

`and[not[not[x1]],x1]==x1`

Proof

We start by taking Critical Pair Lemma 37, and apply the substitution:

`and[x1_,1]→x1`

which follows from Substitution Lemma 2.

Substitution Lemma 48

It can be shown that:

`and[x1,not[not[x1]]]==x1`

Proof

We start by taking Substitution Lemma 47, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Axiom 3.

Critical Pair Lemma 38

The following expressions are equivalent:

`and[not[x1],x2]==and[not[x1],or[x2,x1]]`

Proof

Note that the input for the rule:

`and[not[x1_],or[x1_,x2_]]→and[not[x1],x2]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Substitution Lemma 37 and Axiom 7 respectively.

Out[∘]=

Critical Pair Lemma 39

The following expressions are equivalent:

`nand[not[x1],or[x1,x2]]==nand[not[x1],and[not[x1],x2]]`

Proof

Note that the input for the rule:

`nand[x1_,and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[not[x1_],or[x1_,x2_]]→and[not[x1],x2]`

where these rules follow from Substitution Lemma 45 and Substitution Lemma 37 respectively.

Substitution Lemma 49

It can be shown that:

`nand[not[x1],or[x1,x2]]==nand[not[x1],x2]`

Proof

We start by taking Critical Pair Lemma 39, and apply the substitution:

`nand[x1_,and[x1_,x2_]]→nand[x1,x2]`

which follows from Substitution Lemma 45.

Critical Pair Lemma 40

The following expressions are equivalent:

`and[not[not[x1]],x1]==and[not[not[x1]],1]`

Proof

Note that the input for the rule:

`and[not[x1_],or[x2_,x1_]]→and[not[x1],x2]`

contains a subpattern of the form:

`or[x2_,x1_]`

which can be unified with the input for the rule:

`or[x1_,not[x1_]]→1`

where these rules follow from Critical Pair Lemma 38 and Axiom 8 respectively.

Substitution Lemma 50

It can be shown that:

`and[not[not[x1]],x1]==not[not[x1]]`

Proof

We start by taking Critical Pair Lemma 40, and apply the substitution:

`and[x1_,1]→x1`

which follows from Substitution Lemma 2.

Critical Pair Lemma 41

The following expressions are equivalent:

`nand[not[x1],or[x2,x1]]==nand[not[x1],and[not[x1],x2]]`

Proof

Note that the input for the rule:

`nand[x1_,and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[not[x1_],or[x2_,x1_]]→and[not[x1],x2]`

where these rules follow from Substitution Lemma 45 and Critical Pair Lemma 38 respectively.

Substitution Lemma 51

It can be shown that:

`nand[not[x1],or[x2,x1]]==nand[not[x1],x2]`

Proof

We start by taking Critical Pair Lemma 41, and apply the substitution:

`nand[x1_,and[x1_,x2_]]→nand[x1,x2]`

which follows from Substitution Lemma 45.

Substitution Lemma 52

It can be shown that:

`and[x1,not[not[x1]]]==not[not[x1]]`

Proof

We start by taking Substitution Lemma 50, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Axiom 3.

Substitution Lemma 53

It can be shown that:

`x1==not[not[x1]]`

Proof

We start by taking Substitution Lemma 52, and apply the substitution:

`and[x1_,not[not[x1_]]]→x1`

which follows from Substitution Lemma 48.

Substitution Lemma 54

It can be shown that:

`not[nand[a,a]]==a`

Proof

We start by taking Hypothesis 2, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Critical Pair Lemma 22.

Substitution Lemma 55

It can be shown that:

`not[not[a]]==a`

Proof

We start by taking Substitution Lemma 54, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Critical Pair Lemma 22.

Conclusion 2

We obtain the conclusion:

`True`

Proof

Take Substitution Lemma 55, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 53.

Critical Pair Lemma 42

The following expressions are equivalent:

`and[x1,x2]==not[nand[x1,x2]]`

Proof

Note that the input for the rule:

`not[not[x1_]]→x1`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

which can be unified with the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

where these rules follow from Substitution Lemma 53 and Axiom 9 respectively.

Critical Pair Lemma 43

The following expressions are equivalent:

`nand[not[and[x1,x2]],nand[x2,x1]]==nand[not[and[x1,x2]],1]`

Proof

Note that the input for the rule:

`nand[not[x1_],or[x2_,x1_]]→nand[not[x1],x2]`

contains a subpattern of the form:

`or[x2_,x1_]`

which can be unified with the input for the rule:

`or[nand[x1_,x2_],and[x2_,x1_]]→1`

where these rules follow from Substitution Lemma 51 and Critical Pair Lemma 20 respectively.

Substitution Lemma 56

It can be shown that:

`nand[nand[x1,x2],nand[x2,x1]]==nand[not[and[x1,x2]],1]`

Proof

We start by taking Critical Pair Lemma 43, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Axiom 9.

Substitution Lemma 57

It can be shown that:

`nand[nand[x1,x2],nand[x2,x1]]==not[not[and[x1,x2]]]`

Proof

We start by taking Substitution Lemma 56, and apply the substitution:

`nand[x1_,1]→not[x1]`

which follows from Critical Pair Lemma 11.

Substitution Lemma 58

It can be shown that:

`nand[nand[x1,x2],nand[x2,x1]]==and[x1,x2]`

Proof

We start by taking Substitution Lemma 57, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 53.

Substitution Lemma 59

It can be shown that:

`or[x1,and[not[x1],x2]]==or[x1,x2]`

Proof

We start by taking Critical Pair Lemma 4, and apply the substitution:

`and[1,x1_]→x1`

which follows from Critical Pair Lemma 10.

Critical Pair Lemma 44

The following expressions are equivalent:

`or[x1,or[not[x1],x2]]==or[x1,not[x1]]`

Proof

Note that the input for the rule:

`or[x1_,and[not[x1_],x2_]]→or[x1,x2]`

contains a subpattern of the form:

`and[not[x1_],x2_]`

which can be unified with the input for the rule:

`Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[543,or[x1_,0]→x`

where these rules follow from Substitution Lemma 59 and Substitution Lemma 32 respectively.

Substitution Lemma 60

It can be shown that:

`or[x1,or[not[x1],x2]]==1`

Proof

We start by taking Critical Pair Lemma 44, and apply the substitution:

`or[x1_,not[x1_]]→1`

which follows from Axiom 8.

Critical Pair Lemma 45

The following expressions are equivalent:

`or[x1,or[x2,not[x1]]]==or[x1,not[x1]]`

Proof

Note that the input for the rule:

`or[x1_,and[not[x1_],x2_]]→or[x1,x2]`

contains a subpattern of the form:

`and[not[x1_],x2_]`

which can be unified with the input for the rule:

`Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[541,or[x1_,0]→x`

where these rules follow from Substitution Lemma 59 and Substitution Lemma 30 respectively.

Substitution Lemma 61

It can be shown that:

`or[x1,or[x2,not[x1]]]==1`

Proof

We start by taking Critical Pair Lemma 45, and apply the substitution:

```
or[x1_,not[x1_]]→1
```

which follows from Axiom 8.

Critical Pair Lemma 46

The following expressions are equivalent:

```
1==or[not[x1],or[x1,x2]]
```

Proof

Note that the input for the rule:

```
or[x1_,or[not[x1_],x2_]]→1
```

contains a subpattern of the form:

```
not[x1_]
```

which can be unified with the input for the rule:

```
not[not[x1_]]→x1
```

where these rules follow from Substitution Lemma 60 and Substitution Lemma 53 respectively.

Critical Pair Lemma 47

The following expressions are equivalent:

```
1==or[not[x1],or[x2,x1]]
```

Proof

Note that the input for the rule:

```
or[x1_,or[x2_,not[x1_]]]→1
```

contains a subpattern of the form:

```
not[x1_]
```

which can be unified with the input for the rule:

```
not[not[x1_]]→x1
```

where these rules follow from Substitution Lemma 61 and Substitution Lemma 53 respectively.

Critical Pair Lemma 48

The following expressions are equivalent:

```
nand[not[or[x1,not[x2]]],x2]==nand[not[or[x1,not[x2]]],1]
```

Proof

Note that the input for the rule:

```
nand[not[x1_],or[x2_,x1_]]→nand[not[x1],x2]
```

contains a subpattern of the form:

```
or[x2_,x1_]
```

which can be unified with the input for the rule:

```
or[x1_,or[x2_,not[x1_]]]→1
```

where these rules follow from Substitution Lemma 51 and Substitution Lemma 61 respectively.

Substitution Lemma 62

It can be shown that:

`nand[not[or[x1,not[x2]]],x2]==not[not[or[x1,not[x2]]]]`

PROOF

We start by taking Critical Pair Lemma 48, and apply the substitution:

`nand[x1_,1]→not[x1]`

which follows from Critical Pair Lemma 11.

Substitution Lemma 63

It can be shown that:

`nand[not[or[x1,not[x2]]],x2]==or[x1,not[x2]]`

PROOF

We start by taking Substitution Lemma 62, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 53.

Critical Pair Lemma 49

The following expressions are equivalent:

`nand[not[or[x1,x2]],not[x1]]==nand[not[or[x1,x2]],1]`

PROOF

Note that the input for the rule:

`nand[not[x1_],or[x2_,x1_]]→nand[not[x1],x2]`

contains a subpattern of the form:

`or[x2_,x1_]`

which can be unified with the input for the rule:

`or[not[x1_],or[x1_,x2_]]→1`

where these rules follow from Substitution Lemma 51 and Critical Pair Lemma 46 respectively.

Substitution Lemma 64

It can be shown that:

`nand[not[or[x1,x2]],not[x1]]==not[not[or[x1,x2]]]`

PROOF

We start by taking Critical Pair Lemma 49, and apply the substitution:

`nand[x1_,1]→not[x1]`

which follows from Critical Pair Lemma 11.

Substitution Lemma 65

It can be shown that:

`nand[not[or[x1,x2]],not[x1]]==or[x1,x2]`

PROOF

We start by taking Substitution Lemma 64, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 53.

Critical Pair Lemma 50

The following expressions are equivalent:

`1==or[not[and[and[x1,x2],x3]],x2]`

Proof

Note that the input for the rule:

`or[not[x1_],or[x2_,x1_]]→1`

contains a subpattern of the form:

`or[x2_,x1_]`

which can be unified with the input for the rule:

`or[x1_,and[and[x2_,x1_],x3_]]→x1`

where these rules follow from Critical Pair Lemma 47 and Substitution Lemma 39 respectively.

Substitution Lemma 66

It can be shown that:

`1==or[nand[and[x1,x2],x3],x2]`

Proof

We start by taking Critical Pair Lemma 50, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Axiom 9.

Critical Pair Lemma 51

The following expressions are equivalent:

`1==or[not[and[x1,x2]],x1]`

Proof

Note that the input for the rule:

`or[not[x1_],or[x2_,x1_]]→1`

contains a subpattern of the form:

`or[x2_,x1_]`

which can be unified with the input for the rule:

`or[x1_,and[x1_,x2_]]→x1`

where these rules follow from Critical Pair Lemma 47 and Substitution Lemma 36 respectively.

Substitution Lemma 67

It can be shown that:

`1==or[nand[x1,x2],x1]`

Proof

We start by taking Critical Pair Lemma 51, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Axiom 9.

Substitution Lemma 68

It can be shown that:

`1==or[x1,nand[x1,x2]]`

Proof

We start by taking Substitution Lemma 67, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 7.

Critical Pair Lemma 52

The following expressions are equivalent:

`and[nand[not[x1],x2],x1]==and[x1,1]`

Proof

Note that the input for the rule:

`and[x1_,or[not[x1_],x2_]]→and[x2,x1]`

contains a subpattern of the form:

`or[not[x1_],x2_]`

which can be unified with the input for the rule:

`or[x1_,nand[x1_,x2_]]→1`

where these rules follow from Critical Pair Lemma 35 and Substitution Lemma 68 respectively.

Substitution Lemma 69

It can be shown that:

`and[nand[not[x1],x2],x1]==x1`

Proof

We start by taking Critical Pair Lemma 52, and apply the substitution:

`and[x1_,1]→x1`

which follows from Substitution Lemma 2.

Substitution Lemma 70

It can be shown that:

`and[x1,nand[not[x1],x2]]==x1`

Proof

We start by taking Substitution Lemma 69, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Axiom 3.

Critical Pair Lemma 53

The following expressions are equivalent:

`nand[not[x1],x2]==or[nand[not[x1],x2],x1]`

Proof

Note that the input for the rule:

`or[x1_,and[x2_,x1_]]→x1`

contains a subpattern of the form:

contains a subpattern of the form:

`and[x2_,x1_]`

which can be unified with the input for the rule:

`and[x1_,nand[not[x1_],x2_]]→x1`

where these rules follow from Substitution Lemma 34 and Substitution Lemma 70 respectively.

Substitution Lemma 71

It can be shown that:

`1==or[x1,nand[and[x2,x1],x3]]`

Proof

We start by taking Substitution Lemma 66, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 7.

Critical Pair Lemma 54

The following expressions are equivalent:

`1==or[or[x1,x2],nand[x2,x3]]`

Proof

Note that the input for the rule:

`or[x1_,nand[and[x2_,x1_],x3_]]→1`

contains a subpattern of the form:

`and[x2_,x1_]`

which can be unified with the input for the rule:

`Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[541,or[x1_,0]→x`

where these rules follow from Substitution Lemma 71 and Substitution Lemma 30 respectively.

Critical Pair Lemma 55

The following expressions are equivalent:

`nand[not[nand[x1,x2]],or[x3,x1]]==nand[not[nand[x1,x2]],1]`

Proof

Note that the input for the rule:

`nand[not[x1_],or[x2_,x1_]]→nand[not[x1],x2]`

contains a subpattern of the form:

`or[x2_,x1_]`

which can be unified with the input for the rule:

`or[or[x1_,x2_],nand[x2_,x3_]]→1`

where these rules follow from Substitution Lemma 51 and Critical Pair Lemma 54 respectively.

Substitution Lemma 72

It can be shown that:

`nand[and[x1,x2],or[x3,x1]]==nand[not[nand[x1,x2]],1]`

Proof

We start by taking Critical Pair Lemma 55, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Critical Pair Lemma 42.

Substitution Lemma 73

It can be shown that:

`nand[and[x1,x2],or[x3,x1]]==not[not[nand[x1,x2]]]`

Proof

We start by taking Substitution Lemma 72, and apply the substitution:

`nand[x1_,1]→not[x1]`

which follows from Critical Pair Lemma 11.

Substitution Lemma 74

It can be shown that:

`nand[and[x1,x2],or[x3,x1]]==nand[x1,x2]`

Proof

We start by taking Substitution Lemma 73, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 53.

Substitution Lemma 75

It can be shown that:

`nand[not[x1],x2]==or[x1,nand[not[x1],x2]]`

Proof

We start by taking Critical Pair Lemma 53, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 7.

Substitution Lemma 76

It can be shown that:

`nand[not[x1],not[or[x1,x2]]]==or[x1,x2]`

Proof

We start by taking Substitution Lemma 65, and apply the substitution:

`nand[x1_,x2_]→nand[x2,x1]`

which follows from Substitution Lemma 3.

Critical Pair Lemma 56

The following expressions are equivalent:

`nand[x1,x2]==nand[or[x3,x1],and[x1,x2]]`

Proof

Note that the input for the rule:

`nand[and[x1_,x2_],or[x3_,x1_]]→nand[x1,x2]`

contains a subpattern of the form:

`nand[and[x1_,x2_],or[x3_,x1_]]`

which can be unified with the input for the rule:

`nand[x1_,x2_]↔nand[x2_,x1_]`

where these rules follow from Substitution Lemma 74 and Substitution Lemma 3 respectively.

Substitution Lemma 77

It can be shown that:

`nand[x1,not[or[x2,not[x1]]]]==or[x2,not[x1]]`

Proof

We start by taking Substitution Lemma 63, and apply the substitution:

`nand[x1_,x2_]→nand[x2,x1]`

which follows from Substitution Lemma 3.

Critical Pair Lemma 57

The following expressions are equivalent:

`and[x1,or[x2,and[x3,x2]]]==and[x1,and[x2,or[x1,x3]]]`

Proof

Note that the input for the rule:

`and[x1_,or[and[x2_,x1_],x3_]]→and[x1,or[x2,x3]]`

contains a subpattern of the form:

`or[and[x2_,x1_],x3_]`

which can be unified with the input for the rule:

`or[and[x1_,x2_],and[x3_,x1_]]→and[x1,or[x2,x3]]`

where these rules follow from Substitution Lemma 42 and Critical Pair Lemma 2 respectively.

Substitution Lemma 78

It can be shown that:

`and[x1,x2]==and[x1,and[x2,or[x1,x3]]]`

Proof

We start by taking Critical Pair Lemma 57, and apply the substitution:

`or[x1_,and[x2_,x1_]]→x1`

which follows from Substitution Lemma 34.

Critical Pair Lemma 58

The following expressions are equivalent:

`and[x1,x2]==and[x1,and[or[x1,x3],x2]]`

Proof

Note that the input for the rule:

`and[x1_,and[x2_,or[x1_,x3_]]]→and[x1,x2]`

contains a subpattern of the form:

`and[x2_,or[x1_,x3_]]`

and[x2_,or[x1_,x3_]]

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Substitution Lemma 78 and Axiom 3 respectively.

Critical Pair Lemma 59

The following expressions are equivalent:

`and[x1,x2]==and[x1,and[x2,or[x3,x1]]]`

Proof

Note that the input for the rule:

`and[x1_,and[x2_,or[x1_,x3_]]]→and[x1,x2]`

contains a subpattern of the form:

`or[x1_,x3_]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,x1_]]→or[x2,x1]`

where these rules follow from Substitution Lemma 78 and Substitution Lemma 40 respectively.

Critical Pair Lemma 60

The following expressions are equivalent:

`and[x1,x2]==and[x1,and[or[x3,x1],x2]]`

Proof

Note that the input for the rule:

`and[x1_,and[or[x1_,x2_],x3_]]→and[x1,x3]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,x1_]]→or[x2,x1]`

where these rules follow from Critical Pair Lemma 58 and Substitution Lemma 40 respectively.

Critical Pair Lemma 61

The following expressions are equivalent:

`nand[x1,and[or[x1,x2],x3]]==nand[or[x4,x1],and[x1,x3]]`

Proof

Note that the input for the rule:

`nand[or[x1_,x2_],and[x2_,x3_]]→nand[x2,x3]`

contains a subpattern of the form:

`and[x2_,x3_]`

which can be unified with the input for the rule:

`and[x1_,and[or[x1_,x2_],x3_]]→and[x1,x3]`

where these rules follow from Critical Pair Lemma 56 and Critical Pair Lemma 58 respectively.

Substitution Lemma 79

It can be shown that

It can be shown that:

`nand[x1,and[or[x1,x2],x3]]==nand[x1,x3]`

PROOF

We start by taking Critical Pair Lemma 61, and apply the substitution:

`nand[or[x1_,x2_],and[x2_,x3_]]→nand[x2,x3]`

which follows from Critical Pair Lemma 56.

Critical Pair Lemma 62

The following expressions are equivalent:

`and[and[x1,x2],x3]==and[and[x1,x2],and[x3,x1]]`

PROOF

Note that the input for the rule:

`and[x1_,and[x2_,or[x3_,x1_]]]→and[x1,x2]`

contains a subpattern of the form:

`or[x3_,x1_]`

which can be unified with the input for the rule:

`or[x1_,and[x1_,x2_]]→x1`

where these rules follow from Critical Pair Lemma 59 and Substitution Lemma 36 respectively.

Critical Pair Lemma 63

The following expressions are equivalent:

`and[and[x1,x2],x3]==and[and[x1,x2],and[x2,x3]]`

PROOF

Note that the input for the rule:

`and[x1_,and[or[x2_,x1_],x3_]]→and[x1,x3]`

contains a subpattern of the form:

`or[x2_,x1_]`

which can be unified with the input for the rule:

`or[x1_,and[x2_,x1_]]→x1`

where these rules follow from Critical Pair Lemma 60 and Substitution Lemma 34 respectively.

Critical Pair Lemma 64

The following expressions are equivalent:

`nand[x1,or[x2,x3]]==nand[x1,or[x2,and[x1,x3]]]`

PROOF

Note that the input for the rule:

`nand[x1_,and[or[x1_,x2_],x3_]]→nand[x1,x3]`

contains a subpattern of the form:

`and[or[x1_,x2_],x3_]`

which can be unified with the input for the rule:

`and[or[x1_,x2_],or[x2_,x3_]]→or[x2,and[x1,x3]]`

where these rules follow from Substitution Lemma 79 and Critical Pair Lemma 3 respectively.

where these rules follow from Substitution Lemma 79 and Critical Pair Lemma 3 respectively.

Critical Pair Lemma 65

The following expressions are equivalent:

`and[and[x1,x2],x3]==and[and[x3,x1],and[x1,x2]]`

Proof

Note that the input for the rule:

`and[and[x1_,x2_],and[x3_,x1_]]→and[and[x1,x2],x3]`

contains a subpattern of the form:

`and[and[x1_,x2_],and[x3_,x1_]]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Critical Pair Lemma 62 and Axiom 3 respectively.

Substitution Lemma 80

It can be shown that:

`and[and[x1,x2],x3]==and[and[x3,x1],x2]`

Proof

We start by taking Critical Pair Lemma 65, and apply the substitution:

`and[and[x1_,x2_],and[x2_,x3_]]→and[and[x1,x2],x3]`

which follows from Critical Pair Lemma 63.

Critical Pair Lemma 66

The following expressions are equivalent:

`and[and[x1,x2],x3]==and[x1,and[x2,x3]]`

Proof

Note that the input for the rule:

`and[and[x1_,x2_],x3_]↔and[and[x3_,x1_],x2_]`

contains a subpattern of the form:

`and[and[x1_,x2_],x3_]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Substitution Lemma 80 and Axiom 3 respectively.

Critical Pair Lemma 67

The following expressions are equivalent:

`and[x1,and[x2,not[and[x1,x2]]]]==0`

Proof

Note that the input for the rule:

`and[and[x1_,x2_],x3_]→and[x1,and[x2,x3]]`

contains a subpattern of the form:

`and[and[x1_,x2_],x3_]`

which can be unified with the input for the rule:

`and[x1_,not[x1_]]→0`

where these rules follow from Critical Pair Lemma 66 and Axiom 5 respectively.

Substitution Lemma 81

It can be shown that:

`and[x1,and[x2,nand[x1,x2]]]==0`

Proof

We start by taking Critical Pair Lemma 67, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Axiom 9.

Critical Pair Lemma 68

The following expressions are equivalent:

`or[x1,and[x2,nand[not[x1],x2]]]==or[x1,0]`

Proof

Note that the input for the rule:

`or[x1_,and[not[x1_],x2_]]→or[x1,x2]`

contains a subpattern of the form:

`and[not[x1_],x2_]`

which can be unified with the input for the rule:

`and[x1_,and[x2_,nand[x1_,x2_]]]→0`

where these rules follow from Substitution Lemma 59 and Substitution Lemma 81 respectively.

Substitution Lemma 82

It can be shown that:

`or[x1,and[x2,nand[not[x1],x2]]]==x1`

Proof

We start by taking Critical Pair Lemma 68, and apply the substitution:

`or[x1_,0]→x1`

which follows from Substitution Lemma 1.

Critical Pair Lemma 69

The following expressions are equivalent:

`nand[x1,or[x2,nand[not[x2],x1]]]==nand[x1,x2]`

Proof

Note that the input for the rule:

`nand[x1_,or[x2_,and[x1_,x3_]]]→nand[x1,or[x2,x3]]`

contains a subpattern of the form:

`or[x2_,and[x1_,x3_]]`

which can be unified with the input for the rule:

`or[x1_,and[x2_,nand[not[x1_],x2_]]]→x1`

where these rules follow from Critical Pair Lemma 64 and Substitution Lemma 82 respectively.

Substitution Lemma 83

It can be shown that:

`nand[x1,nand[not[x2],x1]]==nand[x1,x2]`

Proof

We start by taking Critical Pair Lemma 69, and apply the substitution:

`or[x1_,nand[not[x1_],x2_]]→nand[not[x1],x2]`

which follows from Substitution Lemma 75.

Critical Pair Lemma 70

The following expressions are equivalent:

`nand[x1,not[x2]]==nand[x1,nand[x2,x1]]`

Proof

Note that the input for the rule:

`nand[x1_,nand[not[x2_],x1_]]→nand[x1,x2]`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Substitution Lemma 83 and Substitution Lemma 53 respectively.

Critical Pair Lemma 71

The following expressions are equivalent:

`nand[x1,not[x2]]==nand[x1,nand[x1,x2]]`

Proof

Note that the input for the rule:

`nand[x1_,nand[x2_,x1_]]→nand[x1,not[x2]]`

contains a subpattern of the form:

`nand[x2_,x1_]`

which can be unified with the input for the rule:

`nand[x1_,x2_]↔nand[x2_,x1_]`

where these rules follow from Critical Pair Lemma 70 and Substitution Lemma 3 respectively.

Critical Pair Lemma 72

The following expressions are equivalent:

`nand[x1,not[or[x2,not[x1]]]]==nand[x1,nand[x2,x1]]`

Proof

Note that the input for the rule:

`nand[x1_,nand[x1_,x2_]]→nand[x1,not[x2]]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,or[x2_,not[x1_]]]→nand[x2,x1]`

where these rules follow from Critical Pair Lemma 71 and Substitution Lemma 46 respectively.

Substitution Lemma 84

It can be shown that:

`or[x1,not[x2]]==nand[x2,nand[x1,x2]]`

Proof

We start by taking Critical Pair Lemma 72, and apply the substitution:

`nand[x1_,not[or[x2_,not[x1_]]]]→or[x2,not[x1]]`

which follows from Substitution Lemma 77.

Substitution Lemma 85

It can be shown that:

`or[x1,not[x2]]==nand[x2,not[x1]]`

Proof

We start by taking Substitution Lemma 84, and apply the substitution:

`nand[x1_,nand[x2_,x1_]]→nand[x1,not[x2]]`

which follows from Critical Pair Lemma 70.

Critical Pair Lemma 73

The following expressions are equivalent:

`nand[not[x1],not[or[x1,x2]]]==nand[not[x1],nand[not[x1],x2]]`

Proof

Note that the input for the rule:

`nand[x1_,nand[x1_,x2_]]→nand[x1,not[x2]]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[not[x1_],or[x1_,x2_]]→nand[not[x1],x2]`

where these rules follow from Critical Pair Lemma 71 and Substitution Lemma 49 respectively.

Substitution Lemma 86

It can be shown that:

`or[x1,x2]==nand[not[x1],nand[not[x1],x2]]`

Proof

We start by taking Critical Pair Lemma 73, and apply the substitution:

`nand[not[x1_],not[or[x1_,x2_]]]→or[x1,x2]`

which follows from Substitution Lemma 76.

Substitution Lemma 87

It can be shown that:

It can be shown that:

`or[x1,x2]==nand[not[x1],not[x2]]`

PROOF

We start by taking Substitution Lemma 86, and apply the substitution:

`nand[x1_,nand[x1_,x2_]]→nand[x1,not[x2]]`

which follows from Critical Pair Lemma 71.

Critical Pair Lemma 74

The following expressions are equivalent:

`and[not[x1],not[x2]]==not[or[x1,x2]]`

PROOF

Note that the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[not[x1_],not[x2_]]→or[x1,x2]`

where these rules follow from Critical Pair Lemma 42 and Substitution Lemma 87 respectively.

Critical Pair Lemma 75

The following expressions are equivalent:

`not[or[and[x1,x2],x3]]==and[nand[x1,x2],not[x3]]`

PROOF

Note that the input for the rule:

`and[not[x1_],not[x2_]]→not[or[x1,x2]]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

where these rules follow from Critical Pair Lemma 74 and Axiom 9 respectively.

Critical Pair Lemma 76

The following expressions are equivalent:

`and[nand[x1,x2],not[and[x3,x1]]]==not[and[x1,or[x2,x3]]]`

PROOF

Note that the input for the rule:

`not[or[and[x1_,x2_],x3_]]→and[nand[x1,x2],not[x3]]`

contains a subpattern of the form:

`or[and[x1_,x2_],x3_]`

which can be unified with the input for the rule:

`or[and[x1_,x2_],and[x3_,x1_]]→and[x1,or[x2,x3]]`

where these rules follow from Critical Pair Lemma 75 and Critical Pair Lemma 2 respectively.

where these rules follow from Critical Pair Lemma 75 and Critical Pair Lemma 2 respectively.

Substitution Lemma 88

It can be shown that:

`and[nand[x1,x2],nand[x3,x1]]==not[and[x1,or[x2,x3]]]`

Proof

We start by taking Critical Pair Lemma 76, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Axiom 9.

Substitution Lemma 89

It can be shown that:

`and[nand[x1,x2],nand[x3,x1]]==nand[x1,or[x2,x3]]`

Proof

We start by taking Substitution Lemma 88, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Axiom 9.

Critical Pair Lemma 77

The following expressions are equivalent:

`nand[nand[x1,x2],nand[x3,x1]]==not[nand[x1,or[x2,x3]]]`

Proof

Note that the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[nand[x1_,x2_],nand[x3_,x1_]]→nand[x1,or[x2,x3]]`

where these rules follow from Axiom 9 and Substitution Lemma 89 respectively.

Substitution Lemma 90

It can be shown that:

`nand[nand[x1,x2],nand[x3,x1]]==and[x1,or[x2,x3]]`

Proof

We start by taking Critical Pair Lemma 77, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Critical Pair Lemma 42.

Substitution Lemma 91

It can be shown that:

`nand[nand[nand[b,b],a],nand[a,nand[c,c]]]==nand[nand[a,nand[b,c]],nand[a,nand[b,c]]]`

Proof

We start by taking Hypothesis 1, and apply the substitution:

`nand[x1_,x2_]→nand[x2,x1]`

which follows from Substitution Lemma 3.

Substitution Lemma 92

It can be shown that:

`nand[nand[nand[b,b],a],nand[a,nand[c,c]]]==nand[nand[nand[b,c],a],nand[a,nand[b,c]]]`

Proof

We start by taking Substitution Lemma 91, and apply the substitution:

`nand[x1_,x2_]→nand[x2,x1]`

which follows from Substitution Lemma 3.

Substitution Lemma 93

It can be shown that:

`nand[nand[a,nand[c,c]],nand[nand[b,b],a]]==nand[nand[nand[b,c],a],nand[a,nand[b,c]]]`

Proof

We start by taking Substitution Lemma 92, and apply the substitution:

`nand[x1_,x2_]→nand[x2,x1]`

which follows from Substitution Lemma 3.

Substitution Lemma 94

It can be shown that:

`nand[nand[a,nand[c,c]],nand[not[b],a]]==nand[nand[nand[b,c],a],nand[a,nand[b,c]]]`

Proof

We start by taking Substitution Lemma 93, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Critical Pair Lemma 22.

Substitution Lemma 95

It can be shown that:

`nand[nand[a,not[c]],nand[not[b],a]]==nand[nand[nand[b,c],a],nand[a,nand[b,c]]]`

Proof

We start by taking Substitution Lemma 94, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Critical Pair Lemma 22.

Substitution Lemma 96

It can be shown that:

`nand[nand[a,not[c]],nand[not[b],a]]==and[nand[b,c],a]`

Proof

We start by taking Substitution Lemma 95, and apply the substitution:

`nand[nand[x1_,x2_],nand[x2_,x1_]]→and[x1,x2]`

which follows from Substitution Lemma 58.

Substitution Lemma 97

It can be shown that:

`and[a,or[not[c],not[b]]]==and[nand[b,c],a]`

Proof

We start by taking Substitution Lemma 96, and apply the substitution:

`nand[nand[x1_,x2_],nand[x3_,x1_]]→and[x1,or[x2,x3]]`

which follows from Substitution Lemma 90.

Substitution Lemma 98

It can be shown that:

`and[or[not[c],not[b]],a]==and[nand[b,c],a]`

Proof

We start by taking Substitution Lemma 97, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Axiom 3.

Substitution Lemma 99

It can be shown that:

`and[nand[b,not[not[c]]],a]==and[nand[b,c],a]`

Proof

We start by taking Substitution Lemma 98, and apply the substitution:

`or[x1_,not[x2_]]→nand[x2,not[x1]]`

which follows from Substitution Lemma 85.

Conclusion 3

We obtain the conclusion:

`True`

Proof

Take Substitution Lemma 99, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 53.

```
In[∘]:= Clear[proofShefferfromBoolean]
```

Appendix 11. Derivation of equational Boolean logic from Sheffer logic.

In[ ]:= `proofAxB1fromSheffer["ProofNotebook"]`

Axiom 1

We are given that:

`x1==nand[nand[x1,x1],nand[x1,x1]]`

Axiom 2

We are given that:

`nand[x1,x1]==not[x1]`

Axiom 3

We are given that:

`nand[nand[x1,x1],nand[x2,x2]]==or[x1,x2]`

Axiom 4

We are given that:

`nand[nand[x1,nand[x2,x3]],nand[x1,nand[x2,x3]]]==nand[nand[nand[x2,x2],x1],nand[nand[x3,`

Axiom 5

We are given that:

`not[nand[x1,x2]]==and[x1,x2]`

Hypothesis 1

We would like to show that:

`and[a,b]==and[b,a]`

Substitution Lemma 1

It can be shown that:

`not[nand[x1,x1]]==x1`

Proof

We start by taking Axiom 1, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Substitution Lemma 2

It can be shown that:

`not[not[x1]]==x1`

Proof

We start by taking Substitution Lemma 1, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Substitution Lemma 3

It can be shown that:

`nand[not[x1],nand[x2,x2]]==or[x1,x2]`

Proof

We start by taking Axiom 3, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Substitution Lemma 4

It can be shown that:

`nand[not[x1],not[x2]]==or[x1,x2]`

Proof

We start by taking Substitution Lemma 3, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Substitution Lemma 5

It can be shown that:

`not[nand[x1,nand[x2,x3]]]==nand[nand[nand[x2,x2],x1],nand[nand[x3,x3],x1]]`

Proof

We start by taking Axiom 4, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Substitution Lemma 6

It can be shown that:

`not[nand[x1,nand[x2,x3]]]==nand[nand[not[x2],x1],nand[nand[x3,x3],x1]]`

Proof

We start by taking Substitution Lemma 5, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Substitution Lemma 7

It can be shown that:

`not[nand[x1,nand[x2,x3]]]==nand[nand[not[x2],x1],nand[not[x3],x1]]`

Proof

We start by taking Substitution Lemma 6, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Substitution Lemma 8

Substitution Lemma 8

It can be shown that:

`nand[nand[not[x1_],x2_],nand[not[x3_],x2_]]→and[x2,nand[x1,x3]]`

PROOF

We start by taking Substitution Lemma 7, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 5.

Critical Pair Lemma 1

The following expressions are equivalent:

`or[not[x1],x2]==nand[x1,not[x2]]`

PROOF

Note that the input for the rule:

`nand[not[x1_],not[x2_]]→or[x1,x2]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Substitution Lemma 4 and Substitution Lemma 2 respectively.

Critical Pair Lemma 2

The following expressions are equivalent:

`or[not[x1],not[x2]]==nand[x1,x2]`

PROOF

Out[ ]=

Note that the input for the rule:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Critical Pair Lemma 1 and Substitution Lemma 2 respectively.

Critical Pair Lemma 3

The following expressions are equivalent:

`nand[not[x1],x2]==or[x1,not[x2]]`

PROOF

Note that the input for the rule:

`or[not[x1_],not[x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

`not[not[x1_]]→x1`

where these rules follow from Critical Pair Lemma 2 and Substitution Lemma 2 respectively.

Critical Pair Lemma 4

The following expressions are equivalent:

`and[not[x1],x2]==not[or[x1,not[x2]]]`

Proof

Note that the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

where these rules follow from Axiom 5 and Critical Pair Lemma 3 respectively.

Substitution Lemma 9

It can be shown that:

`nand[or[x1,not[x2]],nand[not[x3],x2]]==and[x2,nand[x1,x3]]`

Proof

We start by taking Substitution Lemma 8, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 3.

Substitution Lemma 10

It can be shown that:

`nand[or[x1,not[x2]],or[x3,not[x2]]]==and[x2,nand[x1,x3]]`

Proof

We start by taking Substitution Lemma 9, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 3.

Critical Pair Lemma 5

The following expressions are equivalent:

`and[x1,nand[x2,x2]]==not[or[x2,not[x1]]]`

Proof

Note that the input for the rule:

`nand[or[x1_,not[x2_]],or[x3_,not[x2_]]]→and[x2,nand[x1,x3]]`

contains a subpattern of the form:

`nand[or[x1_,not[x2_]],or[x3_,not[x2_]]]`

which can be unified with the input for the rule:

`nand[x1_,x1_]→not[x1]`

where these rules follow from Substitution Lemma 10 and Axiom 2 respectively.

Substitution Lemma 11

It can be shown that:

`and[x1,not[x2]]==not[or[x2,not[x1]]]`

Proof

We start by taking Critical Pair Lemma 5, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Substitution Lemma 12

It can be shown that:

`and[x1,not[x2]]==and[not[x2],x1]`

Proof

We start by taking Substitution Lemma 11, and apply the substitution:

`not[or[x1_,not[x2_]]]→and[not[x1],x2]`

which follows from Critical Pair Lemma 4.

Critical Pair Lemma 6

The following expressions are equivalent:

`and[not[not[x1]],x2]==and[x2,x1]`

Proof

Note that the input for the rule:

`and[x1_,not[x2_]]↔and[not[x2_],x1_]`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Substitution Lemma 12 and Substitution Lemma 2 respectively.

Substitution Lemma 13

It can be shown that:

`and[x1,x2]==and[x2,x1]`

Proof

We start by taking Critical Pair Lemma 6, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 2.

Conclusion 1

We obtain the conclusion:

`True`

Proof

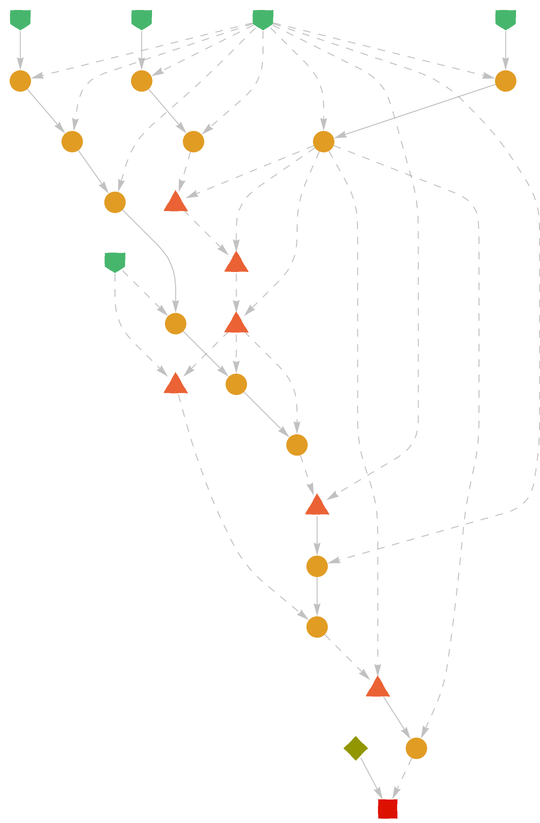
Take Hypothesis 1, and apply the substitution:

and[x1_,x2_]→and[x2,x1]

which follows from Substitution Lemma 13.

In[∘]:= proofAxB1fromSheffer["ProofGraph"]

Out[∘]=



In[∘]:= Clear[proofAxB1fromSheffer]

In[∘]:= proofAxB2fromSheffer["ProofNotebook"]

Axiom 1

We are given that:

x1==nand[nand[x1,x1],nand[x1,x1]]

Axiom 2

We are given that:

nand[x1,x1]==not[x1]

Axiom 3

We are given that:

nand[nand[x1,x1],nand[x2,x2]]==or[x1,x2]

Axiom 4

We are given that:

We are given that:

```
nand[nand[x1,nand[x2,x3]],nand[x1,nand[x2,x3]]]==nand[nand[nand[x2,x2],x1],nand[nand[x3,
```

Axiom 5

We are given that:

```
not[nand[x1,x2]]==and[x1,x2]
```

Hypothesis 1

We would like to show that:

```
or[a,b]==or[b,a]
```

Substitution Lemma 1

It can be shown that:

```
not[nand[x1,x1]]==x1
```

Proof

We start by taking Axiom 1, and apply the substitution:

```
nand[x1_,x1_]→not[x1]
```

which follows from Axiom 2.

Substitution Lemma 2

It can be shown that:

```
not[not[x1]]==x1
```

Proof

We start by taking Substitution Lemma 1, and apply the substitution:

```
nand[x1_,x1_]→not[x1]
```

which follows from Axiom 2.

Substitution Lemma 3

It can be shown that:

```
nand[not[x1],nand[x2,x2]]==or[x1,x2]
```

Proof

We start by taking Axiom 3, and apply the substitution:

```
nand[x1_,x1_]→not[x1]
```

which follows from Axiom 2.

Substitution Lemma 4

It can be shown that:

```
nand[not[x1],not[x2]]==or[x1,x2]
```

Proof

We start by taking Substitution Lemma 3, and apply the substitution:

```
nand[x1_,x1_]→not[x1]
```

which follows from Axiom 2.

Substitution Lemma 5

It can be shown that:

`not[nand[x1,nand[x2,x3]]]==nand[nand[nand[x2,x2],x1],nand[nand[x3,x3],x1]]`

Proof

We start by taking Axiom 4, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Substitution Lemma 6

It can be shown that:

`not[nand[x1,nand[x2,x3]]]==nand[nand[not[x2],x1],nand[nand[x3,x3],x1]]`

Proof

We start by taking Substitution Lemma 5, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Substitution Lemma 7

It can be shown that:

`not[nand[x1,nand[x2,x3]]]==nand[nand[not[x2],x1],nand[not[x3],x1]]`

Proof

We start by taking Substitution Lemma 6, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Substitution Lemma 8

It can be shown that:

`nand[nand[not[x1_],x2_],nand[not[x3_],x2_]]→and[x2,nand[x1,x3]]`

Proof

We start by taking Substitution Lemma 7, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 5.

Critical Pair Lemma 1

The following expressions are equivalent:

`or[not[x1],x2]==nand[x1,not[x2]]`

Proof

Note that the input for the rule:

`nand[not[x1_],not[x2_]]→or[x1,x2]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Substitution Lemma 4 and Substitution Lemma 2 respectively.

where these rules follow from Substitution Lemma 4 and Substitution Lemma 2 respectively.

Critical Pair Lemma 2

The following expressions are equivalent:

`or[not[x1],not[x2]]==nand[x1,x2]`

Proof

Note that the input for the rule:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Critical Pair Lemma 1 and Substitution Lemma 2 respectively.

Critical Pair Lemma 3

The following expressions are equivalent:

`and[x1,not[x2]]==not[or[not[x1],x2]]`

Proof

Note that the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

where these rules follow from Axiom 5 and Critical Pair Lemma 1 respectively.

Critical Pair Lemma 4

The following expressions are equivalent:

`nand[not[x1],x2]==or[x1,not[x2]]`

Proof

Note that the input for the rule:

`or[not[x1_],not[x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Critical Pair Lemma 2 and Substitution Lemma 2 respectively.

Out[ ]=

Critical Pair Lemma 5

The following expressions are equivalent:

`and[not[x1],x2]==not[or[x1,not[x2]]]`

Proof

Note that the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

where these rules follow from Axiom 5 and Critical Pair Lemma 4 respectively.

Substitution Lemma 9

It can be shown that:

`nand[or[x1,not[x2]],nand[not[x3],x2]]==and[x2,nand[x1,x3]]`

Proof

We start by taking Substitution Lemma 8, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 4.

Substitution Lemma 10

It can be shown that:

`nand[or[x1,not[x2]],or[x3,not[x2]]]==and[x2,nand[x1,x3]]`

Proof

We start by taking Substitution Lemma 9, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 4.

Critical Pair Lemma 6

The following expressions are equivalent:

`and[not[x1],not[x2]]==not[or[x1,x2]]`

Proof

Note that the input for the rule:

`not[or[not[x1_],x2_]]→and[x1,not[x2]]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Critical Pair Lemma 3 and Substitution Lemma 2 respectively.

Critical Pair Lemma 7

The following expressions are equivalent:

`and[x1,nand[x2,x2]]==not[or[x2,not[x1]]]`

Proof

Note that the input for the rule:

`nand[or[x1_,not[x2_]],or[x3_,not[x2_]]]→and[x2,nand[x1,x3]]` `nand[or[x1_,not[x2_]],or[x3_,not[x2_]]]→and[x2,nand[x1,x3]]`

contains a subpattern of the form:

`nand[or[x1_,not[x2_]],or[x3_,not[x2_]]]`

which can be unified with the input for the rule:

`nand[x1_,x1_]→not[x1]`

where these rules follow from Substitution Lemma 10 and Axiom 2 respectively.

Substitution Lemma 11

It can be shown that:

`and[x1,not[x2]]==not[or[x2,not[x1]]]`

Proof

We start by taking Critical Pair Lemma 7, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Substitution Lemma 12

It can be shown that:

`and[x1,not[x2]]==and[not[x2],x1]`

Proof

We start by taking Substitution Lemma 11, and apply the substitution:

`not[or[x1_,not[x2_]]]→and[not[x1],x2]`

which follows from Critical Pair Lemma 5.

Critical Pair Lemma 8

The following expressions are equivalent:

`and[not[not[x1]],x2]==and[x2,x1]`

Proof

Note that the input for the rule:

`and[x1_,not[x2_]]↔and[not[x2_],x1_]`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Substitution Lemma 12 and Substitution Lemma 2 respectively.

Substitution Lemma 13

It can be shown that:

`and[x1,x2]==and[x2,x1]`

Proof

We start by taking Critical Pair Lemma 8, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 2.

Critical Pair Lemma 9

The following expressions are equivalent:

`and[not[x1],not[x2]]==not[or[x2,x1]]`

Proof

Note that the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[not[x1_],not[x2_]]→not[or[x1,x2]]`

where these rules follow from Substitution Lemma 13 and Critical Pair Lemma 6 respectively.

Substitution Lemma 14

It can be shown that:

`not[or[x1,x2]]==not[or[x2,x1]]`

Proof

We start by taking Critical Pair Lemma 9, and apply the substitution:

`and[not[x1_],not[x2_]]→not[or[x1,x2]]`

which follows from Critical Pair Lemma 6.

Critical Pair Lemma 10

The following expressions are equivalent:

`or[x1,x2]==not[not[or[x2,x1]]]`

Proof

Note that the input for the rule:

`not[not[x1_]]→x1`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[or[x1_,x2_]]↔not[or[x2_,x1_]]`

where these rules follow from Substitution Lemma 2 and Substitution Lemma 14 respectively.

Substitution Lemma 15

It can be shown that:

`or[x1,x2]==or[x2,x1]`

Proof

We start by taking Critical Pair Lemma 10, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 2.

Conclusion 1

We obtain the conclusion:

```
True
```

Proof

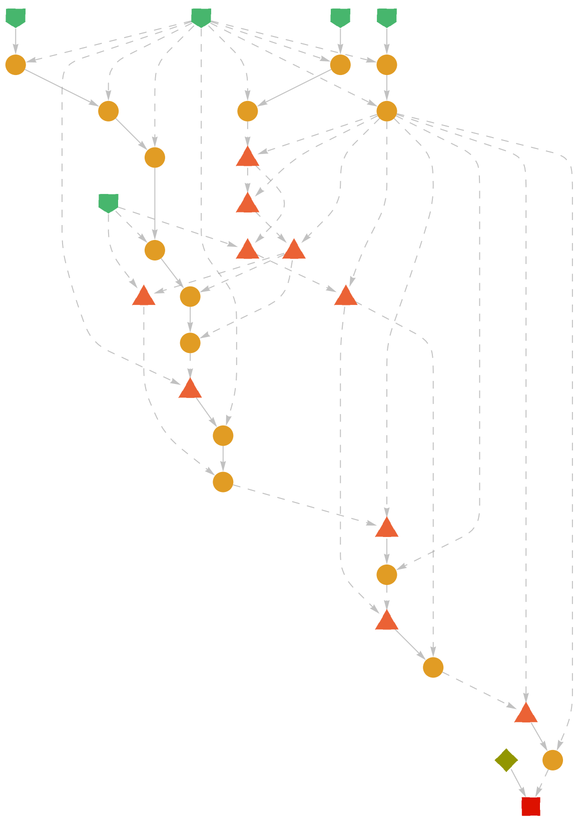
Take Hypothesis 1, and apply the substitution:

```
or[x1_,x2_]→or[x2,x1]
```

which follows from Substitution Lemma 15.

In[ ]:=
```
proofAxB2fromSheffer["ProofGraph"]
```

Out[ ]=



In[ ]:=
```
Clear[proofAxB2fromSheffer]
```

In[ ]:=
```
proofAxB3fromSheffer["ProofNotebook"]
```

Axiom 1

We are given that:

```
x1==nand[nand[x1,x1],nand[x1,x1]]
```

Axiom 2

We are given that:

```
nand[x1,x1]==nand[x1,nand[x2,nand[x2,x2]]]
```

Axiom 3

We are given that:

`nand[x1,x1]==not[x1]`

Axiom 4

We are given that:

`nand[nand[x1,x1],nand[x2,x2]]==or[x1,x2]`

Axiom 5

We are given that:

`not[nand[x1,x2]]==and[x1,x2]`

Hypothesis 1

We would like to show that:

`and[a,or[b,not[b]]]==a`

Substitution Lemma 1

It can be shown that:

`not[x1]==nand[x1,nand[x2,nand[x2,x2]]]`

Proof

We start by taking Axiom 2, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 3.

Substitution Lemma 2

It can be shown that:

`not[x1]==nand[x1,nand[x2,not[x2]]]`

Proof

We start by taking Substitution Lemma 1, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 3.

Substitution Lemma 3

It can be shown that:

`not[nand[x1,x1]]==x1`

Proof

We start by taking Axiom 1, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 3.

Substitution Lemma 4

It can be shown that:

`not[not[x1]]==x1`

Proof

We start by taking Substitution Lemma 3, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 3.

Substitution Lemma 5

It can be shown that:

`nand[not[x1],nand[x2,x2]]==or[x1,x2]`

Proof

We start by taking Axiom 4, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 3.

Substitution Lemma 6

It can be shown that:

`nand[not[x1],not[x2]]==or[x1,x2]`

Proof

We start by taking Substitution Lemma 5, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 3.

Critical Pair Lemma 1

The following expressions are equivalent:

`or[not[x1],x2]==nand[x1,not[x2]]`

Proof

Note that the input for the rule:

`nand[not[x1_],not[x2_]]→or[x1,x2]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

Out[ ]= where these rules follow from Substitution Lemma 6 and Substitution Lemma 4 respectively.

Critical Pair Lemma 2

The following expressions are equivalent:

`or[not[x1],not[x2]]==nand[x1,x2]`

Proof

Note that the input for the rule:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Critical Pair Lemma 1 and Substitution Lemma 4 respectively.

where these rules follow from Critical Pair Lemma 1 and Substitution Lemma 4 respectively.

Critical Pair Lemma 3

The following expressions are equivalent:

`nand[not[x1],x2]==or[x1,not[x2]]`

Proof

Note that the input for the rule:

`or[not[x1_],not[x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Critical Pair Lemma 2 and Substitution Lemma 4 respectively.

Substitution Lemma 7

It can be shown that:

`not[x1]==nand[x1,or[not[x2],x2]]`

Proof

We start by taking Substitution Lemma 2, and apply the substitution:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 1.

Critical Pair Lemma 4

The following expressions are equivalent:

`and[x1,or[not[x2],x2]]==not[not[x1]]`

Proof

Note that the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,or[not[x2_],x2_]]→not[x1]`

where these rules follow from Axiom 5 and Substitution Lemma 7 respectively.

Substitution Lemma 8

It can be shown that:

`and[x1,or[not[x2],x2]]==x1`

Proof

We start by taking Critical Pair Lemma 4, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 4.

Critical Pair Lemma 5

The following expressions are equivalent:

```
x1==and[x1,nand[not[x2],x2]]
```

Proof

Note that the input for the rule:

```
and[x1_,or[not[x2_],x2_]]→x1
```

contains a subpattern of the form:

```
or[not[x2_],x2_]
```

which can be unified with the input for the rule:

```
or[not[x1_],not[x2_]]→nand[x1,x2]
```

where these rules follow from Substitution Lemma 8 and Critical Pair Lemma 2 respectively.

Substitution Lemma 9

It can be shown that:

```
x1==and[x1,or[x2,not[x2]]]
```

Proof

We start by taking Critical Pair Lemma 5, and apply the substitution:

```
nand[not[x1_],x2_]→or[x1,not[x2]]
```

which follows from Critical Pair Lemma 3.

Conclusion 1

We obtain the conclusion:

```
True
```

Proof

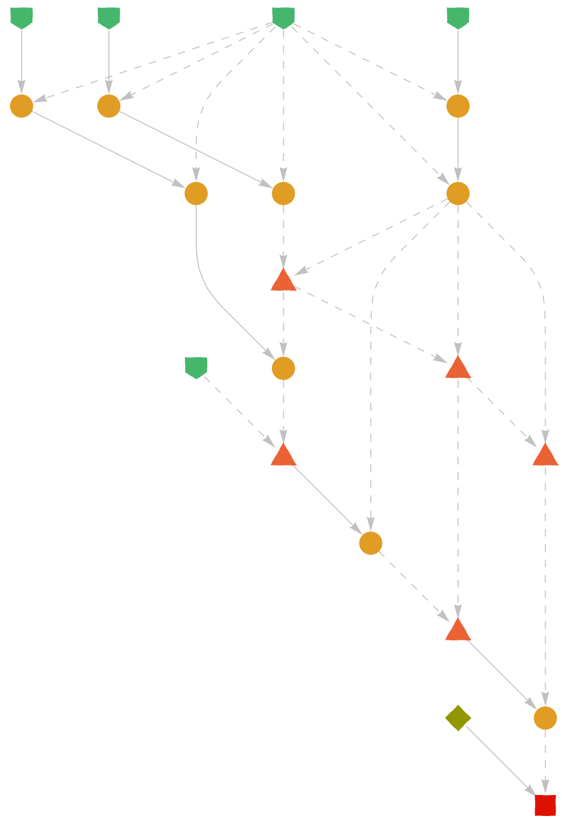
Take Hypothesis 1, and apply the substitution:

```
and[x1_,or[x2_,not[x2_]]]→x1
```

which follows from Substitution Lemma 9.

```
In[◦]:= proofAxB3fromSheffer["ProofGraph"]
```

Out[∘]=



In[∘]:=
```
Clear[proofAxB3fromSheffer]
```

In[∘]:=
```
proofAxB4fromSheffer["ProofNotebook"]
```

Axiom 1

We are given that:

```
x1==nand[nand[x1,x1],nand[x1,x1]]
```

Axiom 2

We are given that:

```
nand[x1,x1]==nand[x1,nand[x2,nand[x2,x2]]]
```

Axiom 3

We are given that:

```
nand[x1,x1]==not[x1]
```

Axiom 4

We are given that:

```
nand[nand[x1,x1],nand[x2,x2]]==or[x1,x2]
```

Axiom 5

We are given that:

```
not[nand[x1,x2]]==and[x1,x2]
```

Hypothesis 1

We would like to show that:

`or[a,and[b,not[b]]]==a`

Substitution Lemma 1

It can be shown that:

`not[x1]==nand[x1,nand[x2,nand[x2,x2]]]`

Proof

We start by taking Axiom 2, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 3.

Substitution Lemma 2

It can be shown that:

`not[x1]==nand[x1,nand[x2,not[x2]]]`

Proof

We start by taking Substitution Lemma 1, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 3.

Substitution Lemma 3

It can be shown that:

`not[nand[x1,x1]]==x1`

Proof

We start by taking Axiom 1, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 3.

Substitution Lemma 4

It can be shown that:

`not[not[x1]]==x1`

Proof

We start by taking Substitution Lemma 3, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 3.

Substitution Lemma 5

It can be shown that:

`nand[not[x1],nand[x2,x2]]==or[x1,x2]`

Proof

We start by taking Axiom 4, and apply the substitution:

```
nand[x1_,x1_]→not[x1]
```

which follows from Axiom 3.

Substitution Lemma 6

It can be shown that:

```
nand[not[x1],not[x2]]==or[x1,x2]
```

Proof

We start by taking Substitution Lemma 5, and apply the substitution:

```
nand[x1_,x1_]→not[x1]
```

which follows from Axiom 3.

Critical Pair Lemma 1

The following expressions are equivalent:

```
or[not[x1],x2]==nand[x1,not[x2]]
```

Proof

Note that the input for the rule:

```
nand[not[x1_],not[x2_]]→or[x1,x2]
```

contains a subpattern of the form:

```
not[x1_]
```

which can be unified with the input for the rule:

```
not[not[x1_]]→x1
```

Out[ ]= where these rules follow from Substitution Lemma 6 and Substitution Lemma 4 respectively.

Critical Pair Lemma 2

The following expressions are equivalent:

```
or[not[x1],not[x2]]==nand[x1,x2]
```

Proof

Note that the input for the rule:

```
nand[x1_,not[x2_]]→or[not[x1],x2]
```

contains a subpattern of the form:

```
not[x2_]
```

which can be unified with the input for the rule:

```
not[not[x1_]]→x1
```

where these rules follow from Critical Pair Lemma 1 and Substitution Lemma 4 respectively.

Critical Pair Lemma 3

The following expressions are equivalent:

```
and[x1,not[x2]]==not[or[not[x1],x2]]
```

Proof

Note that the input for the rule:

```
not[nand[x1_,x2_]]→and[x1,x2]
```

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

where these rules follow from Axiom 5 and Critical Pair Lemma 1 respectively.

Critical Pair Lemma 4

The following expressions are equivalent:

`nand[not[x1],x2]==or[x1,not[x2]]`

Proof

Note that the input for the rule:

`or[not[x1_],not[x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Critical Pair Lemma 2 and Substitution Lemma 4 respectively.

Substitution Lemma 7

It can be shown that:

`not[x1]==nand[x1,or[not[x2],x2]]`

Proof

We start by taking Substitution Lemma 2, and apply the substitution:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 1.

Critical Pair Lemma 5

The following expressions are equivalent:

`not[not[x1]]==or[x1,not[or[not[x2],x2]]]`

Proof

Note that the input for the rule:

`nand[x1_,or[not[x2_],x2_]]→not[x1]`

contains a subpattern of the form:

`nand[x1_,or[not[x2_],x2_]]`

which can be unified with the input for the rule:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

where these rules follow from Substitution Lemma 7 and Critical Pair Lemma 4 respectively.

Substitution Lemma 8

It can be shown that:

`x1==or[x1,not[or[not[x2],x2]]]`

Proof

We start by taking Critical Pair Lemma 5, and apply the substitution:

We start by taking Critical Pair Lemma 5, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 4.

Substitution Lemma 9

It can be shown that:

`x1==or[x1,and[x2,not[x2]]]`

Proof

We start by taking Substitution Lemma 8, and apply the substitution:

`not[or[not[x1_],x2_]]→and[x1,not[x2]]`

which follows from Critical Pair Lemma 3.

Conclusion 1

We obtain the conclusion:

`True`

Proof

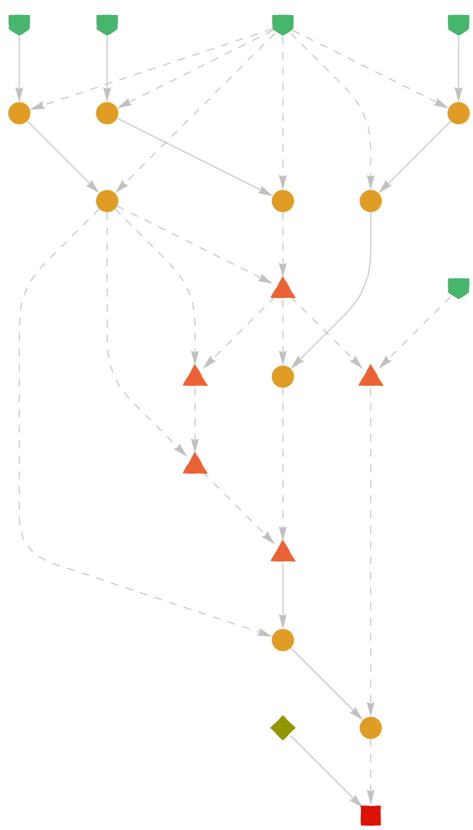
Take Hypothesis 1, and apply the substitution:

`or[x1_,and[x2_,not[x2_]]]→x1`

which follows from Substitution Lemma 9.

In[∘]:= `proofAxB4fromSheffer["ProofGraph"]`

Out[∘]=



In[ ]:= `Clear[proofAxB4fromSheffer]`

In[ ]:= `proofAxB5fromSheffer["ProofNotebook"]`

Axiom 1

We are given that:

`x1==nand[nand[x1,x1],nand[x1,x1]]`

Axiom 2

We are given that:

`nand[x1,x1]==nand[x1,nand[x2,nand[x2,x2]]]`

Axiom 3

We are given that:

`nand[x1,x1]==not[x1]`

Axiom 4

We are given that:

`nand[nand[x1,x1],nand[x2,x2]]==or[x1,x2]`

Axiom 5

We are given that:

`nand[nand[x1,nand[x2,x3]],nand[x1,nand[x2,x3]]]==nand[nand[nand[x2,x2],x1],nand[nand[x3,`

Axiom 6

We are given that:

`not[nand[x1,x2]]==and[x1,x2]`

Hypothesis 1

We would like to show that:

`or[and[a,b],and[a,c]]==and[a,or[b,c]]`

Substitution Lemma 1

It can be shown that:

`not[x1]==nand[x1,nand[x2,nand[x2,x2]]]`

Proof

We start by taking Axiom 2, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 3.

Substitution Lemma 2

It can be shown that:

`not[x1]==nand[x1,nand[x2,not[x2]]]`

Proof

We start by taking Substitution Lemma 1, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 3.

Substitution Lemma 3

It can be shown that:

`not[nand[x1,x1]]==x1`

Proof

We start by taking Axiom 1, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 3.

Substitution Lemma 4

It can be shown that:

`not[not[x1]]==x1`

Proof

We start by taking Substitution Lemma 3, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 3.

Substitution Lemma 5

It can be shown that:

`nand[not[x1],nand[x2,x2]]==or[x1,x2]`

Proof

We start by taking Axiom 4, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 3.

Substitution Lemma 6

It can be shown that:

`nand[not[x1],not[x2]]==or[x1,x2]`

Proof

We start by taking Substitution Lemma 5, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 3.

Substitution Lemma 7

It can be shown that:

`not[nand[x1,nand[x2,x3]]]==nand[nand[nand[x2,x2],x1],nand[nand[x3,x3],x1]]`

Proof

We start by taking Axiom 5, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 3.

which follows from Axiom 3.

Substitution Lemma 8

It can be shown that:

`not[nand[x1,nand[x2,x3]]]==nand[nand[not[x2],x1],nand[nand[x3,x3],x1]]`

Proof

We start by taking Substitution Lemma 7, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 3.

Substitution Lemma 9

It can be shown that:

`not[nand[x1,nand[x2,x3]]]==nand[nand[not[x2],x1],nand[not[x3],x1]]`

Proof

We start by taking Substitution Lemma 8, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 3.

Substitution Lemma 10

It can be shown that:

`nand[nand[not[x1_],x2_],nand[not[x3_],x2_]]→and[x2,nand[x1,x3]]`

Proof

We start by taking Substitution Lemma 9, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Critical Pair Lemma 1

The following expressions are equivalent:

`and[x1,x1]==not[not[x1]]`

Proof

Note that the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,x1_]→not[x1]`

where these rules follow from Axiom 6 and Axiom 3 respectively.

Critical Pair Lemma 2

The following expressions are equivalent:

`nand[x1,x2]==not[and[x1,x2]]`

Proof

Note that the input for the rule:

`not[not[x1_]]→x1`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

where these rules follow from Substitution Lemma 4 and Axiom 6 respectively.

Critical Pair Lemma 3

The following expressions are equivalent:

`or[not[x1],x2]==nand[x1,not[x2]]`

Proof

Note that the input for the rule:

`nand[not[x1_],not[x2_]]→or[x1,x2]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Substitution Lemma 6 and Substitution Lemma 4 respectively.

Critical Pair Lemma 4

The following expressions are equivalent:

`or[not[x1],and[x2,x3]]==nand[x1,nand[x2,x3]]`

Proof

Note that the input for the rule:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

where these rules follow from Critical Pair Lemma 3 and Critical Pair Lemma 2 respectively.

Critical Pair Lemma 5

The following expressions are equivalent:

`or[not[x1],not[x2]]==nand[x1,x2]`

Proof

Note that the input for the rule:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Critical Pair Lemma 3 and Substitution Lemma 4 respectively.

Critical Pair Lemma 6

The following expressions are equivalent:

`and[x1,not[x2]]==not[or[not[x1],x2]]`

Proof

Note that the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

where these rules follow from Axiom 6 and Critical Pair Lemma 3 respectively.

Substitution Lemma 11

It can be shown that:

`nand[x1,or[not[x2],x2]]==not[x1]`

Proof

We start by taking Substitution Lemma 2, and apply the substitution:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 3.

Substitution Lemma 12

It can be shown that:

`and[x1,x1]==x1`

Proof

We start by taking Critical Pair Lemma 1, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 4.

Critical Pair Lemma 7

The following expressions are equivalent:

`nand[and[x1,x2],x3]==or[nand[x1,x2],not[x3]]`

Proof

Note that the input for the rule:

`or[not[x1_],not[x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

where these rules follow from Critical Pair Lemma 5 and Critical Pair Lemma 2 respectively.

Critical Pair Lemma 8

The following expressions are equivalent:

`nand[nand[x1,x2],x3]==or[and[x1,x2],not[x3]]`

Proof

Note that the input for the rule:

`or[not[x1_],not[x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

where these rules follow from Critical Pair Lemma 5 and Axiom 6 respectively.

Critical Pair Lemma 9

The following expressions are equivalent:

`nand[not[x1],x2]==or[x1,not[x2]]`

Proof

Note that the input for the rule:

`or[not[x1_],not[x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Critical Pair Lemma 5 and Substitution Lemma 4 respectively.

Critical Pair Lemma 10

The following expressions are equivalent:

`and[not[x1],x2]==not[or[x1,not[x2]]]`

Proof

Note that the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

where these rules follow from Axiom 6 and Critical Pair Lemma 9 respectively.

Substitution Lemma 13

It can be shown that:

`nand[or[x1,not[x2]],nand[not[x3],x2]]==and[x2,nand[x1,x3]]`

Proof

We start by taking Substitution Lemma 10, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 9.

Substitution Lemma 14

It can be shown that:

`nand[or[x1,not[x2]],or[x3,not[x2]]]==and[x2,nand[x1,x3]]`

Proof

We start by taking Substitution Lemma 13, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 9.

Critical Pair Lemma 11

The following expressions are equivalent:

`and[not[x1],not[x2]]==not[or[x1,x2]]`

Proof

Note that the input for the rule:

`not[or[not[x1_],x2_]]→and[x1,not[x2]]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Critical Pair Lemma 6 and Substitution Lemma 4 respectively.

Critical Pair Lemma 12

The following expressions are equivalent:

`not[not[x1]]==or[x1,not[or[not[x2],x2]]]`

Proof

Note that the input for the rule:

`nand[x1_,or[not[x2_],x2_]]→not[x1]`

contains a subpattern of the form:

`nand[x1_,or[not[x2_],x2_]]`

which can be unified with the input for the rule:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

where these rules follow from Substitution Lemma 11 and Critical Pair Lemma 9 respectively.

Substitution Lemma 15

It can be shown that:

`x1==or[x1,not[or[not[x2],x2]]]`

Proof

We start by taking Critical Pair Lemma 12, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 4.

which follows from Substitution Lemma 4.

Substitution Lemma 16

It can be shown that:

`x1==or[x1,and[x2,not[x2]]]`

Proof

We start by taking Substitution Lemma 15, and apply the substitution:

`not[or[not[x1_],x2_]]→and[x1,not[x2]]`

which follows from Critical Pair Lemma 6.

Critical Pair Lemma 13

The following expressions are equivalent:

`not[x1]==nand[x1,nand[not[x2],x2]]`

Proof

Note that the input for the rule:

`nand[x1_,or[not[x2_],x2_]]→not[x1]`

contains a subpattern of the form:

`or[not[x2_],x2_]`

which can be unified with the input for the rule:

`or[not[x1_],not[x2_]]→nand[x1,x2]`

where these rules follow from Substitution Lemma 11 and Critical Pair Lemma 5 respectively.

Substitution Lemma 17

It can be shown that:

`not[x1]==nand[x1,or[x2,not[x2]]]`

Proof

We start by taking Critical Pair Lemma 13, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 9.

Critical Pair Lemma 14

The following expressions are equivalent:

`x1==or[x1,and[and[x2,x3],nand[x2,x3]]]`

Proof

Note that the input for the rule:

`or[x1_,and[x2_,not[x2_]]]→x1`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

where these rules follow from Substitution Lemma 16 and Critical Pair Lemma 2 respectively.

Critical Pair Lemma 15

The following expressions are equivalent:

`nand[nand[x1,x2],nand[x3,x4]]==or[and[x1,x2],and[x3,x4]]`

Proof

Note that the input for the rule:

`or[not[x1_],and[x2_,x3_]]→nand[x1,nand[x2,x3]]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

where these rules follow from Critical Pair Lemma 4 and Axiom 6 respectively.

Critical Pair Lemma 16

The following expressions are equivalent:

`and[x1,nand[x2,x1]]==not[or[x2,not[x1]]]`

Proof

Note that the input for the rule:

`nand[or[x1_,not[x2_]],or[x3_,not[x2_]]]→and[x2,nand[x1,x3]]`

contains a subpattern of the form:

`nand[or[x1_,not[x2_]],or[x3_,not[x2_]]]`

which can be unified with the input for the rule:

`nand[x1_,or[x2_,not[x2_]]]→not[x1]`

where these rules follow from Substitution Lemma 14 and Substitution Lemma 17 respectively.

Substitution Lemma 18

It can be shown that:

`and[x1,nand[x2,x1]]==and[not[x2],x1]`

Proof

We start by taking Critical Pair Lemma 16, and apply the substitution:

`not[or[x1_,not[x2_]]]→and[not[x1],x2]`

which follows from Critical Pair Lemma 10.

Critical Pair Lemma 17

The following expressions are equivalent:

`and[x1,nand[x2,x2]]==not[or[x2,not[x1]]]`

Proof

Note that the input for the rule:

`nand[or[x1_,not[x2_]],or[x3_,not[x2_]]]→and[x2,nand[x1,x3]]`

contains a subpattern of the form:

`nand[or[x1_,not[x2_]],or[x3_,not[x2_]]]`

which can be unified with the input for the rule:

`nand[x1_,x1_]→not[x1]`

where these rules follow from Substitution Lemma 14 and Axiom 3 respectively.

Substitution Lemma 19

It can be shown that:

`and[x1,not[x2]]==not[or[x2,not[x1]]]`

Proof

We start by taking Critical Pair Lemma 17, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 3.

Substitution Lemma 20

It can be shown that:

`and[x1,not[x2]]==and[not[x2],x1]`

Proof

We start by taking Substitution Lemma 19, and apply the substitution:

`not[or[x1_,not[x2_]]]→and[not[x1],x2]`

which follows from Critical Pair Lemma 10.

Critical Pair Lemma 18

The following expressions are equivalent:

`and[not[x1],nand[x2,x3]]==nand[or[x2,x1],or[x3,not[not[x1]]]]`

Proof

Note that the input for the rule:

`nand[or[x1_,not[x2_]],or[x3_,not[x2_]]]→and[x2,nand[x1,x3]]`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Substitution Lemma 14 and Substitution Lemma 4 respectively.

Substitution Lemma 21

It can be shown that:

`and[not[x1],nand[x2,x3]]==nand[or[x2,x1],or[x3,x1]]`

Proof

We start by taking Critical Pair Lemma 18, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 4.

Critical Pair Lemma 19

The following expressions are equivalent:

`and[not[x1],x1]==and[x1,not[x1]]`

Proof

PROOF

Note that the input for the rule:

`and[x1_,nand[x2_,x1_]]→and[not[x2],x1]`

contains a subpattern of the form:

`nand[x2_,x1_]`

which can be unified with the input for the rule:

`nand[x1_,x1_]→not[x1]`

where these rules follow from Substitution Lemma 18 and Axiom 3 respectively.

Critical Pair Lemma 20

The following expressions are equivalent:

`and[not[not[x1]],x2]==and[x2,or[x1,not[x2]]]`

PROOF

Note that the input for the rule:

`and[x1_,nand[x2_,x1_]]→and[not[x2],x1]`

contains a subpattern of the form:

`nand[x2_,x1_]`

which can be unified with the input for the rule:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

where these rules follow from Substitution Lemma 18 and Critical Pair Lemma 9 respectively.

Substitution Lemma 22

It can be shown that:

`and[x1,x2]==and[x2,or[x1,not[x2]]]`

PROOF

We start by taking Critical Pair Lemma 20, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 4.

Critical Pair Lemma 21

The following expressions are equivalent:

`nand[x1,nand[x2,x1]]==not[and[not[x2],x1]]`

PROOF

Note that the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,nand[x2_,x1_]]→and[not[x2],x1]`

where these rules follow from Critical Pair Lemma 2 and Substitution Lemma 18 respectively.

Substitution Lemma 23

It can be shown that:

It can be shown that:

`nand[x1,nand[x2,x1]]==nand[not[x2],x1]`

PROOF

We start by taking Critical Pair Lemma 21, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 2.

Substitution Lemma 24

It can be shown that:

`nand[x1,nand[x2,x1]]==or[x2,not[x1]]`

PROOF

We start by taking Substitution Lemma 23, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 9.

Critical Pair Lemma 22

The following expressions are equivalent:

`nand[not[x1],x1]==not[and[x1,not[x1]]]`

PROOF

Note that the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[not[x1_],x1_]→and[x1,not[x1]]`

where these rules follow from Critical Pair Lemma 2 and Critical Pair Lemma 19 respectively.

Substitution Lemma 25

It can be shown that:

`or[x1,not[x1]]==not[and[x1,not[x1]]]`

PROOF

We start by taking Critical Pair Lemma 22, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 9.

Substitution Lemma 26

It can be shown that:

`or[x1,not[x1]]==nand[x1,not[x1]]`

PROOF

We start by taking Substitution Lemma 25, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 2.

which follows from Critical Pair Lemma 2.

Substitution Lemma 27

It can be shown that:

`or[x1,not[x1]]==or[not[x1],x1]`

Proof

We start by taking Substitution Lemma 26, and apply the substitution:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 3.

Critical Pair Lemma 23

The following expressions are equivalent:

`or[x1,not[not[x2]]]==or[x2,not[nand[x1,not[x2]]]]`

Proof

Note that the input for the rule:

`nand[x1_,nand[x2_,x1_]]→or[x2,not[x1]]`

contains a subpattern of the form:

`nand[x1_,nand[x2_,x1_]]`

which can be unified with the input for the rule:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

where these rules follow from Substitution Lemma 24 and Critical Pair Lemma 9 respectively.

Substitution Lemma 28

It can be shown that:

`or[x1,x2]==or[x2,not[nand[x1,not[x2]]]]`

Proof

We start by taking Critical Pair Lemma 23, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 4.

Substitution Lemma 29

It can be shown that:

`or[x1,x2]==or[x2,and[x1,not[x2]]]`

Proof

We start by taking Substitution Lemma 28, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Critical Pair Lemma 24

The following expressions are equivalent:

`or[x1,not[or[x2,not[x2]]]]==nand[or[x2,not[x2]],not[x1]]`

Proof

Note that the input for the rule:

`nand[x1_,nand[x2_,x1_]]→or[x2,not[x1]]`

contains a subpattern of the form:

`nand[x2_,x1_]`

which can be unified with the input for the rule:

`nand[x1_,or[x2_,not[x2_]]]→not[x1]`

where these rules follow from Substitution Lemma 24 and Substitution Lemma 17 respectively.

Substitution Lemma 30

It can be shown that:

`or[x1,and[not[x2],x2]]==nand[or[x2,not[x2]],not[x1]]`

Proof

We start by taking Critical Pair Lemma 24, and apply the substitution:

`not[or[x1_,not[x2_]]]→and[not[x1],x2]`

which follows from Critical Pair Lemma 10.

Substitution Lemma 31

It can be shown that:

`or[x1,and[x2,not[x2]]]==nand[or[x2,not[x2]],not[x1]]`

Proof

We start by taking Substitution Lemma 30, and apply the substitution:

`and[not[x1_],x1_]→and[x1,not[x1]]`

which follows from Critical Pair Lemma 19.

Substitution Lemma 32

It can be shown that:

`x1==nand[or[x2,not[x2]],not[x1]]`

Proof

We start by taking Substitution Lemma 31, and apply the substitution:

`or[x1_,and[x2_,not[x2_]]]→x1`

which follows from Substitution Lemma 16.

Substitution Lemma 33

It can be shown that:

`x1==or[not[or[x2,not[x2]]],x1]`

Proof

We start by taking Substitution Lemma 32, and apply the substitution:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 3.

Substitution Lemma 34

It can be shown that:

`x1==or[and[not[x2],x2],x1]`

```
x1==or[and[not[x2],x2],x1]
```

Proof

We start by taking Substitution Lemma 33, and apply the substitution:

```
not[or[x1_,not[x2_]]]→and[not[x1],x2]
```

which follows from Critical Pair Lemma 10.

Substitution Lemma 35

It can be shown that:

```
x1==or[and[x2,not[x2]],x1]
```

Proof

We start by taking Substitution Lemma 34, and apply the substitution:

```
and[not[x1_],x1_]→and[x1,not[x1]]
```

which follows from Critical Pair Lemma 19.

Critical Pair Lemma 25

The following expressions are equivalent:

```
or[not[x1],not[x2]]==nand[x2,or[x1,not[x2]]]
```

Proof

Note that the input for the rule:

```
nand[x1_,nand[x2_,x1_]]→or[x2,not[x1]]
```

contains a subpattern of the form:

```
nand[x2_,x1_]
```

which can be unified with the input for the rule:

```
nand[not[x1_],x2_]→or[x1,not[x2]]
```

where these rules follow from Substitution Lemma 24 and Critical Pair Lemma 9 respectively.

Substitution Lemma 36

It can be shown that:

```
nand[x1,x2]==nand[x2,or[x1,not[x2]]]
```

Proof

We start by taking Critical Pair Lemma 25, and apply the substitution:

```
or[not[x1_],not[x2_]]→nand[x1,x2]
```

which follows from Critical Pair Lemma 5.

Critical Pair Lemma 26

The following expressions are equivalent:

```
not[x1]==nand[nand[x2,not[x2]],x1]
```

Proof

Note that the input for the rule:

```
or[and[x1_,not[x1_]],x2_]→x2
```

contains a subpattern of the form:

```
or[and[x1_,not[x1_]],x2_]
```

`or[and[x1_,not[x1_]],x2_]`

which can be unified with the input for the rule:

`or[and[x1_,x2_],not[x3_]]→nand[nand[x1,x2],x3]`

where these rules follow from Substitution Lemma 35 and Critical Pair Lemma 8 respectively.

Substitution Lemma 37

It can be shown that:

`not[x1]==nand[or[not[x2],x2],x1]`

Proof

We start by taking Critical Pair Lemma 26, and apply the substitution:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 3.

Substitution Lemma 38

It can be shown that:

`not[x1]==nand[or[x2,not[x2]],x1]`

Proof

Out[]=

We start by taking Substitution Lemma 37, and apply the substitution:

`or[not[x1_],x1_]→or[x1,not[x1]]`

which follows from Substitution Lemma 27.

Critical Pair Lemma 27

The following expressions are equivalent:

`and[not[and[x1,not[x1]]],x2]==not[not[x2]]`

Proof

Note that the input for the rule:

`not[or[x1_,not[x2_]]]→and[not[x1],x2]`

contains a subpattern of the form:

`or[x1_,not[x2_]]`

which can be unified with the input for the rule:

`or[and[x1_,not[x1_]],x2_]→x2`

where these rules follow from Critical Pair Lemma 10 and Substitution Lemma 35 respectively.

Substitution Lemma 39

It can be shown that:

`and[nand[x1,not[x1]],x2]==not[not[x2]]`

Proof

We start by taking Critical Pair Lemma 27, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 2.

Substitution Lemma 40

It can be shown that:

It can be shown that:

`and[or[not[x1],x1],x2]==not[not[x2]]`

Proof

We start by taking Substitution Lemma 39, and apply the substitution:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 3.

Substitution Lemma 41

It can be shown that:

`and[or[x1,not[x1]],x2]==not[not[x2]]`

Proof

We start by taking Substitution Lemma 40, and apply the substitution:

`or[not[x1_],x1_]→or[x1,not[x1]]`

which follows from Substitution Lemma 27.

Substitution Lemma 42

It can be shown that:

`and[or[x1,not[x1]],x2]==x2`

Proof

We start by taking Substitution Lemma 41, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 4.

Critical Pair Lemma 28

The following expressions are equivalent:

`not[or[x1,not[x2]]]==and[x2,nand[x2,x1]]`

Proof

Note that the input for the rule:

`nand[or[x1_,not[x1_]],x2_]→not[x2]`

contains a subpattern of the form:

`nand[or[x1_,not[x1_]],x2_]`

which can be unified with the input for the rule:

`nand[or[x1_,not[x2_]],or[x3_,not[x2_]]]→and[x2,nand[x1,x3]]`

where these rules follow from Substitution Lemma 38 and Substitution Lemma 14 respectively.

Substitution Lemma 43

It can be shown that:

`and[not[x1],x2]==and[x2,nand[x2,x1]]`

Proof

We start by taking Critical Pair Lemma 28, and apply the substitution:

`not[or[x1_,not[x2_]]]→and[not[x1],x2]`

which follows from Critical Pair Lemma 10.

Critical Pair Lemma 29

The following expressions are equivalent:

`and[not[not[x1]],x2]==and[x2,or[not[x2],x1]]`

Proof

Note that the input for the rule:

`and[x1_,nand[x1_,x2_]]→and[not[x2],x1]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

where these rules follow from Substitution Lemma 43 and Critical Pair Lemma 3 respectively.

Substitution Lemma 44

It can be shown that:

`and[x1,x2]==and[x2,or[not[x2],x1]]`

Proof

We start by taking Critical Pair Lemma 29, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 4.

Critical Pair Lemma 30

The following expressions are equivalent:

`nand[x1,nand[x1,x2]]==not[and[not[x2],x1]]`

Proof

Note that the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,nand[x1_,x2_]]→and[not[x2],x1]`

where these rules follow from Critical Pair Lemma 2 and Substitution Lemma 43 respectively.

Substitution Lemma 45

It can be shown that:

`nand[x1,nand[x1,x2]]==nand[not[x2],x1]`

Proof

We start by taking Critical Pair Lemma 30, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 2.

Substitution Lemma 46

It can be shown that:

`nand[x1,nand[x1,x2]]==or[x2,not[x1]]`

Proof

We start by taking Substitution Lemma 45, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 9.

Critical Pair Lemma 31

The following expressions are equivalent:

`or[x1,not[not[x2]]]==or[x2,not[nand[not[x2],x1]]]`

Proof

Note that the input for the rule:

`nand[x1_,nand[x1_,x2_]]→or[x2,not[x1]]`

contains a subpattern of the form:

`nand[x1_,nand[x1_,x2_]]`

which can be unified with the input for the rule:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

where these rules follow from Substitution Lemma 46 and Critical Pair Lemma 9 respectively.

Substitution Lemma 47

It can be shown that:

`or[x1,x2]==or[x2,not[nand[not[x2],x1]]]`

Proof

We start by taking Critical Pair Lemma 31, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 4.

Substitution Lemma 48

It can be shown that:

`or[x1,x2]==or[x2,and[not[x2],x1]]`

Proof

We start by taking Substitution Lemma 47, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Critical Pair Lemma 32

The following expressions are equivalent:

`and[and[x1,not[not[x2]]],x2]==and[x2,or[x1,not[x2]]]`

Proof

Note that the input for the rule:

`and[x1_,or[not[x1_],x2_]]→and[x2,x1]`

contains a subpattern of the form:

`or[not[x1_],x2_]`

which can be unified with the input for the rule:

`or[x1_,and[x2_,not[x1_]]]→or[x2,x1]`

where these rules follow from Substitution Lemma 44 and Substitution Lemma 29 respectively.

Substitution Lemma 49

It can be shown that:

`and[and[x1,x2],x2]==and[x2,or[x1,not[x2]]]`

Proof

We start by taking Critical Pair Lemma 32, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 4.

Substitution Lemma 50

It can be shown that:

`and[and[x1,x2],x2]==and[x1,x2]`

Proof

We start by taking Substitution Lemma 49, and apply the substitution:

`and[x1_,or[x2_,not[x1_]]]→and[x2,x1]`

which follows from Substitution Lemma 22.

Critical Pair Lemma 33

The following expressions are equivalent:

`or[not[x1],x2]==or[x2,not[or[x2,x1]]]`

Proof

Note that the input for the rule:

`or[x1_,and[not[x1_],x2_]]→or[x2,x1]`

contains a subpattern of the form:

`and[not[x1_],x2_]`

which can be unified with the input for the rule:

`and[not[x1_],not[x2_]]→not[or[x1,x2]]`

where these rules follow from Substitution Lemma 48 and Critical Pair Lemma 11 respectively.

Critical Pair Lemma 34

The following expressions are equivalent:

`and[and[not[not[x1]],x2],x1]==and[x1,or[x2,not[x1]]]`

Proof

Note that the input for the rule:

`and[x1_,or[not[x1_],x2_]]→and[x2,x1]`

contains a subpattern of the form:

`or[not[x1_],x2_]`

which can be unified with the input for the rule:

`or[x1_,and[not[x1_],x2_]]→or[x2,x1]`

where these rules follow from Substitution Lemma 44 and Substitution Lemma 48 respectively.

Substitution Lemma 51

It can be shown that:

`and[and[x1,x2],x1]==and[x1,or[x2,not[x1]]]`

Proof

We start by taking Critical Pair Lemma 34, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 4.

Substitution Lemma 52

It can be shown that:

`and[and[x1,x2],x1]==and[x2,x1]`

Proof

We start by taking Substitution Lemma 51, and apply the substitution:

`and[x1_,or[x2_,not[x1_]]]→and[x2,x1]`

which follows from Substitution Lemma 22.

Critical Pair Lemma 35

The following expressions are equivalent:

`and[x1,and[x2,x1]]==and[and[x2,x1],and[x2,x1]]`

Proof

Note that the input for the rule:

`and[and[x1_,x2_],x1_]→and[x2,x1]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[and[x1_,x2_],x2_]→and[x1,x2]`

where these rules follow from Substitution Lemma 52 and Substitution Lemma 50 respectively.

Substitution Lemma 53

It can be shown that:

`and[x1,and[x2,x1]]==and[x2,x1]`

Proof

We start by taking Critical Pair Lemma 35, and apply the substitution:

`and[x1_,x1_]→x1`

which follows from Substitution Lemma 12.

Critical Pair Lemma 36

The following expressions are equivalent:

`x1==or[x1,and[and[x2,x3],nand[x3,and[x2,x3]]]]`

PROOF

Note that the input for the rule:

`or[x1_,and[and[x2_,x3_],nand[x2_,x3_]]]→x1`

contains a subpattern of the form:

`and[x2_,x3_]`

which can be unified with the input for the rule:

`and[x1_,and[x2_,x1_]]→and[x2,x1]`

where these rules follow from Critical Pair Lemma 14 and Substitution Lemma 53 respectively.

Substitution Lemma 54

It can be shown that:

`x1==or[x1,and[not[x2],and[x3,x2]]]`

PROOF

We start by taking Critical Pair Lemma 36, and apply the substitution:

`and[x1_,nand[x2_,x1_]]→and[not[x2],x1]`

which follows from Substitution Lemma 18.

Critical Pair Lemma 37

The following expressions are equivalent:

`and[not[not[x1]],x2]==and[x2,x1]`

PROOF

Note that the input for the rule:

`and[x1_,not[x2_]]↔and[not[x2_],x1_]`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Substitution Lemma 20 and Substitution Lemma 4 respectively.

Substitution Lemma 55

It can be shown that:

`and[x1,x2]==and[x2,x1]`

PROOF

We start by taking Critical Pair Lemma 37, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 4.

Critical Pair Lemma 38

The following expressions are equivalent:

`and[not[x1],not[x2]]==not[or[x2,x1]]`

PROOF

PROOF

Note that the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[not[x1_],not[x2_]]→not[or[x1,x2]]`

where these rules follow from Substitution Lemma 55 and Critical Pair Lemma 11 respectively.

Substitution Lemma 56

It can be shown that:

`not[or[x1,x2]]==not[or[x2,x1]]`

PROOF

We start by taking Critical Pair Lemma 38, and apply the substitution:

`and[not[x1_],not[x2_]]→not[or[x1,x2]]`

which follows from Critical Pair Lemma 11.

Critical Pair Lemma 39

The following expressions are equivalent:

`nand[x1,x2]==not[and[x2,x1]]`

PROOF

Note that the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Critical Pair Lemma 2 and Substitution Lemma 55 respectively.

Substitution Lemma 57

It can be shown that:

`nand[x1,x2]==nand[x2,x1]`

PROOF

We start by taking Critical Pair Lemma 39, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 2.

Critical Pair Lemma 40

The following expressions are equivalent:

`or[x1,x2]==not[not[or[x2,x1]]]`

PROOF

Note that the input for the rule:

`not[not[x1_]]→x1`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[or[x1_,x2_]]↔not[or[x2_,x1_]]`

where these rules follow from Substitution Lemma 4 and Substitution Lemma 56 respectively.

Substitution Lemma 58

It can be shown that:

`or[x1,x2]==or[x2,x1]`

Proof

We start by taking Critical Pair Lemma 40, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 4.

Critical Pair Lemma 41

The following expressions are equivalent:

`x1==or[and[x2,x1],x1]`

Proof

Note that the input for the rule:

`or[x1_,and[not[x2_],and[x3_,x2_]]]→x1`

contains a subpattern of the form:

`or[x1_,and[not[x2_],and[x3_,x2_]]]`

which can be unified with the input for the rule:

`or[x1_,and[not[x1_],x2_]]→or[x2,x1]`

where these rules follow from Substitution Lemma 54 and Substitution Lemma 48 respectively.

Substitution Lemma 59

It can be shown that:

`x1==or[x1,and[x2,x1]]`

Proof

We start by taking Critical Pair Lemma 41, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Substitution Lemma 58.

Critical Pair Lemma 42

The following expressions are equivalent:

`x1==or[x1,and[x1,x2]]`

Proof

Note that the input for the rule:

`or[x1_,and[x2_,x1_]]→x1`

contains a subpattern of the form:

`and[x2_,x1_]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Substitution Lemma 59 and Substitution Lemma 55 respectively.

Critical Pair Lemma 43

The following expressions are equivalent:

`or[not[and[x1,x2]],x2]==or[x2,not[x2]]`

Proof

Note that the input for the rule:

`or[x1_,not[or[x1_,x2_]]]→or[not[x2],x1]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,and[x2_,x1_]]→x1`

where these rules follow from Critical Pair Lemma 33 and Substitution Lemma 59 respectively.

Substitution Lemma 60

It can be shown that:

`or[nand[x1,x2],x2]==or[x2,not[x2]]`

Proof

We start by taking Critical Pair Lemma 43, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 2.

Critical Pair Lemma 44

The following expressions are equivalent:

`not[x1]==or[not[x1],not[or[x1,x2]]]`

Proof

Note that the input for the rule:

`or[x1_,and[x1_,x2_]]→x1`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[not[x1_],not[x2_]]→not[or[x1,x2]]`

where these rules follow from Critical Pair Lemma 42 and Critical Pair Lemma 11 respectively.

Substitution Lemma 61

It can be shown that:

`not[x1]==nand[x1,or[x1,x2]]`

Proof

We start by taking Critical Pair Lemma 44, and apply the substitution:

`or[not[x1_],not[x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 5.

Substitution Lemma 62

It can be shown that:

`or[x1,nand[x2,x1]]==or[x1,not[x1]]`

Proof

We start by taking Substitution Lemma 60, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Substitution Lemma 58.

Critical Pair Lemma 45

The following expressions are equivalent:

`and[not[x1],nand[x2,x3]]==nand[or[x2,x1],or[x1,x3]]`

Proof

Note that the input for the rule:

`nand[or[x1_,x2_],or[x3_,x2_]]→and[not[x2],nand[x1,x3]]`

contains a subpattern of the form:

`or[x3_,x2_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Substitution Lemma 21 and Substitution Lemma 58 respectively.

Critical Pair Lemma 46

The following expressions are equivalent:

`and[or[x1,x2],or[x3,x2]]==not[and[not[x2],nand[x1,x3]]]`

Proof

Note that the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[or[x1_,x2_],or[x3_,x2_]]→and[not[x2],nand[x1,x3]]`

where these rules follow from Axiom 6 and Substitution Lemma 21 respectively.

Substitution Lemma 63

It can be shown that:

`and[or[x1,x2],or[x3,x2]]==nand[not[x2],nand[x1,x3]]`

Proof

We start by taking Critical Pair Lemma 46, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 2.

Substitution Lemma 64

It can be shown that:

`and[or[x1,x2],or[x3,x2]]==or[x2,not[nand[x1,x3]]]`

Proof

We start by taking Substitution Lemma 63, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 9.

Substitution Lemma 65

It can be shown that:

`and[or[x1,x2],or[x3,x2]]==or[x2,and[x1,x3]]`

Proof

We start by taking Substitution Lemma 64, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Critical Pair Lemma 47

The following expressions are equivalent:

`or[nand[x1,x2],and[x2,x3]]==and[or[x2,not[x2]],or[x3,nand[x1,x2]]]`

Proof

Note that the input for the rule:

`and[or[x1_,x2_],or[x3_,x2_]]→or[x2,and[x1,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,nand[x2_,x1_]]→or[x1,not[x1]]`

where these rules follow from Substitution Lemma 65 and Substitution Lemma 62 respectively.

Substitution Lemma 66

It can be shown that:

`or[nand[x1,x2],and[x2,x3]]==or[x3,nand[x1,x2]]`

Proof

We start by taking Critical Pair Lemma 47, and apply the substitution:

`and[or[x1_,not[x1_]],x2_]→x2`

which follows from Substitution Lemma 42.

Critical Pair Lemma 48

The following expressions are equivalent:

`or[x1,nand[x2,or[x2,x3]]]==or[not[x2],and[or[x2,x3],x1]]`

Proof

PROOF

Note that the input for the rule:

`or[nand[x1_,x2_],and[x2_,x3_]]→or[x3,nand[x1,x2]]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,or[x1_,x2_]]→not[x1]`

where these rules follow from Substitution Lemma 66 and Substitution Lemma 61 respectively.

Substitution Lemma 67

It can be shown that:

`or[x1,not[x2]]==or[not[x2],and[or[x2,x3],x1]]`

PROOF

We start by taking Critical Pair Lemma 48, and apply the substitution:

`nand[x1_,or[x1_,x2_]]→not[x1]`

which follows from Substitution Lemma 61.

Substitution Lemma 68

It can be shown that:

`or[x1,not[x2]]==nand[x2,nand[or[x2,x3],x1]]`

PROOF

We start by taking Substitution Lemma 67, and apply the substitution:

`or[not[x1_],and[x2_,x3_]]→nand[x1,nand[x2,x3]]`

which follows from Critical Pair Lemma 4.

Critical Pair Lemma 49

The following expressions are equivalent:

`or[nand[or[x1,x2],x3],not[x1]]==nand[x1,or[x3,not[x1]]]`

PROOF

Note that the input for the rule:

`nand[x1_,nand[x1_,x2_]]→or[x2,not[x1]]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,nand[or[x1_,x2_],x3_]]→or[x3,not[x1]]`

where these rules follow from Substitution Lemma 46 and Substitution Lemma 68 respectively.

Substitution Lemma 69

It can be shown that:

`nand[and[or[x1,x2],x3],x1]==nand[x1,or[x3,not[x1]]]`

PROOF

We start by taking Critical Pair Lemma 49, and apply the substitution:

```
or[nand[x1_,x2_],not[x3_]]→nand[and[x1,x2],x3]
```

which follows from Critical Pair Lemma 7.

Substitution Lemma 70

It can be shown that:

```
nand[and[or[x1,x2],x3],x1]==nand[x3,x1]
```

Proof

We start by taking Substitution Lemma 69, and apply the substitution:

```
nand[x1_,or[x2_,not[x1_]]]→nand[x2,x1]
```

which follows from Substitution Lemma 36.

Substitution Lemma 71

It can be shown that:

```
nand[x1,and[or[x1,x2],x3]]==nand[x3,x1]
```

Proof

We start by taking Substitution Lemma 70, and apply the substitution:

```
nand[x1_,x2_]→nand[x2,x1]
```

which follows from Substitution Lemma 57.

Critical Pair Lemma 50

The following expressions are equivalent:

```
nand[or[x1,x2],x3]==nand[x3,or[x2,and[x3,x1]]]
```

Proof

Note that the input for the rule:

```
nand[x1_,and[or[x1_,x2_],x3_]]→nand[x3,x1]
```

contains a subpattern of the form:

```
and[or[x1_,x2_],x3_]
```

which can be unified with the input for the rule:

```
and[or[x1_,x2_],or[x3_,x2_]]→or[x2,and[x1,x3]]
```

where these rules follow from Substitution Lemma 71 and Substitution Lemma 65 respectively.

Critical Pair Lemma 51

The following expressions are equivalent:

```
nand[or[x1,x2],x3]==nand[x3,or[and[x3,x1],x2]]
```

Proof

Note that the input for the rule:

```
nand[x1_,or[x2_,and[x1_,x3_]]]→nand[or[x3,x2],x1]
```

contains a subpattern of the form:

```
or[x2_,and[x1_,x3_]]
```

which can be unified with the input for the rule:

```
or[x1_,x2_]↔or[x2_,x1_]
```

where these rules follow from Critical Pair Lemma 50 and Substitution Lemma 58 respectively.

where these rules follow from Critical Pair Lemma 50 and Substitution Lemma 58 respectively.

Critical Pair Lemma 52

The following expressions are equivalent:

`and[not[and[x1,x2]],nand[x1,x3]]==nand[x1,or[and[x1,x2],x3]]`

Proof

Note that the input for the rule:

`nand[or[x1_,x2_],or[x2_,x3_]]→and[not[x2],nand[x1,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,and[x1_,x2_]]→x1`

where these rules follow from Critical Pair Lemma 45 and Critical Pair Lemma 42 respectively.

Substitution Lemma 72

It can be shown that:

`and[nand[x1,x2],nand[x1,x3]]==nand[x1,or[and[x1,x2],x3]]`

Proof

We start by taking Critical Pair Lemma 52, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 2.

Substitution Lemma 73

It can be shown that:

`and[nand[x1,x2],nand[x1,x3]]==nand[or[x2,x3],x1]`

Proof

We start by taking Substitution Lemma 72, and apply the substitution:

`nand[x1_,or[and[x1_,x2_],x3_]]→nand[or[x2,x3],x1]`

which follows from Critical Pair Lemma 51.

Critical Pair Lemma 53

The following expressions are equivalent:

`nand[nand[x1,x2],nand[x1,x3]]==not[nand[or[x2,x3],x1]]`

Proof

Note that the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[nand[x1_,x2_],nand[x1_,x3_]]→nand[or[x2,x3],x1]`

where these rules follow from Critical Pair Lemma 2 and Substitution Lemma 73 respectively.

Substitution Lemma 74

It can be shown that:

`or[and[x1,x2],and[x1,x3]]==not[nand[or[x2,x3],x1]]`

PROOF

We start by taking Critical Pair Lemma 53, and apply the substitution:

`nand[nand[x1_,x2_],nand[x3_,x4_]]→or[and[x1,x2],and[x3,x4]]`

which follows from Critical Pair Lemma 15.

Substitution Lemma 75

It can be shown that:

`or[and[x1,x2],and[x1,x3]]==and[or[x2,x3],x1]`

PROOF

We start by taking Substitution Lemma 74, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Substitution Lemma 76

It can be shown that:

`or[and[a,b],and[a,c]]==and[or[b,c],a]`

PROOF

We start by taking Hypothesis 1, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Substitution Lemma 55.

Conclusion 1

We obtain the conclusion:

`True`

PROOF

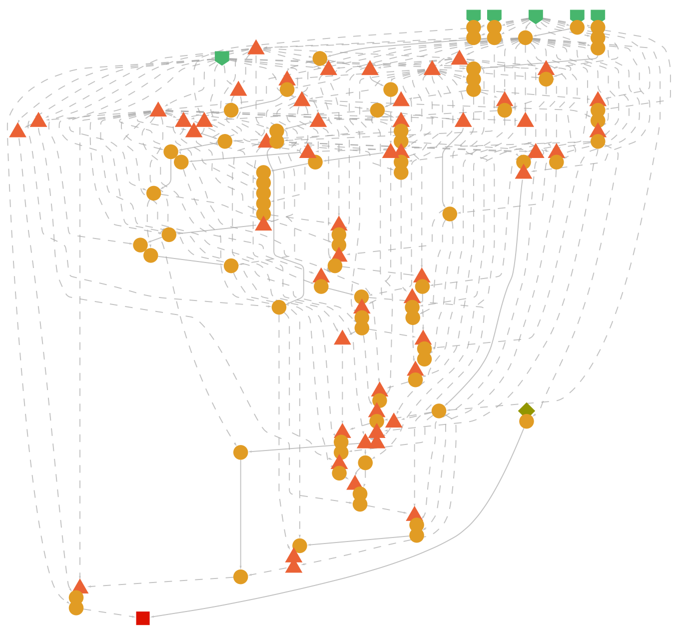
Take Substitution Lemma 76, and apply the substitution:

`or[and[x1_,x2_],and[x1_,x3_]]→and[or[x2,x3],x1]`

which follows from Substitution Lemma 75.

In[ ]:= `proofAxB5fromSheffer["ProofGraph"]`

Out[ ]=



In[ ]:= Clear[proofAxB5fromSheffer]

In[ ]:= proofAxB6fromSheffer["ProofNotebook"]

Axiom 1

We are given that:

`x1==nand[nand[x1,x1],nand[x1,x1]]`

Axiom 2

We are given that:

`nand[x1,x1]==not[x1]`

Axiom 3

We are given that:

`nand[nand[x1,x1],nand[x2,x2]]==or[x1,x2]`

Axiom 4

We are given that:

`nand[nand[x1,nand[x2,x3]],nand[x1,nand[x2,x3]]]==nand[nand[nand[x2,x2],x1],nand[nand[x3,`

Axiom 5

We are given that:

`not[nand[x1,x2]]==and[x1,x2]`

Hypothesis 1

We would like to show that:

`and[or[a,b],or[a,c]]==or[a,and[b,c]]`

Substitution Lemma 1

It can be shown that:

`not[nand[x1,x1]]==x1`

Proof

We start by taking Axiom 1, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Substitution Lemma 2

It can be shown that:

`not[not[x1]]==x1`

Proof

We start by taking Substitution Lemma 1, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Substitution Lemma 3

It can be shown that:

`nand[not[x1],nand[x2,x2]]==or[x1,x2]`

Proof

We start by taking Axiom 3, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Substitution Lemma 4

It can be shown that:

`nand[not[x1],not[x2]]==or[x1,x2]`

Proof

We start by taking Substitution Lemma 3, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Substitution Lemma 5

It can be shown that:

`not[nand[x1,nand[x2,x3]]]==nand[nand[nand[x2,x2],x1],nand[nand[x3,x3],x1]]`

Proof

We start by taking Axiom 4, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Substitution Lemma 6

It can be shown that:

`not[nand[x1,nand[x2,x3]]]==nand[nand[not[x2],x1],nand[nand[x3,x3],x1]]`

PROOF

We start by taking Substitution Lemma 5, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Substitution Lemma 7

It can be shown that:

`not[nand[x1,nand[x2,x3]]]==nand[nand[not[x2],x1],nand[not[x3],x1]]`

PROOF

We start by taking Substitution Lemma 6, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Substitution Lemma 8

It can be shown that:

`nand[nand[not[x1_],x2_],nand[not[x3_],x2_]]→and[x2,nand[x1,x3]]`

PROOF

We start by taking Substitution Lemma 7, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 5.

Critical Pair Lemma 1

The following expressions are equivalent:

`nand[x1,x2]==not[and[x1,x2]]`

PROOF

Note that the input for the rule:

`not[not[x1_]]→x1`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

where these rules follow from Substitution Lemma 2 and Axiom 5 respectively.

Critical Pair Lemma 2

The following expressions are equivalent:

`or[not[x1],x2]==nand[x1,not[x2]]`

PROOF

Note that the input for the rule:

`nand[not[x1_],not[x2_]]→or[x1,x2]`

contains a subpattern of the form:

`not[x1_]`

not[x1_]

which can be unified with the input for the rule:

not[not[x1_]]→x1

where these rules follow from Substitution Lemma 4 and Substitution Lemma 2 respectively.

Critical Pair Lemma 3

The following expressions are equivalent:

or[not[x1],not[x2]]==nand[x1,x2]

Proof

Note that the input for the rule:

nand[x1_,not[x2_]]→or[not[x1],x2]

contains a subpattern of the form:

not[x2_]

which can be unified with the input for the rule:

not[not[x1_]]→x1

where these rules follow from Critical Pair Lemma 2 and Substitution Lemma 2 respectively.

Critical Pair Lemma 4

The following expressions are equivalent:

and[x1,not[x2]]==not[or[not[x1],x2]]

Proof

Note that the input for the rule:

not[nand[x1_,x2_]]→and[x1,x2]

contains a subpattern of the form:

nand[x1_,x2_]

which can be unified with the input for the rule:

nand[x1_,not[x2_]]→or[not[x1],x2]

where these rules follow from Axiom 5 and Critical Pair Lemma 2 respectively.

Critical Pair Lemma 5

The following expressions are equivalent:

nand[not[x1],x2]==or[x1,not[x2]]

Proof

Note that the input for the rule:

or[not[x1_],not[x2_]]→nand[x1,x2]

contains a subpattern of the form:

not[x1_]

which can be unified with the input for the rule:

not[not[x1_]]→x1

where these rules follow from Critical Pair Lemma 3 and Substitution Lemma 2 respectively.

Critical Pair Lemma 6

The following expressions are equivalent:

The following expressions are equivalent:

`and[not[x1],x2]==not[or[x1,not[x2]]]`

PROOF

Note that the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

where these rules follow from Axiom 5 and Critical Pair Lemma 5 respectively.

Substitution Lemma 9

It can be shown that:

`nand[or[x1,not[x2]],nand[not[x3],x2]]==and[x2,nand[x1,x3]]`

PROOF

We start by taking Substitution Lemma 8, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 5.

Substitution Lemma 10

It can be shown that:

`nand[or[x1,not[x2]],or[x3,not[x2]]]==and[x2,nand[x1,x3]]`

PROOF

We start by taking Substitution Lemma 9, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 5.

Critical Pair Lemma 7

The following expressions are equivalent:

`and[not[x1],not[x2]]==not[or[x1,x2]]`

PROOF

Note that the input for the rule:

`not[or[not[x1_],x2_]]→and[x1,not[x2]]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

Out[∘]= where these rules follow from Critical Pair Lemma 4 and Substitution Lemma 2 respectively.

Critical Pair Lemma 8

The following expressions are equivalent:

`and[x1,nand[x2,x2]]==not[or[x2,not[x1]]]`

Proof

Note that the input for the rule:

`nand[or[x1_,not[x2_]],or[x3_,not[x2_]]]→and[x2,nand[x1,x3]]`

contains a subpattern of the form:

`nand[or[x1_,not[x2_]],or[x3_,not[x2_]]]`

which can be unified with the input for the rule:

`nand[x1_,x1_]→not[x1]`

where these rules follow from Substitution Lemma 10 and Axiom 2 respectively.

Substitution Lemma 11

It can be shown that:

`and[x1,not[x2]]==not[or[x2,not[x1]]]`

Proof

We start by taking Critical Pair Lemma 8, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Substitution Lemma 12

It can be shown that:

`and[x1,not[x2]]==and[not[x2],x1]`

Proof

We start by taking Substitution Lemma 11, and apply the substitution:

`not[or[x1_,not[x2_]]]→and[not[x1],x2]`

which follows from Critical Pair Lemma 6.

Critical Pair Lemma 9

The following expressions are equivalent:

`and[not[x1],nand[x2,x3]]==nand[or[x2,x1],or[x3,not[not[x1]]]]`

Proof

Note that the input for the rule:

`nand[or[x1_,not[x2_]],or[x3_,not[x2_]]]→and[x2,nand[x1,x3]]`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Substitution Lemma 10 and Substitution Lemma 2 respectively.

Substitution Lemma 13

It can be shown that:

`and[not[x1],nand[x2,x3]]==nand[or[x2,x1],or[x3,x1]]`

Proof

We start by taking Critical Pair Lemma 9, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 2.

Critical Pair Lemma 10

The following expressions are equivalent:

`and[not[not[x1]],x2]==and[x2,x1]`

Proof

Note that the input for the rule:

`and[x1_,not[x2_]]↔and[not[x2_],x1_]`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Substitution Lemma 12 and Substitution Lemma 2 respectively.

Substitution Lemma 14

It can be shown that:

`and[x1,x2]==and[x2,x1]`

Proof

We start by taking Critical Pair Lemma 10, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 2.

Critical Pair Lemma 11

The following expressions are equivalent:

`and[not[x1],not[x2]]==not[or[x2,x1]]`

Proof

Note that the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[not[x1_],not[x2_]]→not[or[x1,x2]]`

where these rules follow from Substitution Lemma 14 and Critical Pair Lemma 7 respectively.

Substitution Lemma 15

It can be shown that:

`not[or[x1,x2]]==not[or[x2,x1]]`

Proof

We start by taking Critical Pair Lemma 11, and apply the substitution:

`and[not[x1_],not[x2_]]→not[or[x1,x2]]`

which follows from Critical Pair Lemma 7.

Critical Pair Lemma 12

The following expressions are equivalent:

```
or[x1,x2]==not[not[or[x2,x1]]]
```

Proof

Note that the input for the rule:

```
not[not[x1_]]→x1
```

contains a subpattern of the form:

```
not[x1_]
```

which can be unified with the input for the rule:

```
not[or[x1_,x2_]]↔not[or[x2_,x1_]]
```

where these rules follow from Substitution Lemma 2 and Substitution Lemma 15 respectively.

Substitution Lemma 16

It can be shown that:

```
or[x1,x2]==or[x2,x1]
```

Proof

We start by taking Critical Pair Lemma 12, and apply the substitution:

```
not[not[x1_]]→x1
```

which follows from Substitution Lemma 2.

Critical Pair Lemma 13

The following expressions are equivalent:

```
and[or[x1,x2],or[x3,x2]]==not[and[not[x2],nand[x1,x3]]]
```

Proof

Note that the input for the rule:

```
not[nand[x1_,x2_]]→and[x1,x2]
```

contains a subpattern of the form:

```
nand[x1_,x2_]
```

which can be unified with the input for the rule:

```
nand[or[x1_,x2_],or[x3_,x2_]]→and[not[x2],nand[x1,x3]]
```

where these rules follow from Axiom 5 and Substitution Lemma 13 respectively.

Substitution Lemma 17

It can be shown that:

```
and[or[x1,x2],or[x3,x2]]==nand[not[x2],nand[x1,x3]]
```

Proof

We start by taking Critical Pair Lemma 13, and apply the substitution:

```
not[and[x1_,x2_]]→nand[x1,x2]
```

which follows from Critical Pair Lemma 1.

Substitution Lemma 18

It can be shown that:

`and[or[x1,x2],or[x3,x2]]==or[x2,not[nand[x1,x3]]]`

Proof

We start by taking Substitution Lemma 17, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 5.

Substitution Lemma 19

It can be shown that:

`and[or[x1,x2],or[x3,x2]]==or[x2,and[x1,x3]]`

Proof

We start by taking Substitution Lemma 18, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 5.

Substitution Lemma 20

It can be shown that:

`and[or[a,c],or[a,b]]==or[a,and[b,c]]`

Proof

We start by taking Hypothesis 1, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Substitution Lemma 14.

Substitution Lemma 21

It can be shown that:

`and[or[a,c],or[a,b]]==or[a,and[c,b]]`

Proof

We start by taking Substitution Lemma 20, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Substitution Lemma 14.

Substitution Lemma 22

It can be shown that:

`and[or[a,c],or[b,a]]==or[a,and[c,b]]`

Proof

We start by taking Substitution Lemma 21, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Substitution Lemma 16.

Substitution Lemma 23

It can be shown that:

It can be shown that:

`and[or[c,a],or[b,a]]==or[a,and[c,b]]`

Proof

We start by taking Substitution Lemma 22, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Substitution Lemma 16.

Conclusion 1

We obtain the conclusion:

`True`

Proof

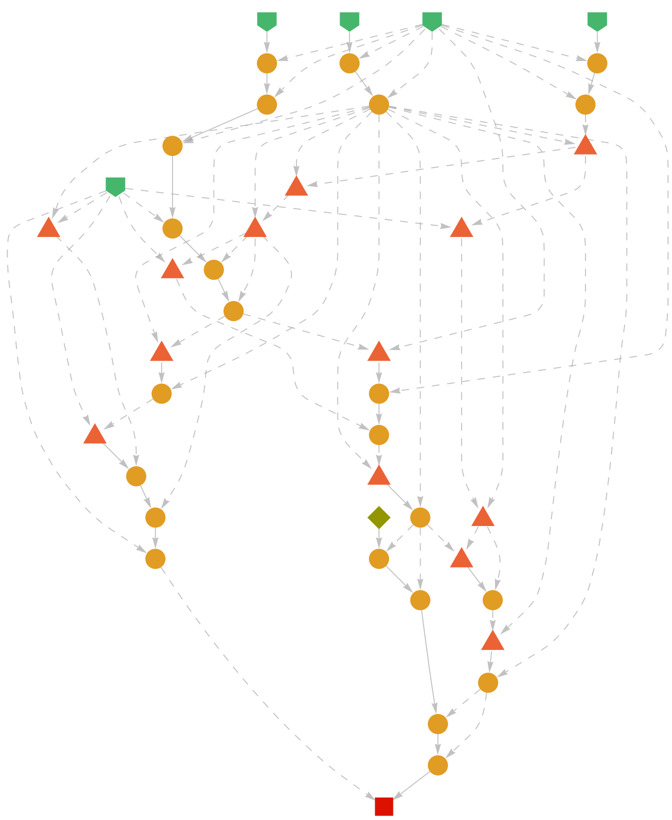
Take Substitution Lemma 23, and apply the substitution:

`and[or[x1_,x2_],or[x3_,x2_]]→or[x2,and[x1,x3]]`

which follows from Substitution Lemma 19.

In[ ]:= `proofAxB6fromSheffer["ProofGraph"]`

Out[ ]=



In[ ]:= `Clear[proofAxB6fromSheffer]`

Appendix 12. Derivation of equational Boolean logic from “short” logic

In[∘]:=
```
proofAxB1fromShort["ProofNotebook"]
```

Axiom 1

We are given that:

`x1==nand[nand[x1,x1],nand[x1,x2]]`

Axiom 2

We are given that:

`nand[x1,x1]==not[x1]`

Axiom 3

We are given that:

`nand[x1,nand[x1,x2]]==nand[x1,nand[x2,x2]]`

Axiom 4

We are given that:

`nand[x1,nand[x1,nand[x2,x3]]]==nand[x2,nand[x2,nand[x1,x3]]]`

Axiom 5

We are given that:

`nand[nand[x1,x1],nand[x2,x2]]==or[x1,x2]`

Axiom 6

We are given that:

`not[nand[x1,x2]]==and[x1,x2]`

Hypothesis 1

We would like to show that:

`and[a,b]==and[b,a]`

Substitution Lemma 1

It can be shown that:

`nand[not[x1],nand[x1,x2]]==x1`

Proof

We start by taking Axiom 1, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Substitution Lemma 2

It can be shown that:

`nand[x1,nand[x1,x2]]==nand[x1,not[x2]]`

Proof

We start by taking Axiom 3, and apply the substitution:

```
nand[x1_,x1_]→not[x1]
```

which follows from Axiom 2.

Substitution Lemma 3

It can be shown that:

```
nand[x1,not[nand[x2,x3]]]==nand[x2,nand[x2,nand[x1,x3]]]
```

Proof

We start by taking Axiom 4, and apply the substitution:

```
nand[x1_,nand[x1_,x2_]]→nand[x1,not[x2]]
```

which follows from Substitution Lemma 2.

Substitution Lemma 4

It can be shown that:

```
nand[x1,not[nand[x2,x3]]]==nand[x2,not[nand[x1,x3]]]
```

Proof

We start by taking Substitution Lemma 3, and apply the substitution:

```
nand[x1_,nand[x1_,x2_]]→nand[x1,not[x2]]
```

which follows from Substitution Lemma 2.

Substitution Lemma 5

It can be shown that:

```
nand[not[x1],nand[x2,x2]]==or[x1,x2]
```

Proof

We start by taking Axiom 5, and apply the substitution:

```
nand[x1_,x1_]→not[x1]
```

which follows from Axiom 2.

Substitution Lemma 6

It can be shown that:

```
nand[not[x1],not[x2]]==or[x1,x2]
```

Proof

We start by taking Substitution Lemma 5, and apply the substitution:

```
nand[x1_,x1_]→not[x1]
```

which follows from Axiom 2.

Critical Pair Lemma 1

The following expressions are equivalent:

```
or[x1,x1]==not[not[x1]]
```

Proof

Note that the input for the rule:

```
nand[not[x1_],not[x2_]]→or[x1,x2]
```

contains a subpattern of the form:

contains a subpattern of the form:

`nand[not[x1_],not[x2_]]`

which can be unified with the input for the rule:

`nand[x1_,x1_]→not[x1]`

where these rules follow from Substitution Lemma 6 and Axiom 2 respectively.

Critical Pair Lemma 2

The following expressions are equivalent:

`and[x1,x1]==not[not[x1]]`

Proof

Note that the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,x1_]→not[x1]`

where these rules follow from Axiom 6 and Axiom 2 respectively.

Critical Pair Lemma 3

The following expressions are equivalent:

`and[x1,nand[x1,x2]]==not[nand[x1,not[x2]]]`

Proof

Note that the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,nand[x1_,x2_]]→nand[x1,not[x2]]`

where these rules follow from Axiom 6 and Substitution Lemma 2 respectively.

Substitution Lemma 7

It can be shown that:

`and[x1,nand[x1,x2]]==and[x1,not[x2]]`

Proof

We start by taking Critical Pair Lemma 3, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Substitution Lemma 8

It can be shown that:

`nand[x1,and[x2,x3]]==nand[x2,not[nand[x1,x3]]]`

Proof

We start by taking Substitution Lemma 4, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Substitution Lemma 9

It can be shown that:

`nand[x1,and[x2,x3]]==nand[x2,and[x1,x3]]`

Proof

We start by taking Substitution Lemma 8, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Critical Pair Lemma 4

The following expressions are equivalent:

`and[x1,x1]==or[x1,x1]`

Proof

Note that the input for the rule:

`and[x1_,x1_]↔not[not[x1_]]`

contains a subpattern of the form:

`not[not[x1_]]`

which can be unified with the input for the rule:

`or[x1_,x1_]↔not[not[x1_]]`

where these rules follow from Critical Pair Lemma 2 and Critical Pair Lemma 1 respectively.

Critical Pair Lemma 5

The following expressions are equivalent:

`x1==nand[not[x1],not[x1]]`

Proof

Note that the input for the rule:

`nand[not[x1_],nand[x1_,x2_]]→x1`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,x1_]→not[x1]`

where these rules follow from Substitution Lemma 1 and Axiom 2 respectively.

Substitution Lemma 10

It can be shown that:

`x1==or[x1,x1]`

Proof

We start by taking Critical Pair Lemma 5, and apply the substitution:

`nand[not[x1_],not[x2_]]→or[x1,x2]`

which follows from Substitution Lemma 6.

Substitution Lemma 11

It can be shown that:

`x1==and[x1,x1]`

Proof

We start by taking Substitution Lemma 10, and apply the substitution:

`or[x1_,x1_]→and[x1,x1]`

which follows from Critical Pair Lemma 4.

Substitution Lemma 12

It can be shown that:

`x1==not[not[x1]]`

Proof

We start by taking Critical Pair Lemma 2, and apply the substitution:

`and[x1_,x1_]→x1`

which follows from Substitution Lemma 11.

Critical Pair Lemma 6

The following expressions are equivalent:

`nand[x1,x2]==not[and[x1,x2]]`

Proof

Note that the input for the rule:

`not[not[x1_]]→x1`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

where these rules follow from Substitution Lemma 12 and Axiom 6 respectively.

Critical Pair Lemma 7

The following expressions are equivalent:

`or[not[x1],x2]==nand[x1,not[x2]]`

Proof

Note that the input for the rule:

`nand[not[x1_],not[x2_]]→or[x1,x2]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Substitution Lemma 6 and Substitution Lemma 12 respectively.

Critical Pair Lemma 8

The following expressions are equivalent:

`or[not[x1],not[x2]]==nand[x1,x2]`

Proof

Note that the input for the rule:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Critical Pair Lemma 7 and Substitution Lemma 12 respectively.

Critical Pair Lemma 9

The following expressions are equivalent:

`and[x1,not[x2]]==not[or[not[x1],x2]]`

Proof

Note that the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

where these rules follow from Axiom 6 and Critical Pair Lemma 7 respectively.

Critical Pair Lemma 10

The following expressions are equivalent:

`nand[not[x1],x2]==or[x1,not[x2]]`

Proof

Note that the input for the rule:

`or[not[x1_],not[x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Critical Pair Lemma 8 and Substitution Lemma 12 respectively.

Substitution Lemma 13

It can be shown that:

`or[x1,not[nand[x1,x2]]]==x1`

Proof

We start by taking Substitution Lemma 1, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 10.

Substitution Lemma 14

It can be shown that:

`or[x1,and[x1,x2]]==x1`

Proof

We start by taking Substitution Lemma 13, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Critical Pair Lemma 11

The following expressions are equivalent:

`and[x1,not[not[x2]]]==and[x1,or[not[x1],x2]]`

Proof

Note that the input for the rule:

`and[x1_,nand[x1_,x2_]]→and[x1,not[x2]]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

where these rules follow from Substitution Lemma 7 and Critical Pair Lemma 7 respectively.

Substitution Lemma 15

It can be shown that:

`and[x1,x2]==and[x1,or[not[x1],x2]]`

Proof

We start by taking Critical Pair Lemma 11, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 12.

Critical Pair Lemma 12

The following expressions are equivalent:

`and[x1,not[and[not[x1],x2]]]==not[not[x1]]`

Proof

Note that the input for the rule:

`not[or[not[x1_],x2_]]→and[x1,not[x2]]`

contains a subpattern of the form:

`or[not[x1_],x2_]`

which can be unified with the input for the rule:

`or[x1_,and[x1_,x2_]]→x1`

where these rules follow from Critical Pair Lemma 9 and Substitution Lemma 14 respectively.

where these rules follow from Critical Pair Lemma 9 and Substitution Lemma 14 respectively.

Substitution Lemma 16

It can be shown that:

`and[x1,nand[not[x1],x2]]==not[not[x1]]`

Proof

We start by taking Critical Pair Lemma 12, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 6.

Substitution Lemma 17

It can be shown that:

`and[x1,or[x1,not[x2]]]==not[not[x1]]`

Proof

We start by taking Substitution Lemma 16, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 10.

Substitution Lemma 18

It can be shown that:

`and[x1,or[x1,not[x2]]]==x1`

Proof

We start by taking Substitution Lemma 17, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 12.

Critical Pair Lemma 13

The following expressions are equivalent:

`and[not[x1],not[x2]]==not[or[x1,x2]]`

Proof

Note that the input for the rule:

`not[or[not[x1_],x2_]]→and[x1,not[x2]]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

Out[•]= where these rules follow from Critical Pair Lemma 9 and Substitution Lemma 12 respectively.

Critical Pair Lemma 14

The following expressions are equivalent:

`x1==and[x1,or[x1,x2]]`

Proof

Note that the input for the rule:

Note that the input for the rule:

`and[x1_,or[x1_,not[x2_]]]→x1`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Substitution Lemma 18 and Substitution Lemma 12 respectively.

Critical Pair Lemma 15

The following expressions are equivalent:

`nand[x1,or[x1,x2]]==not[x1]`

Proof

Note that the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,or[x1_,x2_]]→x1`

where these rules follow from Critical Pair Lemma 6 and Critical Pair Lemma 14 respectively.

Critical Pair Lemma 16

The following expressions are equivalent:

`nand[x1,or[not[x1],x2]]==not[and[x1,x2]]`

Proof

Note that the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,or[not[x1_],x2_]]→and[x1,x2]`

where these rules follow from Critical Pair Lemma 6 and Substitution Lemma 15 respectively.

Substitution Lemma 19

It can be shown that:

`nand[x1,or[not[x1],x2]]==nand[x1,x2]`

Proof

We start by taking Critical Pair Lemma 16, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 6.

Critical Pair Lemma 17

The following expressions are equivalent:

`nand[x1,and[not[x1],x2]]==nand[x1,not[x1]]`

Proof

Note that the input for the rule:

`nand[x1_,or[not[x1_],x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`or[not[x1_],x2_]`

which can be unified with the input for the rule:

`or[x1_,and[x1_,x2_]]→x1`

where these rules follow from Substitution Lemma 19 and Substitution Lemma 14 respectively.

Substitution Lemma 20

It can be shown that:

`nand[x1,and[not[x1],x2]]==or[not[x1],x1]`

Proof

We start by taking Critical Pair Lemma 17, and apply the substitution:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 7.

Critical Pair Lemma 18

The following expressions are equivalent:

`nand[x1,and[x2,x1]]==nand[x2,x1]`

Proof

Note that the input for the rule:

`nand[x1_,and[x2_,x3_]]↔nand[x2_,and[x1_,x3_]]`

contains a subpattern of the form:

`and[x2_,x3_]`

which can be unified with the input for the rule:

`and[x1_,x1_]→x1`

where these rules follow from Substitution Lemma 9 and Substitution Lemma 11 respectively.

Critical Pair Lemma 19

The following expressions are equivalent:

`nand[not[x1],x1]==or[not[x1],x1]`

Proof

Note that the input for the rule:

`nand[x1_,and[x2_,x1_]]→nand[x2,x1]`

contains a subpattern of the form:

`nand[x1_,and[x2_,x1_]]`

which can be unified with the input for the rule:

`nand[x1_,and[not[x1_],x2_]]→or[not[x1],x1]`

where these rules follow from Critical Pair Lemma 18 and Substitution Lemma 20 respectively.

Substitution Lemma 21

It can be shown that:

`or[x1,not[x1]]==or[not[x1],x1]`

Proof

We start by taking Critical Pair Lemma 19, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 10.

Critical Pair Lemma 20

The following expressions are equivalent:

`nand[not[x1],not[x2]]==nand[not[x2],not[or[x1,x2]]]`

Proof

Note that the input for the rule:

`nand[x1_,and[x2_,x1_]]→nand[x2,x1]`

contains a subpattern of the form:

`and[x2_,x1_]`

which can be unified with the input for the rule:

`and[not[x1_],not[x2_]]→not[or[x1,x2]]`

where these rules follow from Critical Pair Lemma 18 and Critical Pair Lemma 13 respectively.

Substitution Lemma 22

It can be shown that:

`or[x1,not[not[x2]]]==nand[not[x2],not[or[x1,x2]]]`

Proof

We start by taking Critical Pair Lemma 20, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 10.

Substitution Lemma 23

It can be shown that:

`or[x1,x2]==nand[not[x2],not[or[x1,x2]]]`

Proof

We start by taking Substitution Lemma 22, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 12.

Substitution Lemma 24

It can be shown that:

`or[x1,x2]==or[x2,not[not[or[x1,x2]]]]`

Proof

We start by taking Substitution Lemma 23, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 10.

Substitution Lemma 25

It can be shown that:

`or[x1,x2]==or[x2,or[x1,x2]]`

Proof

We start by taking Substitution Lemma 24, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 12.

Critical Pair Lemma 21

The following expressions are equivalent:

`not[x1]==nand[x1,or[x2,x1]]`

Proof

Note that the input for the rule:

`nand[x1_,or[x1_,x2_]]→not[x1]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,x1_]]→or[x2,x1]`

where these rules follow from Critical Pair Lemma 15 and Substitution Lemma 25 respectively.

Critical Pair Lemma 22

The following expressions are equivalent:

`not[not[x1]]==nand[not[x1],nand[x2,x1]]`

Proof

Note that the input for the rule:

`nand[x1_,or[x2_,x1_]]→not[x1]`

contains a subpattern of the form:

`or[x2_,x1_]`

which can be unified with the input for the rule:

`or[not[x1_],not[x2_]]→nand[x1,x2]`

where these rules follow from Critical Pair Lemma 21 and Critical Pair Lemma 8 respectively.

Substitution Lemma 26

It can be shown that:

`x1==nand[not[x1],nand[x2,x1]]`

Proof

We start by taking Critical Pair Lemma 22, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 12.

which follows from Substitution Lemma 12.

Substitution Lemma 27

It can be shown that:

`x1==or[x1,not[nand[x2,x1]]]`

Proof

We start by taking Substitution Lemma 26, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 10.

Substitution Lemma 28

It can be shown that:

`x1==or[x1,and[x2,x1]]`

Proof

We start by taking Substitution Lemma 27, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Critical Pair Lemma 23

The following expressions are equivalent:

`nand[x1,and[x2,not[x1]]]==nand[x1,not[x1]]`

Proof

Note that the input for the rule:

`nand[x1_,or[not[x1_],x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`or[not[x1_],x2_]`

which can be unified with the input for the rule:

`or[x1_,and[x2_,x1_]]→x1`

where these rules follow from Substitution Lemma 19 and Substitution Lemma 28 respectively.

Substitution Lemma 29

It can be shown that:

`nand[x1,and[x2,not[x1]]]==or[not[x1],x1]`

Proof

We start by taking Critical Pair Lemma 23, and apply the substitution:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 7.

Substitution Lemma 30

It can be shown that:

`nand[x1,and[x2,not[x1]]]==or[x1,not[x1]]`

Proof

We start by taking Substitution Lemma 29, and apply the substitution:

We start by taking Substitution Lemma 29, and apply the substitution:

`or[not[x1_],x1_]→or[x1,not[x1]]`

which follows from Substitution Lemma 21.

Critical Pair Lemma 24

The following expressions are equivalent:

`or[x1,not[x1]]==nand[x2,and[x1,not[x1]]]`

Proof

Note that the input for the rule:

`nand[x1_,and[x2_,not[x1_]]]→or[x1,not[x1]]`

contains a subpattern of the form:

`nand[x1_,and[x2_,not[x1_]]]`

which can be unified with the input for the rule:

`nand[x1_,and[x2_,x3_]]↔nand[x2_,and[x1_,x3_]]`

where these rules follow from Substitution Lemma 30 and Substitution Lemma 9 respectively.

Critical Pair Lemma 25

The following expressions are equivalent:

`or[x1,not[x1]]==or[x2,not[and[x1,not[x1]]]]`

Proof

Note that the input for the rule:

`nand[x1_,and[x2_,not[x2_]]]→or[x2,not[x2]]`

contains a subpattern of the form:

`nand[x1_,and[x2_,not[x2_]]]`

which can be unified with the input for the rule:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

where these rules follow from Critical Pair Lemma 24 and Critical Pair Lemma 10 respectively.

Substitution Lemma 31

It can be shown that:

`or[x1,not[x1]]==or[x2,nand[x1,not[x1]]]`

Proof

We start by taking Critical Pair Lemma 25, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 6.

Substitution Lemma 32

It can be shown that:

`or[x1,not[x1]]==or[x2,or[not[x1],x1]]`

Proof

We start by taking Substitution Lemma 31, and apply the substitution:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 7.

Substitution Lemma 33

It can be shown that:

`or[x1,not[x1]]==or[x2,or[x1,not[x1]]]`

Proof

We start by taking Substitution Lemma 32, and apply the substitution:

`or[not[x1_],x1_]→or[x1,not[x1]]`

which follows from Substitution Lemma 21.

Critical Pair Lemma 26

The following expressions are equivalent:

`x1==and[x1,or[x2,not[x2]]]`

Proof

Note that the input for the rule:

`and[x1_,or[x1_,x2_]]→x1`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,not[x2_]]]→or[x2,not[x2]]`

where these rules follow from Critical Pair Lemma 14 and Substitution Lemma 33 respectively.

Critical Pair Lemma 27

The following expressions are equivalent:

`nand[x1,and[x2,or[x3,not[x3]]]]==nand[x2,x1]`

Proof

Note that the input for the rule:

`nand[x1_,and[x2_,x3_]]↔nand[x2_,and[x1_,x3_]]`

contains a subpattern of the form:

`and[x2_,x3_]`

which can be unified with the input for the rule:

`and[x1_,or[x2_,not[x2_]]]→x1`

where these rules follow from Substitution Lemma 9 and Critical Pair Lemma 26 respectively.

Substitution Lemma 34

It can be shown that:

`nand[x1,x2]==nand[x2,x1]`

Proof

We start by taking Critical Pair Lemma 27, and apply the substitution:

`and[x1_,or[x2_,not[x2_]]]→x1`

which follows from Critical Pair Lemma 26.

Critical Pair Lemma 28

The following expressions are equivalent:

`and[x1,x2]==not[nand[x2,x1]]`

PROOF

Note that the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,x2_]↔nand[x2_,x1_]`

where these rules follow from Axiom 6 and Substitution Lemma 34 respectively.

Substitution Lemma 35

It can be shown that:

`and[x1,x2]==and[x2,x1]`

PROOF

We start by taking Critical Pair Lemma 28, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Conclusion 1

We obtain the conclusion:

`True`

PROOF

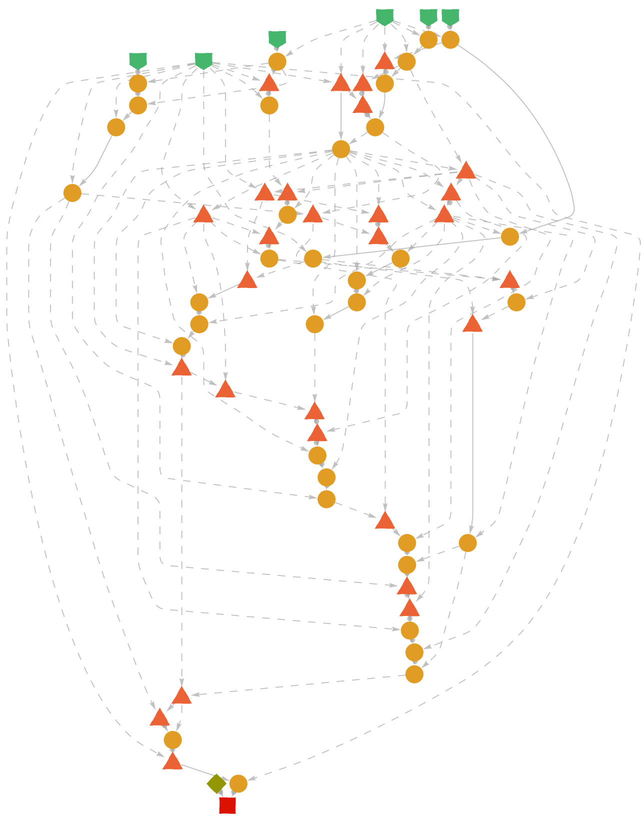
Take Hypothesis 1, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Substitution Lemma 35.

```
In[]:= proofAxB1fromShort["ProofGraph"]
```

Out[ ]=



In[ ]:= Clear[proofAxB1fromShort]

In[ ]:= proofAxB2fromShort["ProofNotebook"]

Axiom 1

We are given that:

`x1==nand[nand[x1,x1],nand[x1,x2]]`

Axiom 2

We are given that:

`nand[x1,x1]==not[x1]`

Axiom 3

We are given that:

`nand[x1,nand[x1,x2]]==nand[x1,nand[x2,x2]]`

Axiom 4

We are given that:

`nand[x1,nand[x1,nand[x2,x3]]]==nand[x2,nand[x2,nand[x1,x3]]]`

Axiom 5

We are given that:

`nand[nand[x1,x1],nand[x2,x2]]==or[x1,x2]`

Axiom 6

We are given that:

`not[nand[x1,x2]]==and[x1,x2]`

Hypothesis 1

We would like to show that:

`or[a,b]==or[b,a]`

Substitution Lemma 1

It can be shown that:

`nand[not[x1],nand[x1,x2]]==x1`

Proof

We start by taking Axiom 1, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Substitution Lemma 2

It can be shown that:

`nand[x1,nand[x1,x2]]==nand[x1,not[x2]]`

Proof

We start by taking Axiom 3, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Critical Pair Lemma 1

The following expressions are equivalent:

`nand[x1,not[nand[x1,x2]]]==nand[x1,nand[x1,not[x2]]]`

Proof

Note that the input for the rule:

`nand[x1_,nand[x1_,x2_]]→nand[x1,not[x2]]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,nand[x1_,x2_]]→nand[x1,not[x2]]`

where these rules follow from Substitution Lemma 2 and Substitution Lemma 2 respectively.

Substitution Lemma 3

It can be shown that:

`nand[x1,not[nand[x1,x2]]]==nand[x1,not[not[x2]]]`

Proof

We start by taking Critical Pair Lemma 1, and apply the substitution:

`nand[x1_,nand[x1_,x2_]]→nand[x1,not[x2]]`

which follows from Substitution Lemma 2.

Substitution Lemma 4

It can be shown that:

`nand[x1,not[nand[x2,x3]]]==nand[x2,nand[x2,nand[x1,x3]]]`

Proof

We start by taking Axiom 4, and apply the substitution:

`nand[x1_,nand[x1_,x2_]]→nand[x1,not[x2]]`

which follows from Substitution Lemma 2.

Substitution Lemma 5

It can be shown that:

`nand[x1,not[nand[x2,x3]]]==nand[x2,not[nand[x1,x3]]]`

Proof

We start by taking Substitution Lemma 4, and apply the substitution:

`nand[x1_,nand[x1_,x2_]]→nand[x1,not[x2]]`

which follows from Substitution Lemma 2.

Substitution Lemma 6

It can be shown that:

`nand[not[x1],nand[x2,x2]]==or[x1,x2]`

Proof

We start by taking Axiom 5, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Substitution Lemma 7

It can be shown that:

`nand[not[x1],not[x2]]==or[x1,x2]`

Proof

We start by taking Substitution Lemma 6, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Critical Pair Lemma 2

The following expressions are equivalent:

`or[x1,x1]==not[not[x1]]`

Proof

Note that the input for the rule:

`nand[not[x1_],not[x2_]]→or[x1,x2]`

contains a subpattern of the form:

`nand[not[x1_],not[x2_]]`

`nand[not[x1_],not[x2_]]`

which can be unified with the input for the rule:

`nand[x1_,x1_]→not[x1]`

where these rules follow from Substitution Lemma 7 and Axiom 2 respectively.

Critical Pair Lemma 3

The following expressions are equivalent:

`and[x1,x1]==not[not[x1]]`

Proof

Note that the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,x1_]→not[x1]`

where these rules follow from Axiom 6 and Axiom 2 respectively.

Critical Pair Lemma 4

The following expressions are equivalent:

`and[x1,nand[x1,x2]]==not[nand[x1,not[x2]]]`

Proof

Note that the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,nand[x1_,x2_]]→nand[x1,not[x2]]`

where these rules follow from Axiom 6 and Substitution Lemma 2 respectively.

Substitution Lemma 8

It can be shown that:

`and[x1,nand[x1,x2]]==and[x1,not[x2]]`

Proof

We start by taking Critical Pair Lemma 4, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Substitution Lemma 9

It can be shown that:

`nand[x1,and[x2,x3]]==nand[x2,not[nand[x1,x3]]]`

Proof

We start by taking Substitution Lemma 5, and apply the substitution:

```
not[nand[x1_,x2_]]→and[x1,x2]
```

which follows from Axiom 6.

Substitution Lemma 10

It can be shown that:

```
nand[x1,and[x2,x3]]==nand[x2,and[x1,x3]]
```

Proof

We start by taking Substitution Lemma 9, and apply the substitution:

```
not[nand[x1_,x2_]]→and[x1,x2]
```

which follows from Axiom 6.

Critical Pair Lemma 5

The following expressions are equivalent:

```
and[x1,x1]==or[x1,x1]
```

Proof

Note that the input for the rule:

```
and[x1_,x1_]↔not[not[x1_]]
```

contains a subpattern of the form:

```
not[not[x1_]]
```

which can be unified with the input for the rule:

```
or[x1_,x1_]↔not[not[x1_]]
```

where these rules follow from Critical Pair Lemma 3 and Critical Pair Lemma 2 respectively.

Critical Pair Lemma 6

The following expressions are equivalent:

```
x1==nand[not[x1],not[x1]]
```

Proof

Note that the input for the rule:

```
nand[not[x1_],nand[x1_,x2_]]→x1
```

contains a subpattern of the form:

```
nand[x1_,x2_]
```

which can be unified with the input for the rule:

```
nand[x1_,x1_]→not[x1]
```

where these rules follow from Substitution Lemma 1 and Axiom 2 respectively.

Substitution Lemma 11

It can be shown that:

```
x1==or[x1,x1]
```

Proof

We start by taking Critical Pair Lemma 6, and apply the substitution:

```
nand[not[x1_],not[x2_]]→or[x1,x2]
```

which follows from Substitution Lemma 7.

Substitution Lemma 12

It can be shown that:

`and[x1_,x1_]→x1`

Proof

We start by taking Critical Pair Lemma 5, and apply the substitution:

`or[x1_,x1_]→x1`

which follows from Substitution Lemma 11.

Substitution Lemma 13

It can be shown that:

`x1==not[not[x1]]`

Proof

We start by taking Critical Pair Lemma 2, and apply the substitution:

`or[x1_,x1_]→x1`

which follows from Substitution Lemma 11.

Critical Pair Lemma 7

The following expressions are equivalent:

`nand[x1,x2]==not[and[x1,x2]]`

Proof

Note that the input for the rule:

`not[not[x1_]]→x1`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

where these rules follow from Substitution Lemma 13 and Axiom 6 respectively.

Critical Pair Lemma 8

The following expressions are equivalent:

`or[not[x1],x2]==nand[x1,not[x2]]`

Proof

Note that the input for the rule:

`nand[not[x1_],not[x2_]]→or[x1,x2]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Substitution Lemma 7 and Substitution Lemma 13 respectively.

Substitution Lemma 14

It can be shown that:

`nand[x1,and[x1,x2]]==nand[x1,not[not[x2]]]`

Proof

We start by taking Substitution Lemma 3, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Substitution Lemma 15

It can be shown that:

`nand[x1,and[x1,x2]]==nand[x1,x2]`

Proof

We start by taking Substitution Lemma 14, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 13.

Substitution Lemma 16

It can be shown that:

`nand[x1_,nand[x1_,x2_]]→or[not[x1],x2]`

Proof

We start by taking Substitution Lemma 2, and apply the substitution:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 8.

Critical Pair Lemma 9

The following expressions are equivalent:

`or[not[x1],not[x2]]==nand[x1,x2]`

Proof

Note that the input for the rule:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Critical Pair Lemma 8 and Substitution Lemma 13 respectively.

Critical Pair Lemma 10

The following expressions are equivalent:

`and[x1,not[x2]]==not[or[not[x1],x2]]`

Proof

Note that the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

where these rules follow from Axiom 6 and Critical Pair Lemma 8 respectively.

Critical Pair Lemma 11

The following expressions are equivalent:

`nand[not[x1],x2]==or[x1,not[x2]]`

Proof

Note that the input for the rule:

`or[not[x1_],not[x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Critical Pair Lemma 9 and Substitution Lemma 13 respectively.

Critical Pair Lemma 12

The following expressions are equivalent:

`or[x1,not[nand[not[x1],x2]]]==or[not[not[x1]],x2]`

Proof

Note that the input for the rule:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

contains a subpattern of the form:

`nand[not[x1_],x2_]`

which can be unified with the input for the rule:

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where these rules follow from Critical Pair Lemma 11 and Substitution Lemma 16 respectively.

Substitution Lemma 17

It can be shown that:

`or[x1,and[not[x1],x2]]==or[not[not[x1]],x2]`

Proof

We start by taking Critical Pair Lemma 12, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Substitution Lemma 18

It can be shown that:

`or[x1,and[not[x1],x2]]==or[x1,x2]`

Proof

We start by taking Substitution Lemma 17, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 13.

Substitution Lemma 19

It can be shown that:

`or[x1,not[nand[x1,x2]]]==x1`

Proof

We start by taking Substitution Lemma 1, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 11.

Substitution Lemma 20

It can be shown that:

`or[x1,and[x1,x2]]==x1`

Proof

We start by taking Substitution Lemma 19, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Critical Pair Lemma 13

The following expressions are equivalent:

`and[x1,not[not[x2]]]==and[x1,or[not[x1],x2]]`

Proof

Note that the input for the rule:

`and[x1_,nand[x1_,x2_]]→and[x1,not[x2]]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

where these rules follow from Substitution Lemma 8 and Critical Pair Lemma 8 respectively.

Substitution Lemma 21

It can be shown that:

`and[x1,x2]==and[x1,or[not[x1],x2]]`

Proof

We start by taking Critical Pair Lemma 13, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 13.

Critical Pair Lemma 14

The following expressions are equivalent:

The following expressions are equivalent:

`and[x1,not[and[not[x1],x2]]]==not[not[x1]]`

PROOF

Note that the input for the rule:

`not[or[not[x1_],x2_]]→and[x1,not[x2]]`

contains a subpattern of the form:

`or[not[x1_],x2_]`

which can be unified with the input for the rule:

`or[x1_,and[x1_,x2_]]→x1`

where these rules follow from Critical Pair Lemma 10 and Substitution Lemma 20 respectively.

Substitution Lemma 22

It can be shown that:

`and[x1,nand[not[x1],x2]]==not[not[x1]]`

PROOF

We start by taking Critical Pair Lemma 14, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 7.

Substitution Lemma 23

It can be shown that:

`and[x1,or[x1,not[x2]]]==not[not[x1]]`

PROOF

We start by taking Substitution Lemma 22, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 11.

Substitution Lemma 24

It can be shown that:

`and[x1,or[x1,not[x2]]]==x1`

PROOF

We start by taking Substitution Lemma 23, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 13.

Critical Pair Lemma 15

The following expressions are equivalent:

`and[not[x1],not[x2]]==not[or[x1,x2]]`

Out[]=

PROOF

Note that the input for the rule:

`not[or[not[x1_],x2_]]→and[x1,not[x2]]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Critical Pair Lemma 10 and Substitution Lemma 13 respectively.

Critical Pair Lemma 16

The following expressions are equivalent:

`x1==and[x1,or[x1,x2]]`

Proof

Note that the input for the rule:

`and[x1_,or[x1_,not[x2_]]]→x1`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Substitution Lemma 24 and Substitution Lemma 13 respectively.

Critical Pair Lemma 17

The following expressions are equivalent:

`nand[x1,or[x1,x2]]==nand[x1,x1]`

Proof

Note that the input for the rule:

`nand[x1_,and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,or[x1_,x2_]]→x1`

where these rules follow from Substitution Lemma 15 and Critical Pair Lemma 16 respectively.

Substitution Lemma 25

It can be shown that:

`nand[x1,or[x1,x2]]==not[x1]`

Proof

We start by taking Critical Pair Lemma 17, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Critical Pair Lemma 18

The following expressions are equivalent:

`nand[x1,or[not[x1],x2]]==nand[x1,and[x1,x2]]`

Proof

Note that the input for the rule:

Note that the input for the rule:

`nand[x1_,and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,or[not[x1_],x2_]]→and[x1,x2]`

where these rules follow from Substitution Lemma 15 and Substitution Lemma 21 respectively.

Substitution Lemma 26

It can be shown that:

`nand[x1,or[not[x1],x2]]==nand[x1,x2]`

Proof

We start by taking Critical Pair Lemma 18, and apply the substitution:

`nand[x1_,and[x1_,x2_]]→nand[x1,x2]`

which follows from Substitution Lemma 15.

Critical Pair Lemma 19

The following expressions are equivalent:

`nand[x1,and[not[x1],x2]]==nand[x1,not[x1]]`

Proof

Note that the input for the rule:

`nand[x1_,or[not[x1_],x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`or[not[x1_],x2_]`

which can be unified with the input for the rule:

`or[x1_,and[x1_,x2_]]→x1`

where these rules follow from Substitution Lemma 26 and Substitution Lemma 20 respectively.

Substitution Lemma 27

It can be shown that:

`nand[x1,and[not[x1],x2]]==or[not[x1],x1]`

Proof

We start by taking Critical Pair Lemma 19, and apply the substitution:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 8.

Critical Pair Lemma 20

The following expressions are equivalent:

`nand[x1,and[x2,x1]]==nand[x2,x1]`

Proof

Note that the input for the rule:

`nand[x1_,and[x2_,x3_]]↔nand[x2_,and[x1_,x3_]]`

contains a subpattern of the form:

`and[x2_,x3_]`

which can be unified with the input for the rule:

`Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[62,or[x1_,x1_]→`

where these rules follow from Substitution Lemma 10 and Substitution Lemma 12 respectively.

Critical Pair Lemma 21

The following expressions are equivalent:

`nand[not[x1],x1]==or[not[x1],x1]`

Proof

Note that the input for the rule:

`nand[x1_,and[x2_,x1_]]→nand[x2,x1]`

contains a subpattern of the form:

`nand[x1_,and[x2_,x1_]]`

which can be unified with the input for the rule:

`nand[x1_,and[not[x1_],x2_]]→or[not[x1],x1]`

where these rules follow from Critical Pair Lemma 20 and Substitution Lemma 27 respectively.

Substitution Lemma 28

It can be shown that:

`or[x1,not[x1]]==or[not[x1],x1]`

Proof

We start by taking Critical Pair Lemma 21, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 11.

Critical Pair Lemma 22

The following expressions are equivalent:

`nand[not[x1],not[x2]]==nand[not[x2],not[or[x1,x2]]]`

Proof

Note that the input for the rule:

`nand[x1_,and[x2_,x1_]]→nand[x2,x1]`

contains a subpattern of the form:

`and[x2_,x1_]`

which can be unified with the input for the rule:

`and[not[x1_],not[x2_]]→not[or[x1,x2]]`

where these rules follow from Critical Pair Lemma 20 and Critical Pair Lemma 15 respectively.

Substitution Lemma 29

It can be shown that:

`or[x1,not[not[x2]]]==nand[not[x2],not[or[x1,x2]]]`

Proof

We start by taking Critical Pair Lemma 22, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 11.

Substitution Lemma 30

It can be shown that:

`or[x1,x2]==nand[not[x2],not[or[x1,x2]]]`

Proof

We start by taking Substitution Lemma 29, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 13.

Substitution Lemma 31

It can be shown that:

`or[x1,x2]==or[x2,not[not[or[x1,x2]]]]`

Proof

We start by taking Substitution Lemma 30, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 11.

Substitution Lemma 32

It can be shown that:

`or[x1,x2]==or[x2,or[x1,x2]]`

Proof

We start by taking Substitution Lemma 31, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 13.

Critical Pair Lemma 23

The following expressions are equivalent:

`not[x1]==nand[x1,or[x2,x1]]`

Proof

Note that the input for the rule:

`nand[x1_,or[x1_,x2_]]→not[x1]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,x1_]]→or[x2,x1]`

where these rules follow from Substitution Lemma 25 and Substitution Lemma 32 respectively.

Critical Pair Lemma 24

The following expressions are equivalent:

```
not[not[x1]]==nand[not[x1],nand[x2,x1]]
```

PROOF

Note that the input for the rule:

```
nand[x1_,or[x2_,x1_]]→not[x1]
```

contains a subpattern of the form:

```
or[x2_,x1_]
```

which can be unified with the input for the rule:

```
or[not[x1_],not[x2_]]→nand[x1,x2]
```

where these rules follow from Critical Pair Lemma 23 and Critical Pair Lemma 9 respectively.

Substitution Lemma 33

It can be shown that:

```
x1==nand[not[x1],nand[x2,x1]]
```

PROOF

We start by taking Critical Pair Lemma 24, and apply the substitution:

```
not[not[x1_]]→x1
```

which follows from Substitution Lemma 13.

Substitution Lemma 34

It can be shown that:

```
x1==or[x1,not[nand[x2,x1]]]
```

PROOF

We start by taking Substitution Lemma 33, and apply the substitution:

```
nand[not[x1_],x2_]→or[x1,not[x2]]
```

which follows from Critical Pair Lemma 11.

Substitution Lemma 35

It can be shown that:

```
x1==or[x1,and[x2,x1]]
```

PROOF

We start by taking Substitution Lemma 34, and apply the substitution:

```
not[nand[x1_,x2_]]→and[x1,x2]
```

which follows from Axiom 6.

Critical Pair Lemma 25

The following expressions are equivalent:

```
nand[x1,and[x2,not[x1]]]==nand[x1,not[x1]]
```

PROOF

Note that the input for the rule:

```
nand[x1_,or[not[x1_],x2_]]→nand[x1,x2]
```

contains a subpattern of the form:

`or[not[x1_],x2_]`

which can be unified with the input for the rule:

`or[x1_,and[x2_,x1_]]→x1`

where these rules follow from Substitution Lemma 26 and Substitution Lemma 35 respectively.

Substitution Lemma 36

It can be shown that:

`nand[x1,and[x2,not[x1]]]==or[not[x1],x1]`

Proof

We start by taking Critical Pair Lemma 25, and apply the substitution:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 8.

Substitution Lemma 37

It can be shown that:

`nand[x1,and[x2,not[x1]]]==or[x1,not[x1]]`

Proof

We start by taking Substitution Lemma 36, and apply the substitution:

`or[not[x1_],x1_]→or[x1,not[x1]]`

which follows from Substitution Lemma 28.

Critical Pair Lemma 26

The following expressions are equivalent:

`or[x1,not[x1]]==nand[x2,and[x1,not[x1]]]`

Proof

Note that the input for the rule:

`nand[x1_,and[x2_,not[x1_]]]→or[x1,not[x1]]`

contains a subpattern of the form:

`nand[x1_,and[x2_,not[x1_]]]`

which can be unified with the input for the rule:

`nand[x1_,and[x2_,x3_]]↔nand[x2_,and[x1_,x3_]]`

where these rules follow from Substitution Lemma 37 and Substitution Lemma 10 respectively.

Critical Pair Lemma 27

The following expressions are equivalent:

`or[x1,not[x1]]==or[x2,not[and[x1,not[x1]]]]`

Proof

Note that the input for the rule:

`nand[x1_,and[x2_,not[x2_]]]→or[x2,not[x2]]`

contains a subpattern of the form:

`nand[x1_,and[x2_,not[x2_]]]`

which can be unified with the input for the rule:

which can be unified with the input for the rule:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

where these rules follow from Critical Pair Lemma 26 and Critical Pair Lemma 11 respectively.

Substitution Lemma 38

It can be shown that:

`or[x1,not[x1]]==or[x2,nand[x1,not[x1]]]`

Proof

We start by taking Critical Pair Lemma 27, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 7.

Substitution Lemma 39

It can be shown that:

`or[x1,not[x1]]==or[x2,or[not[x1],x1]]`

Proof

We start by taking Substitution Lemma 38, and apply the substitution:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 8.

Substitution Lemma 40

It can be shown that:

`or[x1,not[x1]]==or[x2,or[x1,not[x1]]]`

Proof

We start by taking Substitution Lemma 39, and apply the substitution:

`or[not[x1_],x1_]→or[x1,not[x1]]`

which follows from Substitution Lemma 28.

Critical Pair Lemma 28

The following expressions are equivalent:

`x1==and[x1,or[x2,not[x2]]]`

Proof

Note that the input for the rule:

`and[x1_,or[x1_,x2_]]→x1`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,not[x2_]]]→or[x2,not[x2]]`

where these rules follow from Critical Pair Lemma 16 and Substitution Lemma 40 respectively.

Critical Pair Lemma 29

The following expressions are equivalent:

`nand[x1,and[x2,or[x3,not[x3]]]]==nand[x2,x1]`

Proof

Note that the input for the rule:

`nand[x1_,and[x2_,x3_]]↔nand[x2_,and[x1_,x3_]]`

contains a subpattern of the form:

`and[x2_,x3_]`

which can be unified with the input for the rule:

`and[x1_,or[x2_,not[x2_]]]→x1`

where these rules follow from Substitution Lemma 10 and Critical Pair Lemma 28 respectively.

Substitution Lemma 41

It can be shown that:

`nand[x1,x2]==nand[x2,x1]`

Proof

We start by taking Critical Pair Lemma 29, and apply the substitution:

`and[x1_,or[x2_,not[x2_]]]→x1`

which follows from Critical Pair Lemma 28.

Critical Pair Lemma 30

The following expressions are equivalent:

`and[x1,not[x2]]==and[x1,nand[x2,x1]]`

Proof

Note that the input for the rule:

`and[x1_,nand[x1_,x2_]]→and[x1,not[x2]]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,x2_]↔nand[x2_,x1_]`

where these rules follow from Substitution Lemma 8 and Substitution Lemma 41 respectively.

Critical Pair Lemma 31

The following expressions are equivalent:

`and[x1,not[not[x2]]]==and[x1,or[x2,not[x1]]]`

Proof

Note that the input for the rule:

`and[x1_,nand[x2_,x1_]]→and[x1,not[x2]]`

contains a subpattern of the form:

`nand[x2_,x1_]`

which can be unified with the input for the rule:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

where these rules follow from Critical Pair Lemma 30 and Critical Pair Lemma 11 respectively.

Substitution Lemma 42

Substitution Lemma 42

It can be shown that:

`and[x1,x2]==and[x1,or[x2,not[x1]]]`

Proof

We start by taking Critical Pair Lemma 31, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 13.

Critical Pair Lemma 32

The following expressions are equivalent:

`or[x1,or[x2,not[not[x1]]]]==or[x1,and[not[x1],x2]]`

Proof

Note that the input for the rule:

`or[x1_,and[not[x1_],x2_]]→or[x1,x2]`

contains a subpattern of the form:

`and[not[x1_],x2_]`

which can be unified with the input for the rule:

`and[x1_,or[x2_,not[x1_]]]→and[x1,x2]`

where these rules follow from Substitution Lemma 18 and Substitution Lemma 42 respectively.

Substitution Lemma 43

It can be shown that:

`or[x1,or[x2,x1]]==or[x1,and[not[x1],x2]]`

Proof

We start by taking Critical Pair Lemma 32, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 13.

Substitution Lemma 44

It can be shown that:

`or[x1,x2]==or[x2,and[not[x2],x1]]`

Proof

We start by taking Substitution Lemma 43, and apply the substitution:

`or[x1_,or[x2_,x1_]]→or[x2,x1]`

which follows from Substitution Lemma 32.

Substitution Lemma 45

It can be shown that:

`or[x1,x2]==or[x2,x1]`

Proof

We start by taking Substitution Lemma 44, and apply the substitution:

`or[x1_,and[not[x1_],x2_]]→or[x1,x2]`

```
or[x1_,and[not[x1_],x2_]]→or[x1,x2]
```

which follows from Substitution Lemma 18.

Conclusion 1

We obtain the conclusion:

```
True
```

Proof

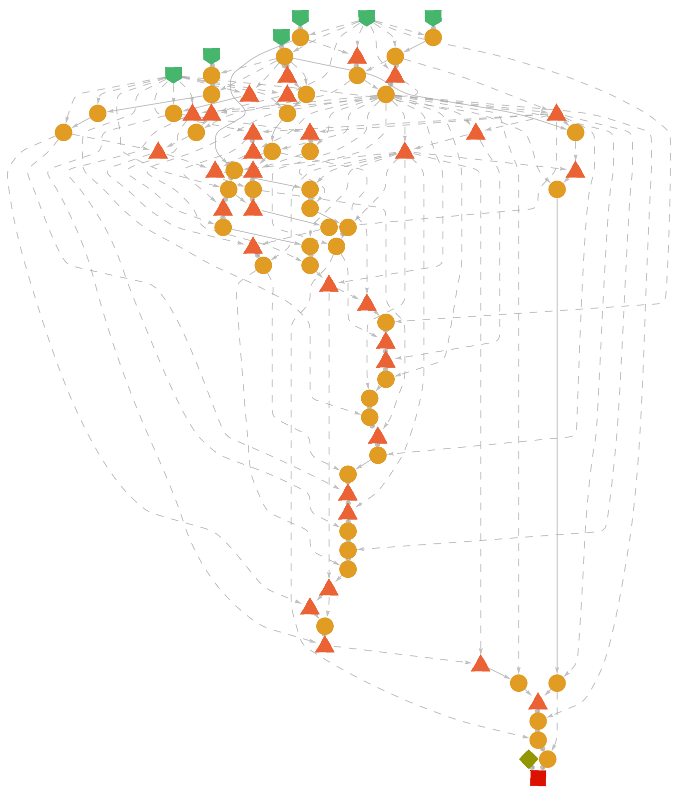
Take Hypothesis 1, and apply the substitution:

```
or[x1_,x2_]→or[x2,x1]
```

which follows from Substitution Lemma 45.

In[∘]:=
```
proofAxB2fromShort["ProofGraph"]
```

Out[∘]=



In[∘]:=
```
Clear[proofAxB2fromShort]
```

In[∘]:=
```
proofAxB3fromShort["ProofNotebook"]
```

Axiom 1

We are given that:

```
x1==nand[nand[x1,x1],nand[x1,x2]]
```

Axiom 2

We are given that:

`nand[x1,x1]==not[x1]`

Axiom 3

We are given that:

`nand[x1,nand[x1,x2]]==nand[x1,nand[x2,x2]]`

Axiom 4

We are given that:

`nand[x1,nand[x1,nand[x2,x3]]]==nand[x2,nand[x2,nand[x1,x3]]]`

Axiom 5

We are given that:

`nand[nand[x1,x1],nand[x2,x2]]==or[x1,x2]`

Axiom 6

We are given that:

`not[nand[x1,x2]]==and[x1,x2]`

Hypothesis 1

We would like to show that:

`and[a,or[b,not[b]]]==a`

Substitution Lemma 1

It can be shown that:

`nand[not[x1],nand[x1,x2]]==x1`

Proof

We start by taking Axiom 1, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Substitution Lemma 2

It can be shown that:

`nand[x1,nand[x1,x2]]==nand[x1,not[x2]]`

Proof

We start by taking Axiom 3, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Substitution Lemma 3

It can be shown that:

`nand[x1,not[nand[x2,x3]]]==nand[x2,nand[x2,nand[x1,x3]]]`

Proof

We start by taking Axiom 4, and apply the substitution:

`nand[x1_,nand[x1_,x2_]]→nand[x1,not[x2]]`

which follows from Substitution Lemma 2.

which follows from Substitution Lemma 2.

Substitution Lemma 4

It can be shown that:

`nand[x1,not[nand[x2,x3]]]==nand[x2,not[nand[x1,x3]]]`

Proof

We start by taking Substitution Lemma 3, and apply the substitution:

`nand[x1_,nand[x1_,x2_]]→nand[x1,not[x2]]`

which follows from Substitution Lemma 2.

Substitution Lemma 5

It can be shown that:

`nand[not[x1],nand[x2,x2]]==or[x1,x2]`

Proof

We start by taking Axiom 5, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Substitution Lemma 6

It can be shown that:

`nand[not[x1],not[x2]]==or[x1,x2]`

Proof

We start by taking Substitution Lemma 5, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Critical Pair Lemma 1

The following expressions are equivalent:

`or[x1,x1]==not[not[x1]]`

Proof

Note that the input for the rule:

`nand[not[x1_],not[x2_]]→or[x1,x2]`

contains a subpattern of the form:

`nand[not[x1_],not[x2_]]`

which can be unified with the input for the rule:

`nand[x1_,x1_]→not[x1]`

where these rules follow from Substitution Lemma 6 and Axiom 2 respectively.

Critical Pair Lemma 2

The following expressions are equivalent:

`and[x1,x1]==not[not[x1]]`

Proof

Note that the input for the rule:

Note that the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,x1_]→not[x1]`

where these rules follow from Axiom 6 and Axiom 2 respectively.

Critical Pair Lemma 3

The following expressions are equivalent:

`and[x1,nand[x1,x2]]==not[nand[x1,not[x2]]]`

Proof

Note that the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,nand[x1_,x2_]]→nand[x1,not[x2]]`

where these rules follow from Axiom 6 and Substitution Lemma 2 respectively.

Substitution Lemma 7

It can be shown that:

`and[x1,nand[x1,x2]]==and[x1,not[x2]]`

Proof

We start by taking Critical Pair Lemma 3, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Substitution Lemma 8

It can be shown that:

`nand[x1,and[x2,x3]]==nand[x2,not[nand[x1,x3]]]`

Proof

We start by taking Substitution Lemma 4, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Substitution Lemma 9

It can be shown that:

`nand[x1,and[x2,x3]]==nand[x2,and[x1,x3]]`

Proof

We start by taking Substitution Lemma 8, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Critical Pair Lemma 4

The following expressions are equivalent:

`and[x1,x1]==or[x1,x1]`

Proof

Note that the input for the rule:

`and[x1_,x1_]↔not[not[x1_]]`

contains a subpattern of the form:

`not[not[x1_]]`

which can be unified with the input for the rule:

`or[x1_,x1_]↔not[not[x1_]]`

where these rules follow from Critical Pair Lemma 2 and Critical Pair Lemma 1 respectively.

Critical Pair Lemma 5

The following expressions are equivalent:

`x1==nand[not[x1],not[x1]]`

Proof

Note that the input for the rule:

`nand[not[x1_],nand[x1_,x2_]]→x1`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,x1_]→not[x1]`

where these rules follow from Substitution Lemma 1 and Axiom 2 respectively.

Substitution Lemma 10

It can be shown that:

`x1==or[x1,x1]`

Proof

We start by taking Critical Pair Lemma 5, and apply the substitution:

`nand[not[x1_],not[x2_]]→or[x1,x2]`

which follows from Substitution Lemma 6.

Substitution Lemma 11

It can be shown that:

`and[x1_,x1_]→x1`

Proof

We start by taking Critical Pair Lemma 4, and apply the substitution:

`or[x1_,x1_]→x1`

which follows from Substitution Lemma 10.

Substitution Lemma 12

Substitution Lemma 12

It can be shown that:

`x1==not[not[x1]]`

Proof

We start by taking Critical Pair Lemma 1, and apply the substitution:

`or[x1_,x1_]→x1`

which follows from Substitution Lemma 10.

Critical Pair Lemma 6

The following expressions are equivalent:

`nand[x1,x2]==not[and[x1,x2]]`

Proof

Note that the input for the rule:

`not[not[x1_]]→x1`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

where these rules follow from Substitution Lemma 12 and Axiom 6 respectively.

Critical Pair Lemma 7

The following expressions are equivalent:

`or[not[x1],x2]==nand[x1,not[x2]]`

Proof

Note that the input for the rule:

`nand[not[x1_],not[x2_]]→or[x1,x2]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Substitution Lemma 6 and Substitution Lemma 12 respectively.

Critical Pair Lemma 8

The following expressions are equivalent:

`or[not[x1],not[x2]]==nand[x1,x2]`

Proof

Note that the input for the rule:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Critical Pair Lemma 7 and Substitution Lemma 12 respectively.

Critical Pair Lemma 9

The following expressions are equivalent:

`and[x1,not[x2]]==not[or[not[x1],x2]]`

Proof

Note that the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

where these rules follow from Axiom 6 and Critical Pair Lemma 7 respectively.

Critical Pair Lemma 10

The following expressions are equivalent:

`nand[not[x1],x2]==or[x1,not[x2]]`

Proof

Note that the input for the rule:

`or[not[x1_],not[x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Critical Pair Lemma 8 and Substitution Lemma 12 respectively.

Substitution Lemma 13

It can be shown that:

`or[x1,not[nand[x1,x2]]]==x1`

Proof

We start by taking Substitution Lemma 1, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 10.

Substitution Lemma 14

It can be shown that:

`or[x1,and[x1,x2]]==x1`

Proof

We start by taking Substitution Lemma 13, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Critical Pair Lemma 11

The following expressions are equivalent:

`and[x1,not[not[x2]]]==and[x1,or[not[x1],x2]]`

Proof

Note that the input for the rule:

`and[x1_,nand[x1_,x2_]]→and[x1,not[x2]]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

where these rules follow from Substitution Lemma 7 and Critical Pair Lemma 7 respectively.

Substitution Lemma 15

It can be shown that:

`and[x1,x2]==and[x1,or[not[x1],x2]]`

Proof

We start by taking Critical Pair Lemma 11, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 12.

Critical Pair Lemma 12

The following expressions are equivalent:

`and[x1,not[and[not[x1],x2]]]==not[not[x1]]`

Proof

Note that the input for the rule:

`not[or[not[x1_],x2_]]→and[x1,not[x2]]`

contains a subpattern of the form:

`or[not[x1_],x2_]`

which can be unified with the input for the rule:

`or[x1_,and[x1_,x2_]]→x1`

where these rules follow from Critical Pair Lemma 9 and Substitution Lemma 14 respectively.

Substitution Lemma 16

It can be shown that:

`and[x1,nand[not[x1],x2]]==not[not[x1]]`

Proof

We start by taking Critical Pair Lemma 12, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 6.

Substitution Lemma 17

Out[ ]=

It can be shown that:

`and[x1,or[x1,not[x2]]]==not[not[x1]]`

Proof

We start by taking Substitution Lemma 16, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 10.

Substitution Lemma 18

It can be shown that:

`and[x1,or[x1,not[x2]]]==x1`

Proof

We start by taking Substitution Lemma 17, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 12.

Critical Pair Lemma 13

The following expressions are equivalent:

`and[not[x1],not[x2]]==not[or[x1,x2]]`

Proof

Note that the input for the rule:

`not[or[not[x1_],x2_]]→and[x1,not[x2]]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Critical Pair Lemma 9 and Substitution Lemma 12 respectively.

Critical Pair Lemma 14

The following expressions are equivalent:

`x1==and[x1,or[x1,x2]]`

Proof

Note that the input for the rule:

`and[x1_,or[x1_,not[x2_]]]→x1`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Substitution Lemma 18 and Substitution Lemma 12 respectively.

Critical Pair Lemma 15

The following expressions are equivalent:

`nand[x1,or[x1,x2]]==not[x1]`

`nand[x1,or[x1,x2]]==not[x1]`

Proof

Note that the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,or[x1_,x2_]]→x1`

where these rules follow from Critical Pair Lemma 6 and Critical Pair Lemma 14 respectively.

Critical Pair Lemma 16

The following expressions are equivalent:

`nand[x1,or[not[x1],x2]]==not[and[x1,x2]]`

Proof

Note that the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,or[not[x1_],x2_]]→and[x1,x2]`

where these rules follow from Critical Pair Lemma 6 and Substitution Lemma 15 respectively.

Substitution Lemma 19

It can be shown that:

`nand[x1,or[not[x1],x2]]==nand[x1,x2]`

Proof

We start by taking Critical Pair Lemma 16, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 6.

Critical Pair Lemma 17

The following expressions are equivalent:

`nand[x1,and[not[x1],x2]]==nand[x1,not[x1]]`

Proof

Note that the input for the rule:

`nand[x1_,or[not[x1_],x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`or[not[x1_],x2_]`

which can be unified with the input for the rule:

`or[x1_,and[x1_,x2_]]→x1`

where these rules follow from Substitution Lemma 19 and Substitution Lemma 14 respectively.

Substitution Lemma 20

Substitution Lemma 20

It can be shown that:

`nand[x1,and[not[x1],x2]]==or[not[x1],x1]`

Proof

We start by taking Critical Pair Lemma 17, and apply the substitution:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 7.

Critical Pair Lemma 18

The following expressions are equivalent:

`nand[x1,and[x2,x1]]==nand[x2,x1]`

Proof

Note that the input for the rule:

`nand[x1_,and[x2_,x3_]]↔nand[x2_,and[x1_,x3_]]`

contains a subpattern of the form:

`and[x2_,x3_]`

which can be unified with the input for the rule:

`Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[62,or[x1_,x1_]→`

where these rules follow from Substitution Lemma 9 and Substitution Lemma 11 respectively.

Critical Pair Lemma 19

The following expressions are equivalent:

`nand[not[x1],x1]==or[not[x1],x1]`

Proof

Note that the input for the rule:

`nand[x1_,and[x2_,x1_]]→nand[x2,x1]`

contains a subpattern of the form:

`nand[x1_,and[x2_,x1_]]`

which can be unified with the input for the rule:

`nand[x1_,and[not[x1_],x2_]]→or[not[x1],x1]`

where these rules follow from Critical Pair Lemma 18 and Substitution Lemma 20 respectively.

Substitution Lemma 21

It can be shown that:

`or[x1,not[x1]]==or[not[x1],x1]`

Proof

We start by taking Critical Pair Lemma 19, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 10.

Critical Pair Lemma 20

The following expressions are equivalent:

`nand[not[x1],not[x2]]==nand[not[x2],not[or[x1,x2]]]`

PROOF

Note that the input for the rule:

`nand[x1_,and[x2_,x1_]]→nand[x2,x1]`

contains a subpattern of the form:

`and[x2_,x1_]`

which can be unified with the input for the rule:

`and[not[x1_],not[x2_]]→not[or[x1,x2]]`

where these rules follow from Critical Pair Lemma 18 and Critical Pair Lemma 13 respectively.

Substitution Lemma 22

It can be shown that:

`or[x1,not[not[x2]]]==nand[not[x2],not[or[x1,x2]]]`

PROOF

We start by taking Critical Pair Lemma 20, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 10.

Substitution Lemma 23

It can be shown that:

`or[x1,x2]==nand[not[x2],not[or[x1,x2]]]`

PROOF

We start by taking Substitution Lemma 22, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 12.

Substitution Lemma 24

It can be shown that:

`or[x1,x2]==or[x2,not[not[or[x1,x2]]]]`

PROOF

We start by taking Substitution Lemma 23, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 10.

Substitution Lemma 25

It can be shown that:

`or[x1,x2]==or[x2,or[x1,x2]]`

PROOF

We start by taking Substitution Lemma 24, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 12.

Critical Pair Lemma 21

The following expressions are equivalent:

`not[x1]==nand[x1,or[x2,x1]]`

Proof

Note that the input for the rule:

`nand[x1_,or[x1_,x2_]]→not[x1]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,x1_]]→or[x2,x1]`

where these rules follow from Critical Pair Lemma 15 and Substitution Lemma 25 respectively.

Critical Pair Lemma 22

The following expressions are equivalent:

`not[not[x1]]==nand[not[x1],nand[x2,x1]]`

Proof

Note that the input for the rule:

`nand[x1_,or[x2_,x1_]]→not[x1]`

contains a subpattern of the form:

`or[x2_,x1_]`

which can be unified with the input for the rule:

`or[not[x1_],not[x2_]]→nand[x1,x2]`

where these rules follow from Critical Pair Lemma 21 and Critical Pair Lemma 8 respectively.

Substitution Lemma 26

It can be shown that:

`x1==nand[not[x1],nand[x2,x1]]`

Proof

We start by taking Critical Pair Lemma 22, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 12.

Substitution Lemma 27

It can be shown that:

`x1==or[x1,not[nand[x2,x1]]]`

Proof

We start by taking Substitution Lemma 26, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 10.

Substitution Lemma 28

It can be shown that:

`x1==or[x1,and[x2,x1]]`

Proof

We start by taking Substitution Lemma 27, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Critical Pair Lemma 23

The following expressions are equivalent:

`nand[x1,and[x2,not[x1]]]==nand[x1,not[x1]]`

Proof

Note that the input for the rule:

`nand[x1_,or[not[x1_],x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`or[not[x1_],x2_]`

which can be unified with the input for the rule:

`or[x1_,and[x2_,x1_]]→x1`

where these rules follow from Substitution Lemma 19 and Substitution Lemma 28 respectively.

Substitution Lemma 29

It can be shown that:

`nand[x1,and[x2,not[x1]]]==or[not[x1],x1]`

Proof

We start by taking Critical Pair Lemma 23, and apply the substitution:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 7.

Substitution Lemma 30

It can be shown that:

`nand[x1,and[x2,not[x1]]]==or[x1,not[x1]]`

Proof

We start by taking Substitution Lemma 29, and apply the substitution:

`or[not[x1_],x1_]→or[x1,not[x1]]`

which follows from Substitution Lemma 21.

Critical Pair Lemma 24

The following expressions are equivalent:

`or[x1,not[x1]]==nand[x2,and[x1,not[x1]]]`

Proof

Note that the input for the rule:

`nand[x1_,and[x2_,not[x1_]]]→or[x1,not[x1]]`

contains a subpattern of the form:

`nand[x1_,and[x2_,not[x1_]]]`

which can be unified with the input for the rule:

`nand[x1_,and[x2_,x3_]]↔nand[x2_,and[x1_,x3_]]`

where these rules follow from Substitution Lemma 30 and Substitution Lemma 9 respectively.

Critical Pair Lemma 25

The following expressions are equivalent:

`or[x1,not[x1]]==or[x2,not[and[x1,not[x1]]]]`

Proof

Note that the input for the rule:

`nand[x1_,and[x2_,not[x2_]]]→or[x2,not[x2]]`

contains a subpattern of the form:

`nand[x1_,and[x2_,not[x2_]]]`

which can be unified with the input for the rule:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

where these rules follow from Critical Pair Lemma 24 and Critical Pair Lemma 10 respectively.

Substitution Lemma 31

It can be shown that:

`or[x1,not[x1]]==or[x2,nand[x1,not[x1]]]`

Proof

We start by taking Critical Pair Lemma 25, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 6.

Substitution Lemma 32

It can be shown that:

`or[x1,not[x1]]==or[x2,or[not[x1],x1]]`

Proof

We start by taking Substitution Lemma 31, and apply the substitution:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 7.

Substitution Lemma 33

It can be shown that:

`or[x1,not[x1]]==or[x2,or[x1,not[x1]]]`

Proof

We start by taking Substitution Lemma 32, and apply the substitution:

`or[not[x1_],x1_]→or[x1,not[x1]]`

which follows from Substitution Lemma 21.

Critical Pair Lemma 26

Critical Pair Lemma 26

The following expressions are equivalent:

`x1==and[x1,or[x2,not[x2]]]`

Proof

Note that the input for the rule:

`and[x1_,or[x1_,x2_]]→x1`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,not[x2_]]]→or[x2,not[x2]]`

where these rules follow from Critical Pair Lemma 14 and Substitution Lemma 33 respectively.

Conclusion 1

We obtain the conclusion:

`True`

Proof

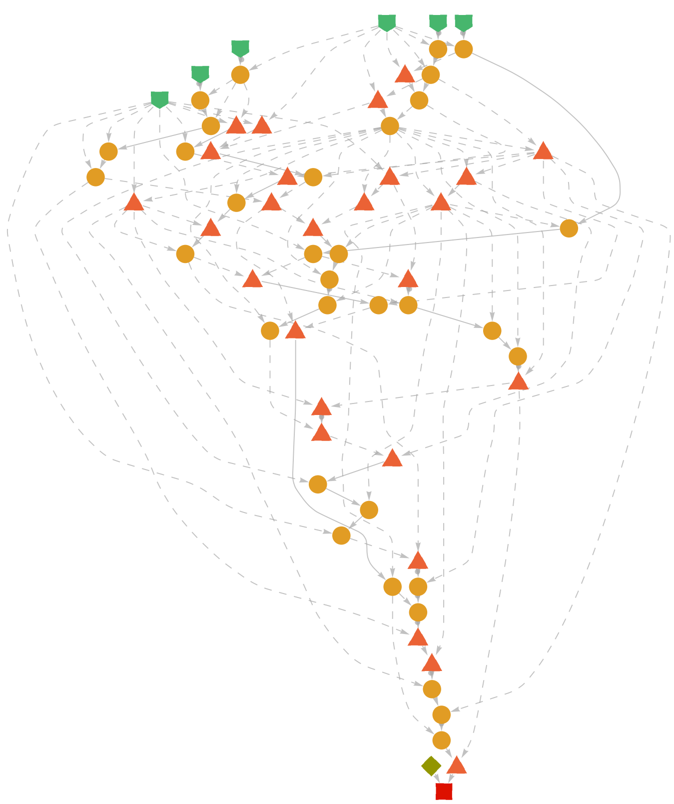
Take Hypothesis 1, and apply the substitution:

`and[x1_,or[x2_,not[x2_]]]→x1`

which follows from Critical Pair Lemma 26.

In[ ]:= `proofAxB3fromShort["ProofGraph"]`

Out[ ]=



```
In[]:= Clear[proofAxB3fromShort]
In[]:= proofAxB4fromShort["ProofNotebook"]
```

Axiom 1

We are given that:

`x1==nand[nand[x1,x1],nand[x1,x2]]`

Axiom 2

We are given that:

`nand[x1,x1]==not[x1]`

Axiom 3

We are given that:

`nand[x1,nand[x1,x2]]==nand[x1,nand[x2,x2]]`

Axiom 4

We are given that:

`nand[x1,nand[x1,nand[x2,x3]]]==nand[x2,nand[x2,nand[x1,x3]]]`

Axiom 5

We are given that:

`nand[nand[x1,x1],nand[x2,x2]]==or[x1,x2]`

Axiom 6

We are given that:

`not[nand[x1,x2]]==and[x1,x2]`

Hypothesis 1

We would like to show that:

`or[a,and[b,not[b]]]==a`

Substitution Lemma 1

It can be shown that:

`nand[not[x1],nand[x1,x2]]==x1`

Proof

We start by taking Axiom 1, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Substitution Lemma 2

It can be shown that:

`nand[x1,nand[x1,x2]]==nand[x1,not[x2]]`

Proof

We start by taking Axiom 3, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Critical Pair Lemma 1

The following expressions are equivalent:

`nand[x1,not[nand[x1,x2]]]==nand[x1,nand[x1,not[x2]]]`

Proof

Note that the input for the rule:

`nand[x1_,nand[x1_,x2_]]→nand[x1,not[x2]]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,nand[x1_,x2_]]→nand[x1,not[x2]]`

where these rules follow from Substitution Lemma 2 and Substitution Lemma 2 respectively.

Substitution Lemma 3

It can be shown that:

`nand[x1,not[nand[x1,x2]]]==nand[x1,not[not[x2]]]`

Proof

We start by taking Critical Pair Lemma 1, and apply the substitution:

`nand[x1_,nand[x1_,x2_]]→nand[x1,not[x2]]`

which follows from Substitution Lemma 2.

Substitution Lemma 4

It can be shown that:

`nand[x1,not[nand[x2,x3]]]==nand[x2,nand[x2,nand[x1,x3]]]`

Proof

We start by taking Axiom 4, and apply the substitution:

`nand[x1_,nand[x1_,x2_]]→nand[x1,not[x2]]`

which follows from Substitution Lemma 2.

Substitution Lemma 5

It can be shown that:

`nand[x1,not[nand[x2,x3]]]==nand[x2,not[nand[x1,x3]]]`

Proof

We start by taking Substitution Lemma 4, and apply the substitution:

`nand[x1_,nand[x1_,x2_]]→nand[x1,not[x2]]`

which follows from Substitution Lemma 2.

Substitution Lemma 6

It can be shown that:

`nand[not[x1],nand[x2,x2]]==or[x1,x2]`

Proof

We start by taking Axiom 5, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Substitution Lemma 7

It can be shown that:

`nand[not[x1],not[x2]]==or[x1,x2]`

Proof

We start by taking Substitution Lemma 6, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Critical Pair Lemma 2

The following expressions are equivalent:

`or[x1,x1]==not[not[x1]]`

Proof

Note that the input for the rule:

`nand[not[x1_],not[x2_]]→or[x1,x2]`

contains a subpattern of the form:

`nand[not[x1_],not[x2_]]`

which can be unified with the input for the rule:

`nand[x1_,x1_]→not[x1]`

where these rules follow from Substitution Lemma 7 and Axiom 2 respectively.

Critical Pair Lemma 3

The following expressions are equivalent:

`and[x1,x1]==not[not[x1]]`

Proof

Note that the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,x1_]→not[x1]`

where these rules follow from Axiom 6 and Axiom 2 respectively.

Critical Pair Lemma 4

The following expressions are equivalent:

`and[x1,nand[x1,x2]]==not[nand[x1,not[x2]]]`

Proof

Note that the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,nand[x1_,x2_]]→nand[x1,not[x2]]`

where these rules follow from Axiom 6 and Substitution Lemma 2 respectively.

Substitution Lemma 8

It can be shown that:

`and[x1,nand[x1,x2]]==and[x1,not[x2]]`

Proof

We start by taking Critical Pair Lemma 4, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Substitution Lemma 9

It can be shown that:

`nand[x1,and[x2,x3]]==nand[x2,not[nand[x1,x3]]]`

Proof

We start by taking Substitution Lemma 5, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Substitution Lemma 10

It can be shown that:

`nand[x1,and[x2,x3]]==nand[x2,and[x1,x3]]`

Proof

We start by taking Substitution Lemma 9, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Critical Pair Lemma 5

The following expressions are equivalent:

`and[x1,x1]==or[x1,x1]`

Proof

Note that the input for the rule:

`and[x1_,x1_]↔not[not[x1_]]`

contains a subpattern of the form:

`not[not[x1_]]`

which can be unified with the input for the rule:

`or[x1_,x1_]↔not[not[x1_]]`

where these rules follow from Critical Pair Lemma 3 and Critical Pair Lemma 2 respectively.

Critical Pair Lemma 6

The following expressions are equivalent:

`x1==nand[not[x1],not[x1]]`

Proof

Note that the input for the rule:

`nand[not[x1_],nand[x1_,x2_]]→x1`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,x1_]→not[x1]`

where these rules follow from Substitution Lemma 1 and Axiom 2 respectively.

Substitution Lemma 11

It can be shown that:

`x1==or[x1,x1]`

Proof

We start by taking Critical Pair Lemma 6, and apply the substitution:

`nand[not[x1_],not[x2_]]→or[x1,x2]`

which follows from Substitution Lemma 7.

Substitution Lemma 12

It can be shown that:

`x1==and[x1,x1]`

Proof

We start by taking Substitution Lemma 11, and apply the substitution:

`or[x1_,x1_]→and[x1,x1]`

which follows from Critical Pair Lemma 5.

Substitution Lemma 13

It can be shown that:

`x1==not[not[x1]]`

Proof

We start by taking Critical Pair Lemma 3, and apply the substitution:

`and[x1_,x1_]→x1`

which follows from Substitution Lemma 12.

Critical Pair Lemma 7

The following expressions are equivalent:

`nand[x1,x2]==not[and[x1,x2]]`

Proof

PROOF

Note that the input for the rule:

`not[not[x1_]]→x1`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

where these rules follow from Substitution Lemma 13 and Axiom 6 respectively.

Critical Pair Lemma 8

The following expressions are equivalent:

`or[not[x1],x2]==nand[x1,not[x2]]`

PROOF

Note that the input for the rule:

`nand[not[x1_],not[x2_]]→or[x1,x2]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Substitution Lemma 7 and Substitution Lemma 13 respectively.

Critical Pair Lemma 9

The following expressions are equivalent:

`or[not[x1],not[x2]]==nand[x1,x2]`

PROOF

Note that the input for the rule:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Critical Pair Lemma 8 and Substitution Lemma 13 respectively.

Critical Pair Lemma 10

The following expressions are equivalent:

`and[x1,not[x2]]==not[or[not[x1],x2]]`

PROOF

Note that the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

```
nand[x1_,not[x2_]]→or[not[x1],x2]
```

where these rules follow from Axiom 6 and Critical Pair Lemma 8 respectively.

Critical Pair Lemma 11

The following expressions are equivalent:

```
nand[not[x1],x2]==or[x1,not[x2]]
```

Proof

Note that the input for the rule:

```
or[not[x1_],not[x2_]]→nand[x1,x2]
```

contains a subpattern of the form:

```
not[x1_]
```

which can be unified with the input for the rule:

```
not[not[x1_]]→x1
```

where these rules follow from Critical Pair Lemma 9 and Substitution Lemma 13 respectively.

Substitution Lemma 14

It can be shown that:

```
or[not[x1],nand[x1,x2]]==nand[x1,not[not[x2]]]
```

Proof

We start by taking Substitution Lemma 3, and apply the substitution:

```
nand[x1_,not[x2_]]→or[not[x1],x2]
```

which follows from Critical Pair Lemma 8.

Substitution Lemma 15

It can be shown that:

```
or[not[x1],nand[x1,x2]]==or[not[x1],not[x2]]
```

Proof

We start by taking Substitution Lemma 14, and apply the substitution:

```
nand[x1_,not[x2_]]→or[not[x1],x2]
```

which follows from Critical Pair Lemma 8.

Substitution Lemma 16

It can be shown that:

```
or[not[x1],nand[x1,x2]]==nand[x1,x2]
```

Proof

We start by taking Substitution Lemma 15, and apply the substitution:

```
or[not[x1_],not[x2_]]→nand[x1,x2]
```

which follows from Critical Pair Lemma 9.

Critical Pair Lemma 12

The following expressions are equivalent:

```
and[not[x1],x2]==not[or[x1,not[x2]]]
```

PROOF

Note that the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

where these rules follow from Axiom 6 and Critical Pair Lemma 11 respectively.

Substitution Lemma 17

It can be shown that:

`or[x1,not[nand[x1,x2]]]==x1`

PROOF

We start by taking Substitution Lemma 1, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 11.

Substitution Lemma 18

It can be shown that:

`or[x1,and[x1,x2]]==x1`

PROOF

We start by taking Substitution Lemma 17, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Critical Pair Lemma 13

The following expressions are equivalent:

`and[x1,not[not[x2]]]==and[x1,or[not[x1],x2]]`

PROOF

Note that the input for the rule:

`and[x1_,nand[x1_,x2_]]→and[x1,not[x2]]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

where these rules follow from Substitution Lemma 8 and Critical Pair Lemma 8 respectively.

Substitution Lemma 19

It can be shown that:

`and[x1,x2]==and[x1,or[not[x1],x2]]`

PROOF

We start by taking Critical Pair Lemma 13, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 13.

Critical Pair Lemma 14

The following expressions are equivalent:

`and[x1,not[nand[x1,x2]]]==not[nand[x1,x2]]`

Proof

Note that the input for the rule:

`not[or[not[x1_],x2_]]→and[x1,not[x2]]`

contains a subpattern of the form:

`or[not[x1_],x2_]`

which can be unified with the input for the rule:

`or[not[x1_],nand[x1_,x2_]]→nand[x1,x2]`

where these rules follow from Critical Pair Lemma 10 and Substitution Lemma 16 respectively.

Substitution Lemma 20

It can be shown that:

`and[x1,and[x1,x2]]==not[nand[x1,x2]]`

Proof

We start by taking Critical Pair Lemma 14, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Substitution Lemma 21

It can be shown that:

`and[x1,and[x1,x2]]==and[x1,x2]`

Proof

We start by taking Substitution Lemma 20, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Critical Pair Lemma 15

The following expressions are equivalent:

`and[x1,not[and[not[x1],x2]]]==not[not[x1]]`

Proof

Note that the input for the rule:

`not[or[not[x1_],x2_]]→and[x1,not[x2]]`

contains a subpattern of the form:

`or[not[x1_],x2_]`

which can be unified with the input for the rule:

`or[x1_,and[x1_,x2_]]→x1`

where these rules follow from Critical Pair Lemma 10 and Substitution Lemma 18 respectively.

Substitution Lemma 22

It can be shown that:

`and[x1,nand[not[x1],x2]]==not[not[x1]]`

Proof

We start by taking Critical Pair Lemma 15, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 7.

Substitution Lemma 23

It can be shown that:

`and[x1,or[x1,not[x2]]]==not[not[x1]]`

Proof

We start by taking Substitution Lemma 22, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 11.

Substitution Lemma 24

It can be shown that:

`and[x1,or[x1,not[x2]]]==x1`

Proof

We start by taking Substitution Lemma 23, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 13.

Critical Pair Lemma 16

The following expressions are equivalent:

`and[not[x1],not[x2]]==not[or[x1,x2]]`

Proof

Note that the input for the rule:

`not[or[not[x1_],x2_]]→and[x1,not[x2]]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Critical Pair Lemma 10 and Substitution Lemma 13 respectively.

Critical Pair Lemma 17

The following expressions are equivalent:

`nand[x1,and[x1,x2]]==not[and[x1,x2]]`

Proof

Proof

Note that the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,and[x1_,x2_]]→and[x1,x2]`

where these rules follow from Critical Pair Lemma 7 and Substitution Lemma 21 respectively.

Substitution Lemma 25

It can be shown that:

`nand[x1,and[x1,x2]]==nand[x1,x2]`

Out[ ]=

Proof

We start by taking Critical Pair Lemma 17, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 7.

Critical Pair Lemma 18

The following expressions are equivalent:

`x1==and[x1,or[x1,x2]]`

Proof

Note that the input for the rule:

`and[x1_,or[x1_,not[x2_]]]→x1`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Substitution Lemma 24 and Substitution Lemma 13 respectively.

Critical Pair Lemma 19

The following expressions are equivalent:

`nand[x1,or[x1,x2]]==nand[x1,x1]`

Proof

Note that the input for the rule:

`nand[x1_,and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,or[x1_,x2_]]→x1`

where these rules follow from Substitution Lemma 25 and Critical Pair Lemma 18 respectively.

Substitution Lemma 26

It can be shown that:

It can be shown that:

`nand[x1,or[x1,x2]]==not[x1]`

Proof

We start by taking Critical Pair Lemma 19, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Critical Pair Lemma 20

The following expressions are equivalent:

`nand[x1,or[not[x1],x2]]==nand[x1,and[x1,x2]]`

Proof

Note that the input for the rule:

`nand[x1_,and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,or[not[x1_],x2_]]→and[x1,x2]`

where these rules follow from Substitution Lemma 25 and Substitution Lemma 19 respectively.

Substitution Lemma 27

It can be shown that:

`nand[x1,or[not[x1],x2]]==nand[x1,x2]`

Proof

We start by taking Critical Pair Lemma 20, and apply the substitution:

`nand[x1_,and[x1_,x2_]]→nand[x1,x2]`

which follows from Substitution Lemma 25.

Critical Pair Lemma 21

The following expressions are equivalent:

`nand[x1,and[not[x1],x2]]==nand[x1,not[x1]]`

Proof

Note that the input for the rule:

`nand[x1_,or[not[x1_],x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`or[not[x1_],x2_]`

which can be unified with the input for the rule:

`or[x1_,and[x1_,x2_]]→x1`

where these rules follow from Substitution Lemma 27 and Substitution Lemma 18 respectively.

Substitution Lemma 28

It can be shown that:

`nand[x1,and[not[x1],x2]]==or[not[x1],x1]`

Proof

We start by taking Critical Pair Lemma 21, and apply the substitution:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 8.

Critical Pair Lemma 22

The following expressions are equivalent:

`nand[x1,and[x2,x1]]==nand[x2,x1]`

Proof

Note that the input for the rule:

`nand[x1_,and[x2_,x3_]]↔nand[x2_,and[x1_,x3_]]`

contains a subpattern of the form:

`and[x2_,x3_]`

which can be unified with the input for the rule:

`and[x1_,x1_]→x1`

where these rules follow from Substitution Lemma 10 and Substitution Lemma 12 respectively.

Critical Pair Lemma 23

The following expressions are equivalent:

`nand[not[x1],x1]==or[not[x1],x1]`

Proof

Note that the input for the rule:

`nand[x1_,and[x2_,x1_]]→nand[x2,x1]`

contains a subpattern of the form:

`nand[x1_,and[x2_,x1_]]`

which can be unified with the input for the rule:

`nand[x1_,and[not[x1_],x2_]]→or[not[x1],x1]`

where these rules follow from Critical Pair Lemma 22 and Substitution Lemma 28 respectively.

Substitution Lemma 29

It can be shown that:

`or[x1,not[x1]]==or[not[x1],x1]`

Proof

We start by taking Critical Pair Lemma 23, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 11.

Critical Pair Lemma 24

The following expressions are equivalent:

`nand[x1,or[x1,x2]]==nand[or[x1,x2],x1]`

Proof

Note that the input for the rule:

`nand[x1_,and[x2_,x1_]]→nand[x2,x1]`

contains a subpattern of the form:

`and[x2_,x1_]`

which can be unified with the input for the rule:

`and[x1_,or[x1_,x2_]]→x1`

where these rules follow from Critical Pair Lemma 22 and Critical Pair Lemma 18 respectively.

Substitution Lemma 30

It can be shown that:

`not[x1]==nand[or[x1,x2],x1]`

Proof

We start by taking Critical Pair Lemma 24, and apply the substitution:

`nand[x1_,or[x1_,x2_]]→not[x1]`

which follows from Substitution Lemma 26.

Critical Pair Lemma 25

The following expressions are equivalent:

`nand[not[x1],not[x2]]==nand[not[x2],not[or[x1,x2]]]`

Proof

Note that the input for the rule:

`nand[x1_,and[x2_,x1_]]→nand[x2,x1]`

contains a subpattern of the form:

`and[x2_,x1_]`

which can be unified with the input for the rule:

`and[not[x1_],not[x2_]]→not[or[x1,x2]]`

where these rules follow from Critical Pair Lemma 22 and Critical Pair Lemma 16 respectively.

Substitution Lemma 31

It can be shown that:

`or[x1,not[not[x2]]]==nand[not[x2],not[or[x1,x2]]]`

Proof

We start by taking Critical Pair Lemma 25, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 11.

Substitution Lemma 32

It can be shown that:

`or[x1,x2]==nand[not[x2],not[or[x1,x2]]]`

Proof

We start by taking Substitution Lemma 31, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 13.

Substitution Lemma 33

It can be shown that:

`or[x1,x2]==or[x2,not[not[or[x1,x2]]]]`

Proof

We start by taking Substitution Lemma 32, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 11.

Substitution Lemma 34

It can be shown that:

`or[x1,x2]==or[x2,or[x1,x2]]`

Proof

We start by taking Substitution Lemma 33, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 13.

Critical Pair Lemma 26

The following expressions are equivalent:

`and[x1,not[x1]]==not[or[x1,not[x1]]]`

Proof

Note that the input for the rule:

`not[or[not[x1_],x2_]]→and[x1,not[x2]]`

contains a subpattern of the form:

`or[not[x1_],x2_]`

which can be unified with the input for the rule:

`or[not[x1_],x1_]→or[x1,not[x1]]`

where these rules follow from Critical Pair Lemma 10 and Substitution Lemma 29 respectively.

Substitution Lemma 35

It can be shown that:

`and[x1,not[x1]]==and[not[x1],x1]`

Proof

We start by taking Critical Pair Lemma 26, and apply the substitution:

`not[or[x1_,not[x2_]]]→and[not[x1],x2]`

which follows from Critical Pair Lemma 12.

Critical Pair Lemma 27

The following expressions are equivalent:

`and[or[x1,x2],x1]==not[not[x1]]`

Proof

Note that the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[or[x1_,x2_],x1_]→not[x1]`

where these rules follow from Axiom 6 and Substitution Lemma 30 respectively.

Substitution Lemma 36

It can be shown that:

`and[or[x1,x2],x1]==x1`

Proof

We start by taking Critical Pair Lemma 27, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 13.

Critical Pair Lemma 28

The following expressions are equivalent:

`or[x1,x2]==or[or[x1,x2],x1]`

Proof

Note that the input for the rule:

`or[x1_,and[x1_,x2_]]→x1`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[or[x1_,x2_],x1_]→x1`

where these rules follow from Substitution Lemma 18 and Substitution Lemma 36 respectively.

Critical Pair Lemma 29

The following expressions are equivalent:

`not[x1]==nand[x1,or[x2,x1]]`

Proof

Note that the input for the rule:

`nand[x1_,or[x1_,x2_]]→not[x1]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,x1_]]→or[x2,x1]`

where these rules follow from Substitution Lemma 26 and Substitution Lemma 34 respectively.

Critical Pair Lemma 30

The following expressions are equivalent:

The following expressions are equivalent:

`not[not[x1]]==nand[not[x1],nand[x2,x1]]`

PROOF

Note that the input for the rule:

`nand[x1_,or[x2_,x1_]]→not[x1]`

contains a subpattern of the form:

`or[x2_,x1_]`

which can be unified with the input for the rule:

`or[not[x1_],not[x2_]]→nand[x1,x2]`

where these rules follow from Critical Pair Lemma 29 and Critical Pair Lemma 9 respectively.

Substitution Lemma 37

It can be shown that:

`x1==nand[not[x1],nand[x2,x1]]`

PROOF

We start by taking Critical Pair Lemma 30, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 13.

Substitution Lemma 38

It can be shown that:

`x1==or[x1,not[nand[x2,x1]]]`

PROOF

We start by taking Substitution Lemma 37, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 11.

Substitution Lemma 39

It can be shown that:

`x1==or[x1,and[x2,x1]]`

PROOF

We start by taking Substitution Lemma 38, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Critical Pair Lemma 31

The following expressions are equivalent:

`nand[x1,and[x2,not[x1]]]==nand[x1,not[x1]]`

PROOF

Note that the input for the rule:

`nand[x1_,or[not[x1_],x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`or[not[x1_],x2_]`

which can be unified with the input for the rule:

`or[x1_,and[x2_,x1_]]→x1`

where these rules follow from Substitution Lemma 27 and Substitution Lemma 39 respectively.

Substitution Lemma 40

It can be shown that:

`nand[x1,and[x2,not[x1]]]==or[not[x1],x1]`

Proof

We start by taking Critical Pair Lemma 31, and apply the substitution:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 8.

Substitution Lemma 41

It can be shown that:

`nand[x1,and[x2,not[x1]]]==or[x1,not[x1]]`

Proof

We start by taking Substitution Lemma 40, and apply the substitution:

`or[not[x1_],x1_]→or[x1,not[x1]]`

which follows from Substitution Lemma 29.

Critical Pair Lemma 32

The following expressions are equivalent:

`or[x1,not[x1]]==nand[x2,and[x1,not[x1]]]`

Proof

Note that the input for the rule:

`nand[x1_,and[x2_,not[x1_]]]→or[x1,not[x1]]`

contains a subpattern of the form:

`nand[x1_,and[x2_,not[x1_]]]`

which can be unified with the input for the rule:

`nand[x1_,and[x2_,x3_]]↔nand[x2_,and[x1_,x3_]]`

where these rules follow from Substitution Lemma 41 and Substitution Lemma 10 respectively.

Critical Pair Lemma 33

The following expressions are equivalent:

`or[x1,not[x1]]==or[x2,not[and[x1,not[x1]]]]`

Proof

Note that the input for the rule:

`nand[x1_,and[x2_,not[x2_]]]→or[x2,not[x2]]`

contains a subpattern of the form:

`nand[x1_,and[x2_,not[x2_]]]`

which can be unified with the input for the rule:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

where these rules follow from Critical Pair Lemma 32 and Critical Pair Lemma 11 respectively.

Substitution Lemma 42

It can be shown that:

`or[x1,not[x1]]==or[x2,nand[x1,not[x1]]]`

Proof

We start by taking Critical Pair Lemma 33, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 7.

Substitution Lemma 43

It can be shown that:

`or[x1,not[x1]]==or[x2,or[not[x1],x1]]`

Proof

We start by taking Substitution Lemma 42, and apply the substitution:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 8.

Substitution Lemma 44

It can be shown that:

`or[x1,not[x1]]==or[x2,or[x1,not[x1]]]`

Proof

We start by taking Substitution Lemma 43, and apply the substitution:

`or[not[x1_],x1_]→or[x1,not[x1]]`

which follows from Substitution Lemma 29.

Critical Pair Lemma 34

The following expressions are equivalent:

`x1==and[or[x2,not[x2]],x1]`

Proof

Note that the input for the rule:

`and[or[x1_,x2_],x1_]→x1`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,not[x2_]]]→or[x2,not[x2]]`

where these rules follow from Substitution Lemma 36 and Substitution Lemma 44 respectively.

Critical Pair Lemma 35

The following expressions are equivalent:

`or[not[or[x1,not[x1]]],x2]==and[or[x1,not[x1]],x2]` `or[not[or[x1,not[x1]]],x2]⩵and[or[x1,not[x1]],x2]`

PROOF

Note that the input for the rule:

`and[or[x1_,not[x1_]],x2_]→x2`

contains a subpattern of the form:

`and[or[x1_,not[x1_]],x2_]`

which can be unified with the input for the rule:

`and[x1_,or[not[x1_],x2_]]→and[x1,x2]`

where these rules follow from Critical Pair Lemma 34 and Substitution Lemma 19 respectively.

Substitution Lemma 45

It can be shown that:

`or[and[not[x1],x1],x2]⩵and[or[x1,not[x1]],x2]`

PROOF

We start by taking Critical Pair Lemma 35, and apply the substitution:

`not[or[x1_,not[x2_]]]→and[not[x1],x2]`

which follows from Critical Pair Lemma 12.

Substitution Lemma 46

It can be shown that:

`or[and[x1,not[x1]],x2]⩵and[or[x1,not[x1]],x2]`

PROOF

We start by taking Substitution Lemma 45, and apply the substitution:

`and[not[x1_],x1_]→and[x1,not[x1]]`

which follows from Substitution Lemma 35.

Substitution Lemma 47

It can be shown that:

`or[and[x1,not[x1]],x2]⩵x2`

PROOF

We start by taking Substitution Lemma 46, and apply the substitution:

`and[or[x1_,not[x1_]],x2_]→x2`

which follows from Critical Pair Lemma 34.

Critical Pair Lemma 36

The following expressions are equivalent:

`or[and[x1,not[x1]],x2]⩵or[x2,and[x1,not[x1]]]`

PROOF

Note that the input for the rule:

`or[or[x1_,x2_],x1_]→or[x1,x2]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[and[x1_,not[x1_]],x2_]→x2`

where these rules follow from Critical Pair Lemma 28 and Substitution Lemma 47 respectively.

Substitution Lemma 48

It can be shown that:

`x1==or[x1,and[x2,not[x2]]]`

Proof

We start by taking Critical Pair Lemma 36, and apply the substitution:

`or[and[x1_,not[x1_]],x2_]→x2`

which follows from Substitution Lemma 47.

Conclusion 1

We obtain the conclusion:

`True`

Proof

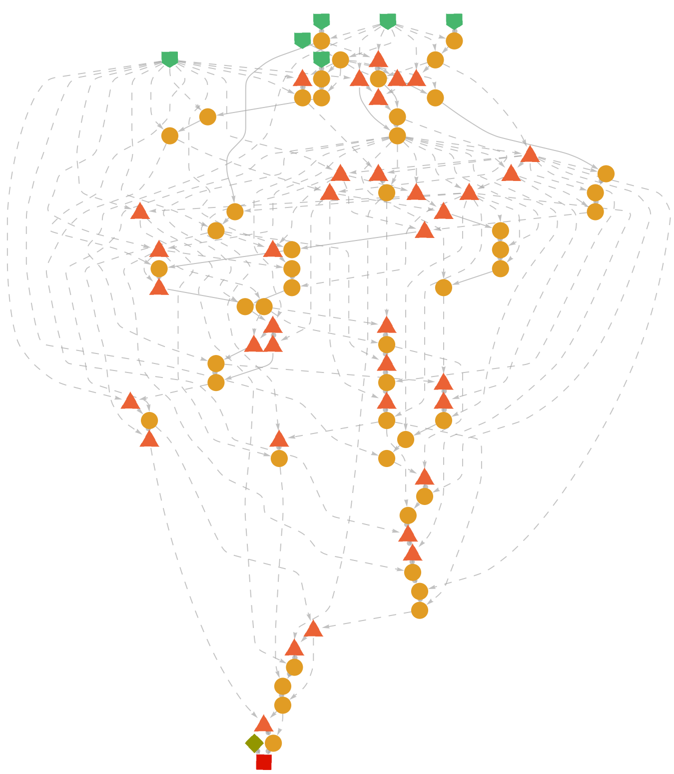
Take Hypothesis 1, and apply the substitution:

`or[x1_,and[x2_,not[x2_]]]→x1`

which follows from Substitution Lemma 48.

In[∘]:= `proofAxB4fromShort["ProofGraph"]`

Out[∘]=



In[ ]:= `Clear[proofAxB4fromShort]`

In[ ]:= `proofAxB5fromShort["ProofNotebook"]`

Axiom 1

We are given that:

`x1==nand[nand[x1,x1],nand[x1,x2]]`

Axiom 2

We are given that:

`nand[x1,x1]==not[x1]`

Axiom 3

We are given that:

`nand[x1,nand[x1,x2]]==nand[x1,nand[x2,x2]]`

Axiom 4

We are given that:

`nand[x1,nand[x1,nand[x2,x3]]]==nand[x2,nand[x2,nand[x1,x3]]]`

Axiom 5

We are given that:

`nand[nand[x1,x1],nand[x2,x2]]==or[x1,x2]`

Axiom 6

We are given that:

`not[nand[x1,x2]]==and[x1,x2]`

Hypothesis 1

We would like to show that:

`or[and[a,b],and[a,c]]==and[a,or[b,c]]`

Substitution Lemma 1

It can be shown that:

`nand[not[x1],nand[x1,x2]]==x1`

Proof

We start by taking Axiom 1, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Substitution Lemma 2

It can be shown that:

`nand[x1,nand[x1,x2]]==nand[x1,not[x2]]`

Proof

We start by taking Axiom 3, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Critical Pair Lemma 1

The following expressions are equivalent:

`nand[x1,not[nand[x1,x2]]]==nand[x1,nand[x1,not[x2]]]`

Proof

Note that the input for the rule:

`nand[x1_,nand[x1_,x2_]]→nand[x1,not[x2]]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,nand[x1_,x2_]]→nand[x1,not[x2]]`

where these rules follow from Substitution Lemma 2 and Substitution Lemma 2 respectively.

Substitution Lemma 3

It can be shown that:

`nand[x1,not[nand[x1,x2]]]==nand[x1,not[not[x2]]]`

Proof

We start by taking Critical Pair Lemma 1, and apply the substitution:

`nand[x1_,nand[x1_,x2_]]→nand[x1,not[x2]]`

which follows from Substitution Lemma 2.

Substitution Lemma 4

It can be shown that:

`nand[x1,not[nand[x2,x3]]]==nand[x2,nand[x2,nand[x1,x3]]]`

Proof

We start by taking Axiom 4, and apply the substitution:

`nand[x1_,nand[x1_,x2_]]→nand[x1,not[x2]]`

which follows from Substitution Lemma 2.

Substitution Lemma 5

It can be shown that:

`nand[x1,not[nand[x2,x3]]]==nand[x2,not[nand[x1,x3]]]`

Proof

We start by taking Substitution Lemma 4, and apply the substitution:

`nand[x1_,nand[x1_,x2_]]→nand[x1,not[x2]]`

which follows from Substitution Lemma 2.

Substitution Lemma 6

It can be shown that:

`nand[not[x1],nand[x2,x2]]==or[x1,x2]`

Proof

We start by taking Axiom 5, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Substitution Lemma 7

It can be shown that:

`nand[not[x1],not[x2]]==or[x1,x2]`

Proof

We start by taking Substitution Lemma 6, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Critical Pair Lemma 2

The following expressions are equivalent:

`or[x1,x1]==not[not[x1]]`

Proof

Note that the input for the rule:

`nand[not[x1_],not[x2_]]→or[x1,x2]`

contains a subpattern of the form:

`nand[not[x1_],not[x2_]]`

which can be unified with the input for the rule:

`nand[x1_,x1_]→not[x1]`

where these rules follow from Substitution Lemma 7 and Axiom 2 respectively.

Critical Pair Lemma 3

The following expressions are equivalent:

`and[x1,x1]==not[not[x1]]`

Proof

Note that the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,x1_]→not[x1]`

where these rules follow from Axiom 6 and Axiom 2 respectively.

Critical Pair Lemma 4

The following expressions are equivalent:

`and[x1,nand[x1,x2]]==not[nand[x1,not[x2]]]`

Proof

PROOF

Note that the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,nand[x1_,x2_]]→nand[x1,not[x2]]`

where these rules follow from Axiom 6 and Substitution Lemma 2 respectively.

Substitution Lemma 8

It can be shown that:

`and[x1,nand[x1,x2]]==and[x1,not[x2]]`

PROOF

We start by taking Critical Pair Lemma 4, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Substitution Lemma 9

It can be shown that:

`nand[x1,and[x2,x3]]==nand[x2,not[nand[x1,x3]]]`

PROOF

We start by taking Substitution Lemma 5, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Substitution Lemma 10

It can be shown that:

`nand[x1,and[x2,x3]]==nand[x2,and[x1,x3]]`

PROOF

We start by taking Substitution Lemma 9, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Critical Pair Lemma 5

The following expressions are equivalent:

`and[x1,x1]==or[x1,x1]`

PROOF

Note that the input for the rule:

`and[x1_,x1_]↔not[not[x1_]]`

contains a subpattern of the form:

`not[not[x1_]]`

which can be unified with the input for the rule:

```
or[x1_,x1_]↔not[not[x1_]]
```

where these rules follow from Critical Pair Lemma 3 and Critical Pair Lemma 2 respectively.

Critical Pair Lemma 6

The following expressions are equivalent:

```
x1==nand[not[x1],not[x1]]
```

Proof

Note that the input for the rule:

```
nand[not[x1_],nand[x1_,x2_]]→x1
```

contains a subpattern of the form:

```
nand[x1_,x2_]
```

which can be unified with the input for the rule:

```
nand[x1_,x1_]→not[x1]
```

where these rules follow from Substitution Lemma 1 and Axiom 2 respectively.

Substitution Lemma 11

It can be shown that:

```
x1==or[x1,x1]
```

Proof

We start by taking Critical Pair Lemma 6, and apply the substitution:

```
nand[not[x1_],not[x2_]]→or[x1,x2]
```

which follows from Substitution Lemma 7.

Substitution Lemma 12

It can be shown that:

```
x1==and[x1,x1]
```

Proof

We start by taking Substitution Lemma 11, and apply the substitution:

```
or[x1_,x1_]→and[x1,x1]
```

which follows from Critical Pair Lemma 5.

Substitution Lemma 13

It can be shown that:

```
or[x1_,x1_]→x1
```

Proof

We start by taking Critical Pair Lemma 5, and apply the substitution:

```
and[x1_,x1_]→x1
```

which follows from Substitution Lemma 12.

Substitution Lemma 14

It can be shown that:

```
x1==not[not[x1]]
```

Proof

We start by taking Critical Pair Lemma 3, and apply the substitution:

`and[x1_,x1_]→x1`

which follows from Substitution Lemma 12.

Critical Pair Lemma 7

The following expressions are equivalent:

`nand[x1,x2]==not[and[x1,x2]]`

Proof

Note that the input for the rule:

`not[not[x1_]]→x1`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

where these rules follow from Substitution Lemma 14 and Axiom 6 respectively.

Critical Pair Lemma 8

The following expressions are equivalent:

`or[not[x1],x2]==nand[x1,not[x2]]`

Proof

Note that the input for the rule:

`nand[not[x1_],not[x2_]]→or[x1,x2]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Substitution Lemma 7 and Substitution Lemma 14 respectively.

Substitution Lemma 15

It can be shown that:

`nand[x1_,nand[x1_,x2_]]→or[not[x1],x2]`

Proof

We start by taking Substitution Lemma 2, and apply the substitution:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 8.

Critical Pair Lemma 9

The following expressions are equivalent:

`or[not[x1],and[x2,x3]]==nand[x1,nand[x2,x3]]`

Proof

Note that the input for the rule:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

where these rules follow from Critical Pair Lemma 8 and Critical Pair Lemma 7 respectively.

Critical Pair Lemma 10

The following expressions are equivalent:

`or[not[x1],nand[x2,x3]]==nand[x1,and[x2,x3]]`

Proof

Note that the input for the rule:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

where these rules follow from Critical Pair Lemma 8 and Axiom 6 respectively.

Critical Pair Lemma 11

The following expressions are equivalent:

`or[not[x1],not[x2]]==nand[x1,x2]`

Proof

Note that the input for the rule:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Critical Pair Lemma 8 and Substitution Lemma 14 respectively.

Critical Pair Lemma 12

The following expressions are equivalent:

`and[x1,not[x2]]==not[or[not[x1],x2]]`

Proof

Note that the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

`nand[x1_,not[x2_]]→or[not[x1],x2]`

where these rules follow from Axiom 6 and Critical Pair Lemma 8 respectively.

Substitution Lemma 16

It can be shown that:

`or[not[x1],nand[x1,x2]]==nand[x1,not[not[x2]]]`

Proof

We start by taking Substitution Lemma 3, and apply the substitution:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 8.

Substitution Lemma 17

It can be shown that:

`or[not[x1],nand[x1,x2]]==or[not[x1],not[x2]]`

Proof

We start by taking Substitution Lemma 16, and apply the substitution:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 8.

Critical Pair Lemma 13

The following expressions are equivalent:

`or[not[not[x1]],not[nand[x1,x2]]]==or[not[not[x1]],x1]`

Proof

Note that the input for the rule:

`or[not[x1_],nand[x1_,x2_]]→or[not[x1],not[x2]]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[not[x1_],nand[x1_,x2_]]→x1`

where these rules follow from Substitution Lemma 17 and Substitution Lemma 1 respectively.

Substitution Lemma 18

It can be shown that:

`or[x1,not[nand[x1,x2]]]==or[not[not[x1]],x1]`

Proof

We start by taking Critical Pair Lemma 13, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 14.

Substitution Lemma 19

It can be shown that:

`or[x1,and[x1,x2]]==or[not[not[x1]],x1]`

Proof

Proof

We start by taking Substitution Lemma 18, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Substitution Lemma 20

It can be shown that:

`or[x1,and[x1,x2]]==or[x1,x1]`

Proof

We start by taking Substitution Lemma 19, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 14.

Substitution Lemma 21

It can be shown that:

`or[x1,and[x1,x2]]==x1`

Proof

We start by taking Substitution Lemma 20, and apply the substitution:

`or[x1_,x1_]→x1`

which follows from Substitution Lemma 13.

Substitution Lemma 22

It can be shown that:

`or[not[x1_],nand[x1_,x2_]]→nand[x1,x2]`

Proof

We start by taking Substitution Lemma 17, and apply the substitution:

`or[not[x1_],not[x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 11.

Critical Pair Lemma 14

The following expressions are equivalent:

`nand[nand[x1,x2],x3]==or[and[x1,x2],not[x3]]`

Proof

Note that the input for the rule:

`or[not[x1_],not[x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

where these rules follow from Critical Pair Lemma 11 and Axiom 6 respectively.

Critical Pair Lemma 15

The following expressions are equivalent:

The following expressions are equivalent:

`nand[not[x1],x2]==or[x1,not[x2]]`

PROOF

Note that the input for the rule:

`or[not[x1_],not[x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Critical Pair Lemma 11 and Substitution Lemma 14 respectively.

Critical Pair Lemma 16

The following expressions are equivalent:

`or[x1,not[nand[not[x1],x2]]]==or[not[not[x1]],x2]`

PROOF

Note that the input for the rule:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

contains a subpattern of the form:

`nand[not[x1_],x2_]`

which can be unified with the input for the rule:

`Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[75,nand[x1_,no`

where these rules follow from Critical Pair Lemma 15 and Substitution Lemma 15 respectively.

Substitution Lemma 23

It can be shown that:

`or[x1,and[not[x1],x2]]==or[not[not[x1]],x2]`

PROOF

We start by taking Critical Pair Lemma 16, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Substitution Lemma 24

It can be shown that:

`or[x1,and[not[x1],x2]]==or[x1,x2]`

PROOF

We start by taking Substitution Lemma 23, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 14.

Critical Pair Lemma 17

The following expressions are equivalent:

`and[x1,not[not[x2]]]==and[x1,or[not[x1],x2]]`

Proof

Note that the input for the rule:

`and[x1_,nand[x1_,x2_]]→and[x1,not[x2]]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

where these rules follow from Substitution Lemma 8 and Critical Pair Lemma 8 respectively.

Substitution Lemma 25

It can be shown that:

`and[x1,x2]==and[x1,or[not[x1],x2]]`

Proof

We start by taking Critical Pair Lemma 17, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 14.

Critical Pair Lemma 18

The following expressions are equivalent:

`and[x1,not[nand[x1,x2]]]==not[nand[x1,x2]]`

Proof

Note that the input for the rule:

`not[or[not[x1_],x2_]]→and[x1,not[x2]]`

contains a subpattern of the form:

`or[not[x1_],x2_]`

which can be unified with the input for the rule:

`` Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[102,or[not[x1_ ``

where these rules follow from Critical Pair Lemma 12 and Substitution Lemma 22 respectively.

Substitution Lemma 26

It can be shown that:

`and[x1,and[x1,x2]]==not[nand[x1,x2]]`

Proof

We start by taking Critical Pair Lemma 18, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Substitution Lemma 27

It can be shown that:

`and[x1,and[x1,x2]]==and[x1,x2]`

Proof

We start by taking Substitution Lemma 26, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Critical Pair Lemma 19

The following expressions are equivalent:

`and[x1,not[and[not[x1],x2]]]==not[not[x1]]`

Proof

Note that the input for the rule:

`not[or[not[x1_],x2_]]→and[x1,not[x2]]`

contains a subpattern of the form:

`or[not[x1_],x2_]`

which can be unified with the input for the rule:

`or[x1_,and[x1_,x2_]]→x1`

where these rules follow from Critical Pair Lemma 12 and Substitution Lemma 21 respectively.

Substitution Lemma 28

It can be shown that:

`and[x1,nand[not[x1],x2]]==not[not[x1]]`

Proof

We start by taking Critical Pair Lemma 19, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 7.

Substitution Lemma 29

It can be shown that:

`and[x1,or[x1,not[x2]]]==not[not[x1]]`

Proof

We start by taking Substitution Lemma 28, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 15.

Substitution Lemma 30

It can be shown that:

`and[x1,or[x1,not[x2]]]==x1`

Proof

We start by taking Substitution Lemma 29, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 14.

Critical Pair Lemma 20

The following expressions are equivalent:

`and[nand[x1,x2],not[x3]]==not[or[and[x1,x2],x3]]`

Proof

Note that the input for the rule:

`not[or[not[x1_],x2_]]→and[x1,not[x2]]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

where these rules follow from Critical Pair Lemma 12 and Axiom 6 respectively.

Critical Pair Lemma 21

The following expressions are equivalent:

`and[not[x1],not[x2]]==not[or[x1,x2]]`

Proof

Note that the input for the rule:

`not[or[not[x1_],x2_]]→and[x1,not[x2]]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Critical Pair Lemma 12 and Substitution Lemma 14 respectively.

Critical Pair Lemma 22

The following expressions are equivalent:

`nand[x1,and[x1,x2]]==not[and[x1,x2]]`

Proof

Note that the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,and[x1_,x2_]]→and[x1,x2]`

where these rules follow from Critical Pair Lemma 7 and Substitution Lemma 27 respectively.

Substitution Lemma 31

It can be shown that:

`nand[x1,and[x1,x2]]==nand[x1,x2]`

Proof

We start by taking Critical Pair Lemma 22, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 7.

Critical Pair Lemma 23

The following expressions are equivalent:

`x1==and[x1,or[x1,x2]]`

Proof

Note that the input for the rule:

`and[x1_,or[x1_,not[x2_]]]→x1`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Substitution Lemma 30 and Substitution Lemma 14 respectively.

Critical Pair Lemma 24

The following expressions are equivalent:

`not[x1]==and[not[x1],nand[x1,x2]]`

Proof

Note that the input for the rule:

`and[x1_,or[x1_,x2_]]→x1`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[102,or[not[x1_`

where these rules follow from Critical Pair Lemma 23 and Substitution Lemma 22 respectively.

Critical Pair Lemma 25

The following expressions are equivalent:

`nand[x1,or[x1,x2]]==nand[x1,x1]`

Proof

Note that the input for the rule:

`nand[x1_,and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,or[x1_,x2_]]→x1`

where these rules follow from Substitution Lemma 31 and Critical Pair Lemma 23 respectively.

Substitution Lemma 32

It can be shown that:

`nand[x1,or[x1,x2]]==not[x1]`

Proof

We start by taking Critical Pair Lemma 25, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Critical Pair Lemma 26

The following expressions are equivalent:

`not[or[x1,or[not[x2],x3]]]==and[not[x1],and[x2,not[x3]]]`

Proof

Note that the input for the rule:

`and[not[x1_],not[x2_]]→not[or[x1,x2]]`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[or[not[x1_],x2_]]→and[x1,not[x2]]`

where these rules follow from Critical Pair Lemma 21 and Critical Pair Lemma 12 respectively.

Critical Pair Lemma 27

The following expressions are equivalent:

`not[and[x1,x2]]==and[nand[x1,x2],nand[and[x1,x2],x3]]`

Proof

Note that the input for the rule:

`and[not[x1_],nand[x1_,x2_]]→not[x1]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

where these rules follow from Critical Pair Lemma 24 and Critical Pair Lemma 7 respectively.

Substitution Lemma 33

It can be shown that:

`nand[x1,x2]==and[nand[x1,x2],nand[and[x1,x2],x3]]`

Proof

We start by taking Critical Pair Lemma 27, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 7.

Critical Pair Lemma 28

The following expressions are equivalent:

`or[x1,not[x2]]==or[x1,not[or[x1,x2]]]`

Proof

Note that the input for the rule:

`or[x1_,and[not[x1_],x2_]]→or[x1,x2]`

contains a subpattern of the form:

`and[not[x1_],x2_]`

which can be unified with the input for the rule:

`and[not[x1_],not[x2_]]→not[or[x1,x2]]`

where these rules follow from Substitution Lemma 24 and Critical Pair Lemma 21 respectively.

Critical Pair Lemma 29

The following expressions are equivalent:

`nand[x1,or[not[x1],x2]]==nand[x1,and[x1,x2]]`

Proof

Note that the input for the rule:

`nand[x1_,and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,or[not[x1_],x2_]]→and[x1,x2]`

where these rules follow from Substitution Lemma 31 and Substitution Lemma 25 respectively.

Substitution Lemma 34

It can be shown that:

`nand[x1,or[not[x1],x2]]==nand[x1,x2]`

Proof

We start by taking Critical Pair Lemma 29, and apply the substitution:

`nand[x1_,and[x1_,x2_]]→nand[x1,x2]`

which follows from Substitution Lemma 31.

Critical Pair Lemma 30

The following expressions are equivalent:

`nand[x1,and[not[x1],x2]]==nand[x1,not[x1]]`

Proof

Note that the input for the rule:

`nand[x1_,or[not[x1_],x2_]]→nand[x1,x2]`

contains a subpattern of the form:

contains a subpattern of the form:

`or[not[x1_],x2_]`

which can be unified with the input for the rule:

`or[x1_,and[x1_,x2_]]→x1`

where these rules follow from Substitution Lemma 34 and Substitution Lemma 21 respectively.

Substitution Lemma 35

It can be shown that:

`nand[x1,and[not[x1],x2]]⩵or[not[x1],x1]`

Proof

We start by taking Critical Pair Lemma 30, and apply the substitution:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 8.

Critical Pair Lemma 31

The following expressions are equivalent:

`nand[nand[x1,x2],nand[x3,x4]]⩵or[and[x1,x2],and[x3,x4]]`

Proof

Note that the input for the rule:

`or[not[x1_],and[x2_,x3_]]→nand[x1,nand[x2,x3]]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

where these rules follow from Critical Pair Lemma 9 and Axiom 6 respectively.

Critical Pair Lemma 32

The following expressions are equivalent:

`nand[nand[x1,x2],and[x3,x4]]⩵or[and[x1,x2],nand[x3,x4]]`

Proof

Note that the input for the rule:

`or[not[x1_],nand[x2_,x3_]]→nand[x1,and[x2,x3]]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

where these rules follow from Critical Pair Lemma 10 and Axiom 6 respectively.

Critical Pair Lemma 33

The following expressions are equivalent:

`nand[x1,and[nand[x2,x3],x4]]⩵or[and[x2,x3],nand[x1,x4]]`

Proof

PROOF

Note that the input for the rule:

`nand[x1_,and[x2_,x3_]]↔nand[x2_,and[x1_,x3_]]`

contains a subpattern of the form:

`nand[x1_,and[x2_,x3_]]`

which can be unified with the input for the rule:

`nand[nand[x1_,x2_],and[x3_,x4_]]→or[and[x1,x2],nand[x3,x4]]`

where these rules follow from Substitution Lemma 10 and Critical Pair Lemma 32 respectively.

Critical Pair Lemma 34

The following expressions are equivalent:

`nand[x1,and[x2,x1]]==nand[x2,x1]`

PROOF

Note that the input for the rule:

`nand[x1_,and[x2_,x3_]]↔nand[x2_,and[x1_,x3_]]`

contains a subpattern of the form:

`and[x2_,x3_]`

which can be unified with the input for the rule:

`and[x1_,x1_]→x1`

where these rules follow from Substitution Lemma 10 and Substitution Lemma 12 respectively.

Critical Pair Lemma 35

The following expressions are equivalent:

`nand[x1,and[x2,or[x1,x3]]]==nand[x2,x1]`

PROOF

Note that the input for the rule:

`nand[x1_,and[x2_,x3_]]↔nand[x2_,and[x1_,x3_]]`

contains a subpattern of the form:

`and[x2_,x3_]`

which can be unified with the input for the rule:

`and[x1_,or[x1_,x2_]]→x1`

where these rules follow from Substitution Lemma 10 and Critical Pair Lemma 23 respectively.

Critical Pair Lemma 36

The following expressions are equivalent:

`or[not[x1],and[x2,x3]]==nand[x1,nand[x2,and[x1,x3]]]`

PROOF

Note that the input for the rule:

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contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,and[x2_,x3_]]↔nand[x2_,and[x1_,x3_]]`

where these rules follow from Substitution Lemma 15 and Substitution Lemma 10 respectively.

Substitution Lemma 36

It can be shown that:

`nand[x1,nand[x2,x3]]==nand[x1,nand[x2,and[x1,x3]]]`

Proof

We start by taking Critical Pair Lemma 36, and apply the substitution:

`or[not[x1_],and[x2_,x3_]]→nand[x1,nand[x2,x3]]`

which follows from Critical Pair Lemma 9.

Critical Pair Lemma 37

The following expressions are equivalent:

`nand[not[x1],x1]==or[not[x1],x1]`

Proof

Note that the input for the rule:

`nand[x1_,and[x2_,x1_]]→nand[x2,x1]`

contains a subpattern of the form:

`nand[x1_,and[x2_,x1_]]`

which can be unified with the input for the rule:

`nand[x1_,and[not[x1_],x2_]]→or[not[x1],x1]`

where these rules follow from Critical Pair Lemma 34 and Substitution Lemma 35 respectively.

Substitution Lemma 37

It can be shown that:

`or[x1,not[x1]]==or[not[x1],x1]`

Proof

We start by taking Critical Pair Lemma 37, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 15.

Critical Pair Lemma 38

The following expressions are equivalent:

`nand[not[x1],not[x2]]==nand[not[x2],not[or[x1,x2]]]`

Proof

Note that the input for the rule:

`nand[x1_,and[x2_,x1_]]→nand[x2,x1]`

nand[x1_,and[x1_,x2_]]→nand[x1,x2]

contains a subpattern of the form:

and[x2_,x1_]

which can be unified with the input for the rule:

and[not[x1_],not[x2_]]→not[or[x1,x2]]

where these rules follow from Critical Pair Lemma 34 and Critical Pair Lemma 21 respectively.

Substitution Lemma 38

It can be shown that:

or[x1,not[not[x2]]]==nand[not[x2],not[or[x1,x2]]]

Proof

We start by taking Critical Pair Lemma 38, and apply the substitution:

nand[not[x1_],x2_]→or[x1,not[x2]]

which follows from Critical Pair Lemma 15.

Substitution Lemma 39

It can be shown that:

or[x1,x2]==nand[not[x2],not[or[x1,x2]]]

Proof

We start by taking Substitution Lemma 38, and apply the substitution:

not[not[x1_]]→x1

which follows from Substitution Lemma 14.

Substitution Lemma 40

It can be shown that:

or[x1,x2]==or[x2,not[not[or[x1,x2]]]]

Proof

We start by taking Substitution Lemma 39, and apply the substitution:

nand[not[x1_],x2_]→or[x1,not[x2]]

which follows from Critical Pair Lemma 15.

Substitution Lemma 41

It can be shown that:

or[x1,x2]==or[x2,or[x1,x2]]

Proof

We start by taking Substitution Lemma 40, and apply the substitution:

not[not[x1_]]→x1

which follows from Substitution Lemma 14.

Critical Pair Lemma 39

The following expressions are equivalent:

and[x1,and[x2,x1]]==not[nand[x2,x1]]

Proof

PROOF

Note that the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,and[x2_,x1_]]→nand[x2,x1]`

where these rules follow from Axiom 6 and Critical Pair Lemma 34 respectively.

Substitution Lemma 42

It can be shown that:

`and[x1,and[x2,x1]]==and[x2,x1]`

PROOF

We start by taking Critical Pair Lemma 39, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Critical Pair Lemma 40

The following expressions are equivalent:

`not[x1]==nand[x1,or[x2,x1]]`

PROOF

Note that the input for the rule:

`nand[x1_,or[x1_,x2_]]→not[x1]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,x1_]]→or[x2,x1]`

where these rules follow from Substitution Lemma 32 and Substitution Lemma 41 respectively.

Critical Pair Lemma 41

The following expressions are equivalent:

`x1==and[x1,or[x2,x1]]`

PROOF

Note that the input for the rule:

`and[x1_,or[x1_,x2_]]→x1`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,x1_]]→or[x2,x1]`

where these rules follow from Critical Pair Lemma 23 and Substitution Lemma 41 respectively.

Critical Pair Lemma 42

The following expressions are equivalent:

`not[not[x1]]==nand[not[x1],nand[x2,x1]]`

Proof

Note that the input for the rule:

`nand[x1_,or[x2_,x1_]]→not[x1]`

contains a subpattern of the form:

`or[x2_,x1_]`

which can be unified with the input for the rule:

`or[not[x1_],not[x2_]]→nand[x1,x2]`

where these rules follow from Critical Pair Lemma 40 and Critical Pair Lemma 11 respectively.

Substitution Lemma 43

It can be shown that:

`x1==nand[not[x1],nand[x2,x1]]`

Proof

We start by taking Critical Pair Lemma 42, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 14.

Substitution Lemma 44

It can be shown that:

`x1==or[x1,not[nand[x2,x1]]]`

Proof

We start by taking Substitution Lemma 43, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 15.

Substitution Lemma 45

It can be shown that:

`x1==or[x1,and[x2,x1]]`

Proof

We start by taking Substitution Lemma 44, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Critical Pair Lemma 43

The following expressions are equivalent:

`nand[x1,and[x2,not[x1]]]==nand[x1,not[x1]]`

Proof

Note that the input for the rule:

`nand[x1_,or[not[x1_],x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`or[not[x1_],x2_]`

which can be unified with the input for the rule:

`or[x1_,and[x2_,x1_]]→x1`

where these rules follow from Substitution Lemma 34 and Substitution Lemma 45 respectively.

Substitution Lemma 46

It can be shown that:

`nand[x1,and[x2,not[x1]]]==or[not[x1],x1]`

Proof

We start by taking Critical Pair Lemma 43, and apply the substitution:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 8.

Substitution Lemma 47

It can be shown that:

`nand[x1,and[x2,not[x1]]]==or[x1,not[x1]]`

Proof

We start by taking Substitution Lemma 46, and apply the substitution:

`or[not[x1_],x1_]→or[x1,not[x1]]`

which follows from Substitution Lemma 37.

Critical Pair Lemma 44

The following expressions are equivalent:

`or[x1,not[x1]]==nand[x2,and[x1,not[x1]]]`

Proof

Note that the input for the rule:

`nand[x1_,and[x2_,not[x1_]]]→or[x1,not[x1]]`

contains a subpattern of the form:

`nand[x1_,and[x2_,not[x1_]]]`

which can be unified with the input for the rule:

`nand[x1_,and[x2_,x3_]]↔nand[x2_,and[x1_,x3_]]`

where these rules follow from Substitution Lemma 47 and Substitution Lemma 10 respectively.

Critical Pair Lemma 45

The following expressions are equivalent:

`or[x1,not[x1]]==or[x2,not[and[x1,not[x1]]]]`

Proof

Note that the input for the rule:

`nand[x1_,and[x2_,not[x2_]]]→or[x2,not[x2]]`

contains a subpattern of the form:

`nand[x1_,and[x2_,not[x2_]]]`

which can be unified with the input for the rule:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

where these rules follow from Critical Pair Lemma 44 and Critical Pair Lemma 15 respectively.

Substitution Lemma 48

It can be shown that:

`or[x1,not[x1]]==or[x2,nand[x1,not[x1]]]`

Proof

We start by taking Critical Pair Lemma 45, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 7.

Substitution Lemma 49

It can be shown that:

`or[x1,not[x1]]==or[x2,or[not[x1],x1]]`

Proof

We start by taking Substitution Lemma 48, and apply the substitution:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 8.

Substitution Lemma 50

It can be shown that:

`or[x1,not[x1]]==or[x2,or[x1,not[x1]]]`

Proof

We start by taking Substitution Lemma 49, and apply the substitution:

`or[not[x1_],x1_]→or[x1,not[x1]]`

which follows from Substitution Lemma 37.

Critical Pair Lemma 46

The following expressions are equivalent:

`x1==and[x1,or[x2,not[x2]]]`

Proof

Note that the input for the rule:

`and[x1_,or[x1_,x2_]]→x1`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,not[x2_]]]→or[x2,not[x2]]`

where these rules follow from Critical Pair Lemma 23 and Substitution Lemma 50 respectively.

Critical Pair Lemma 47

The following expressions are equivalent:

`nand[x1,and[x2,or[x3,not[x3]]]]==nand[x2,x1]`

Proof

Note that the input for the rule:

`nand[x1_,and[x2_,x3_]]↔nand[x2_,and[x1_,x3_]]`

contains a subpattern of the form:

`and[x2_,x3_]`

which can be unified with the input for the rule:

`and[x1_,or[x2_,not[x2_]]]→x1`

where these rules follow from Substitution Lemma 10 and Critical Pair Lemma 46 respectively.

Substitution Lemma 51

It can be shown that:

`nand[x1,x2]==nand[x2,x1]`

Proof

We start by taking Critical Pair Lemma 47, and apply the substitution:

`and[x1_,or[x2_,not[x2_]]]→x1`

which follows from Critical Pair Lemma 46.

Critical Pair Lemma 48

The following expressions are equivalent:

`nand[and[x1,x2],x3]==nand[x1,and[x3,x2]]`

Proof

Note that the input for the rule:

`nand[x1_,x2_]↔nand[x2_,x1_]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,and[x2_,x3_]]↔nand[x2_,and[x1_,x3_]]`

where these rules follow from Substitution Lemma 51 and Substitution Lemma 10 respectively.

Critical Pair Lemma 49

The following expressions are equivalent:

`and[x1,not[x2]]==and[x1,nand[x2,x1]]`

Proof

Note that the input for the rule:

Note that the input for the rule:

`and[x1_,nand[x1_,x2_]]→and[x1,not[x2]]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,x2_]↔nand[x2_,x1_]`

where these rules follow from Substitution Lemma 8 and Substitution Lemma 51 respectively.

Critical Pair Lemma 50

The following expressions are equivalent:

`and[x1,x2]==not[nand[x2,x1]]`

Proof

Note that the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,x2_]↔nand[x2_,x1_]`

where these rules follow from Axiom 6 and Substitution Lemma 51 respectively.

Substitution Lemma 52

It can be shown that:

`and[x1,x2]==and[x2,x1]`

Proof

We start by taking Critical Pair Lemma 50, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Critical Pair Lemma 51

The following expressions are equivalent:

`and[x1,not[not[x2]]]==and[x1,or[x2,not[x1]]]`

Proof

Note that the input for the rule:

`and[x1_,nand[x2_,x1_]]→and[x1,not[x2]]`

contains a subpattern of the form:

`nand[x2_,x1_]`

which can be unified with the input for the rule:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

where these rules follow from Critical Pair Lemma 49 and Critical Pair Lemma 15 respectively.

Substitution Lemma 53

It can be shown that:

`and[x1,x2]==and[x1,or[x2,not[x1]]]`

Proof

We start by taking Critical Pair Lemma 51, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 14.

Critical Pair Lemma 52

The following expressions are equivalent:

`nand[not[x1],and[x2,not[x3]]]==nand[not[or[x1,x3]],x2]`

Proof

Note that the input for the rule:

`nand[and[x1_,x2_],x3_]→nand[x1,and[x3,x2]]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[not[x1_],not[x2_]]→not[or[x1,x2]]`

where these rules follow from Critical Pair Lemma 48 and Critical Pair Lemma 21 respectively.

Substitution Lemma 54

It can be shown that:

`or[x1,not[and[x2,not[x3]]]]==nand[not[or[x1,x3]],x2]`

Proof

We start by taking Critical Pair Lemma 52, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 15.

Substitution Lemma 55

It can be shown that:

`or[x1,nand[x2,not[x3]]]==nand[not[or[x1,x3]],x2]`

Proof

We start by taking Substitution Lemma 54, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 7.

Substitution Lemma 56

It can be shown that:

`or[x1,or[not[x2],x3]]==nand[not[or[x1,x3]],x2]`

Proof

We start by taking Substitution Lemma 55, and apply the substitution:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 8.

Substitution Lemma 57

Substitution Lemma 57

It can be shown that:

`or[x1,or[not[x2],x3]]==or[or[x1,x3],not[x2]]`

Proof

We start by taking Substitution Lemma 56, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 15.

Critical Pair Lemma 53

The following expressions are equivalent:

`and[and[x1,x2],x3]==not[nand[x1,and[x3,x2]]]`

Proof

Note that the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[and[x1_,x2_],x3_]→nand[x1,and[x3,x2]]`

where these rules follow from Axiom 6 and Critical Pair Lemma 48 respectively.

Substitution Lemma 58

It can be shown that:

`and[and[x1,x2],x3]==and[x1,and[x3,x2]]`

Proof

We start by taking Critical Pair Lemma 53, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Critical Pair Lemma 54

The following expressions are equivalent:

`and[x1,and[x2,or[x3,x1]]]==and[x1,x2]`

Proof

Note that the input for the rule:

`and[and[x1_,x2_],x3_]→and[x1,and[x3,x2]]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,or[x2_,x1_]]→x1`

where these rules follow from Substitution Lemma 58 and Critical Pair Lemma 41 respectively.

Critical Pair Lemma 55

The following expressions are equivalent:

`and[not[x1],and[x2,not[x3]]]==and[not[or[x1,x3]],x2]`

Proof

Note that the input for the rule:

`and[and[x1_,x2_],x3_]→and[x1,and[x3,x2]]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[not[x1_],not[x2_]]→not[or[x1,x2]]`

where these rules follow from Substitution Lemma 58 and Critical Pair Lemma 21 respectively.

Critical Pair Lemma 56

The following expressions are equivalent:

`x1==or[x1,and[x2,and[x1,x3]]]`

Proof

Note that the input for the rule:

`or[x1_,and[x2_,x1_]]→x1`

contains a subpattern of the form:

`and[x2_,x1_]`

which can be unified with the input for the rule:

`and[and[x1_,x2_],x3_]→and[x1,and[x3,x2]]`

where these rules follow from Substitution Lemma 45 and Substitution Lemma 58 respectively.

Critical Pair Lemma 57

The following expressions are equivalent:

`and[not[x1],x2]==and[not[x1],or[x2,x1]]`

Proof

Note that the input for the rule:

`and[x1_,or[x2_,not[x1_]]]→and[x1,x2]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Substitution Lemma 53 and Substitution Lemma 14 respectively.

Critical Pair Lemma 58

The following expressions are equivalent:

`or[x1,or[x2,not[not[x1]]]]==or[x1,and[not[x1],x2]]`

Proof

PROOF

Note that the input for the rule:

`or[x1_,and[not[x1_],x2_]]→or[x1,x2]`

contains a subpattern of the form:

`and[not[x1_],x2_]`

which can be unified with the input for the rule:

`and[x1_,or[x2_,not[x1_]]]→and[x1,x2]`

where these rules follow from Substitution Lemma 24 and Substitution Lemma 53 respectively.

Substitution Lemma 59

It can be shown that:

`or[x1,or[x2,x1]]==or[x1,and[not[x1],x2]]`

Proof

We start by taking Critical Pair Lemma 58, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 14.

Substitution Lemma 60

It can be shown that:

`or[x1,x2]==or[x2,and[not[x2],x1]]`

Proof

We start by taking Substitution Lemma 59, and apply the substitution:

`or[x1_,or[x2_,x1_]]→or[x2,x1]`

which follows from Substitution Lemma 41.

Substitution Lemma 61

It can be shown that:

`or[x1,x2]==or[x2,x1]`

Proof

We start by taking Substitution Lemma 60, and apply the substitution:

`or[x1_,and[not[x1_],x2_]]→or[x1,x2]`

which follows from Substitution Lemma 24.

Critical Pair Lemma 59

The following expressions are equivalent:

`x1==or[x1,and[x2,and[x3,x1]]]`

Proof

Note that the input for the rule:

`or[x1_,and[x2_,and[x1_,x3_]]]→x1`

contains a subpattern of the form:

`and[x1_,x3_]`

which can be unified with the input for the rule:

`and[x1_,and[x2_,x1_]]→and[x2,x1]`

where these rules follow from Critical Pair Lemma 56 and Substitution Lemma 42 respectively.

Critical Pair Lemma 60

The following expressions are equivalent:

`or[x1,x2]⩵or[or[x1,x2],and[x3,x2]]`

Proof

Note that the input for the rule:

`or[x1_,and[x2_,and[x3_,x1_]]]→x1`

contains a subpattern of the form:

`and[x3_,x1_]`

which can be unified with the input for the rule:

`and[x1_,or[x2_,x1_]]→x1`

where these rules follow from Critical Pair Lemma 59 and Critical Pair Lemma 41 respectively.

Critical Pair Lemma 61

The following expressions are equivalent:

`nand[not[x1],and[x2,or[x3,x1]]]⩵nand[and[not[x1],x3],x2]`

Proof

Note that the input for the rule:

`nand[and[x1_,x2_],x3_]→nand[x1,and[x3,x2]]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[not[x1_],or[x2_,x1_]]→and[not[x1],x2]`

where these rules follow from Critical Pair Lemma 48 and Critical Pair Lemma 57 respectively.

Substitution Lemma 62

It can be shown that:

`or[x1,not[and[x2,or[x3,x1]]]]⩵nand[and[not[x1],x3],x2]`

Proof

We start by taking Critical Pair Lemma 61, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 15.

Substitution Lemma 63

It can be shown that:

`or[x1,nand[x2,or[x3,x1]]]⩵nand[and[not[x1],x3],x2]`

Proof

We start by taking Substitution Lemma 62, and apply the substitution:

We start by taking Substitution Lemma 62, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 7.

Substitution Lemma 64

It can be shown that:

`or[x1,nand[x2,or[x3,x1]]]==nand[not[x1],and[x2,x3]]`

Proof

We start by taking Substitution Lemma 63, and apply the substitution:

`nand[and[x1_,x2_],x3_]→nand[x1,and[x3,x2]]`

which follows from Critical Pair Lemma 48.

Substitution Lemma 65

It can be shown that:

`or[x1,nand[x2,or[x3,x1]]]==or[x1,not[and[x2,x3]]]`

Proof

We start by taking Substitution Lemma 64, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 15.

Substitution Lemma 66

It can be shown that:

`or[x1,nand[x2,or[x3,x1]]]==or[x1,nand[x2,x3]]`

Proof

We start by taking Substitution Lemma 65, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 7.

Critical Pair Lemma 62

The following expressions are equivalent:

`nand[x1,x2]==nand[x2,and[or[x2,x3],x1]]`

Proof

Note that the input for the rule:

`nand[x1_,and[x2_,or[x1_,x3_]]]→nand[x2,x1]`

contains a subpattern of the form:

`and[x2_,or[x1_,x3_]]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Critical Pair Lemma 35 and Substitution Lemma 52 respectively.

Critical Pair Lemma 63

The following expressions are equivalent:

The following expressions are equivalent:

`and[x1,or[x2,x3]]==or[and[x1,or[x2,x3]],and[x3,x1]]`

PROOF

Note that the input for the rule:

`or[x1_,and[x2_,x1_]]→x1`

contains a subpattern of the form:

`and[x2_,x1_]`

which can be unified with the input for the rule:

`and[x1_,and[x2_,or[x3_,x1_]]]→and[x1,x2]`

where these rules follow from Substitution Lemma 45 and Critical Pair Lemma 54 respectively.

Critical Pair Lemma 64

The following expressions are equivalent:

`or[x1,x2]==or[and[x3,x2],or[x1,x2]]`

PROOF

Note that the input for the rule:

`or[or[x1_,x2_],and[x3_,x2_]]→or[x1,x2]`

contains a subpattern of the form:

`or[or[x1_,x2_],and[x3_,x2_]]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Critical Pair Lemma 60 and Substitution Lemma 61 respectively.

Critical Pair Lemma 65

The following expressions are equivalent:

`or[x1,or[x2,x3]]==or[x3,or[x1,or[x2,x3]]]`

PROOF

Note that the input for the rule:

`or[and[x1_,x2_],or[x3_,x2_]]→or[x3,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,or[x2_,x1_]]→x1`

where these rules follow from Critical Pair Lemma 64 and Critical Pair Lemma 41 respectively.

Critical Pair Lemma 66

The following expressions are equivalent:

`or[x1,or[not[not[x2]],x3]]==or[or[x1,x3],x2]`

PROOF

Note that the input for the rule:

`or[or[x1_,x2_],not[x3_]]→or[x1,or[not[x3],x2]]`

contains a subpattern of the form:

`not[x3_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Substitution Lemma 57 and Substitution Lemma 14 respectively.

Substitution Lemma 67

It can be shown that:

`or[x1,or[x2,x3]]==or[or[x1,x3],x2]`

Proof

We start by taking Critical Pair Lemma 66, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 14.

Critical Pair Lemma 67

The following expressions are equivalent:

`or[x1,or[x2,and[x3,x1]]]==or[x1,x2]`

Proof

Note that the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x3,x2]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,and[x2_,x1_]]→x1`

where these rules follow from Substitution Lemma 67 and Substitution Lemma 45 respectively.

Critical Pair Lemma 68

The following expressions are equivalent:

`or[or[x1,x2],x3]==or[or[x1,x2],or[x3,x1]]`

Proof

Note that the input for the rule:

`or[x1_,or[x2_,and[x3_,x1_]]]→or[x1,x2]`

contains a subpattern of the form:

`and[x3_,x1_]`

which can be unified with the input for the rule:

`and[x1_,or[x1_,x2_]]→x1`

where these rules follow from Critical Pair Lemma 67 and Critical Pair Lemma 23 respectively.

Substitution Lemma 68

It can be shown that:

`or[x1,or[x2,x3]]==or[or[x1,x3],or[x2,x1]]`

PROOF

We start by taking Critical Pair Lemma 68, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x3,x2]]`

which follows from Substitution Lemma 67.

Substitution Lemma 69

It can be shown that:

`or[x1,or[x2,x3]]==or[x1,or[or[x2,x1],x3]]`

PROOF

We start by taking Substitution Lemma 68, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x3,x2]]`

which follows from Substitution Lemma 67.

Substitution Lemma 70

It can be shown that:

`or[x1,or[x2,x3]]==or[x1,or[x2,or[x3,x1]]]`

PROOF

We start by taking Substitution Lemma 69, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x3,x2]]`

which follows from Substitution Lemma 67.

Substitution Lemma 71

It can be shown that:

`nand[x1,x2]==and[nand[x1,x2],nand[x1,and[x3,x2]]]`

PROOF

We start by taking Substitution Lemma 33, and apply the substitution:

`nand[and[x1_,x2_],x3_]→nand[x1,and[x3,x2]]`

which follows from Critical Pair Lemma 48.

Critical Pair Lemma 69

The following expressions are equivalent:

`nand[x1,x2]==and[nand[x1,x2],nand[x2,x1]]`

PROOF

Note that the input for the rule:

`and[nand[x1_,x2_],nand[x1_,and[x3_,x2_]]]→nand[x1,x2]`

contains a subpattern of the form:

`nand[x1_,and[x3_,x2_]]`

which can be unified with the input for the rule:

`nand[x1_,and[or[x1_,x2_],x3_]]→nand[x3,x1]`

where these rules follow from Substitution Lemma 71 and Critical Pair Lemma 62 respectively.

Critical Pair Lemma 70

The following expressions are equivalent:

`nand[nand[x1,x2],nand[x2,x1]]==nand[nand[x1,x2],nand[x1,x2]]`

Proof

Note that the input for the rule:

`nand[x1_,and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[nand[x1_,x2_],nand[x2_,x1_]]→nand[x1,x2]`

where these rules follow from Substitution Lemma 31 and Critical Pair Lemma 69 respectively.

Substitution Lemma 72

It can be shown that:

`or[and[x1,x2],and[x2,x1]]==nand[nand[x1,x2],nand[x1,x2]]`

Proof

We start by taking Critical Pair Lemma 70, and apply the substitution:

`nand[nand[x1_,x2_],nand[x3_,x4_]]→or[and[x1,x2],and[x3,x4]]`

which follows from Critical Pair Lemma 31.

Substitution Lemma 73

It can be shown that:

`or[and[x1,x2],and[x2,x1]]==or[and[x1,x2],and[x1,x2]]`

Proof

We start by taking Substitution Lemma 72, and apply the substitution:

`nand[nand[x1_,x2_],nand[x3_,x4_]]→or[and[x1,x2],and[x3,x4]]`

which follows from Critical Pair Lemma 31.

Substitution Lemma 74

It can be shown that:

`or[and[x1,x2],and[x2,x1]]==and[x1,x2]`

Proof

We start by taking Substitution Lemma 73, and apply the substitution:

`or[x1_,x1_]→x1`

which follows from Substitution Lemma 13.

Critical Pair Lemma 71

The following expressions are equivalent:

`nand[x1,nand[x2,nand[x1,x2]]]==nand[x1,nand[x2,and[x1,not[x2]]]]`

```
nand[x1,nand[x2,nand[x1,x3]]]==nand[x1,nand[x2,and[x1,not[x3]]]]
```

Proof

Note that the input for the rule:

```
nand[x1_,nand[x2_,and[x1_,x3_]]]→nand[x1,nand[x2,x3]]
```

contains a subpattern of the form:

```
and[x1_,x3_]
```

which can be unified with the input for the rule:

```
and[x1_,nand[x1_,x2_]]→and[x1,not[x2]]
```

where these rules follow from Substitution Lemma 36 and Substitution Lemma 8 respectively.

Substitution Lemma 75

It can be shown that:

```
nand[x1,nand[x2,nand[x1,x3]]]==nand[x1,nand[x2,not[x3]]]
```

Proof

We start by taking Critical Pair Lemma 71, and apply the substitution:

```
nand[x1_,nand[x2_,and[x1_,x3_]]]→nand[x1,nand[x2,x3]]
```

which follows from Substitution Lemma 36.

Substitution Lemma 76

It can be shown that:

```
nand[x1,nand[x2,nand[x1,x3]]]==nand[x1,or[not[x2],x3]]
```

Proof

We start by taking Substitution Lemma 75, and apply the substitution:

```
nand[x1_,not[x2_]]→or[not[x1],x2]
```

which follows from Critical Pair Lemma 8.

Critical Pair Lemma 72

The following expressions are equivalent:

```
or[x1,nand[x2,x3]]==or[x1,nand[or[x3,x1],x2]]
```

Proof

Note that the input for the rule:

```
or[x1_,nand[x2_,or[x3_,x1_]]]→or[x1,nand[x2,x3]]
```

contains a subpattern of the form:

```
nand[x2_,or[x3_,x1_]]
```

which can be unified with the input for the rule:

```
nand[x1_,x2_]↔nand[x2_,x1_]
```

where these rules follow from Substitution Lemma 66 and Substitution Lemma 51 respectively.

Substitution Lemma 77

It can be shown that:

```
or[x1,or[x2,x3]]==or[x2,or[x3,x1]]
```

Proof

We start by taking Substitution Lemma 70, and apply the substitution:

`or[x1_,or[x2_,or[x3_,x1_]]]→or[x2,or[x3,x1]]`

which follows from Critical Pair Lemma 65.

Critical Pair Lemma 73

The following expressions are equivalent:

`and[not[x1],and[x2,not[x3]]]==not[or[not[x2],or[x3,x1]]]`

Proof

Note that the input for the rule:

`not[or[x1_,or[not[x2_],x3_]]]→and[not[x1],and[x2,not[x3]]]`

contains a subpattern of the form:

`or[x1_,or[not[x2_],x3_]]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,x3_]]↔or[x2_,or[x3_,x1_]]`

where these rules follow from Critical Pair Lemma 26 and Substitution Lemma 77 respectively.

Substitution Lemma 78

It can be shown that:

`and[not[x1],and[x2,not[x3]]]==and[x2,not[or[x3,x1]]]`

Proof

We start by taking Critical Pair Lemma 73, and apply the substitution:

`not[or[not[x1_],x2_]]→and[x1,not[x2]]`

which follows from Critical Pair Lemma 12.

Critical Pair Lemma 74

The following expressions are equivalent:

`or[x1,not[nand[or[x2,x1],x3]]]==or[x1,not[or[x1,nand[x3,x2]]]]`

Proof

Note that the input for the rule:

`or[x1_,not[or[x1_,x2_]]]→or[x1,not[x2]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,nand[or[x2_,x1_],x3_]]→or[x1,nand[x3,x2]]`

where these rules follow from Critical Pair Lemma 28 and Critical Pair Lemma 72 respectively.

Substitution Lemma 79

It can be shown that:

`or[x1,and[or[x2,x1],x3]]==or[x1,not[or[x1,nand[x3,x2]]]]`

Proof

PROOF

We start by taking Critical Pair Lemma 74, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Substitution Lemma 80

It can be shown that:

`or[x1,and[or[x2,x1],x3]]==or[x1,not[nand[x3,x2]]]`

PROOF

We start by taking Substitution Lemma 79, and apply the substitution:

`or[x1_,not[or[x1_,x2_]]]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 28.

Substitution Lemma 81

It can be shown that:

`or[x1,and[or[x2,x1],x3]]==or[x1,and[x3,x2]]`

PROOF

We start by taking Substitution Lemma 80, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Critical Pair Lemma 75

The following expressions are equivalent:

`nand[x1,or[not[not[x2]],x3]]==nand[x1,or[x2,not[nand[x1,x3]]]]`

PROOF

Note that the input for the rule:

`nand[x1_,nand[x2_,nand[x1_,x3_]]]→nand[x1,or[not[x2],x3]]`

contains a subpattern of the form:

`nand[x2_,nand[x1_,x3_]]`

which can be unified with the input for the rule:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

where these rules follow from Substitution Lemma 76 and Critical Pair Lemma 15 respectively.

Substitution Lemma 82

It can be shown that:

`nand[x1,or[x2,x3]]==nand[x1,or[x2,not[nand[x1,x3]]]]`

PROOF

We start by taking Critical Pair Lemma 75, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 14.

Substitution Lemma 83

Substitution Lemma 83

It can be shown that:

`nand[x1,or[x2,x3]]==nand[x1,or[x2,and[x1,x3]]]`

Proof

We start by taking Substitution Lemma 82, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Critical Pair Lemma 76

The following expressions are equivalent:

`nand[x1,or[x2,x3]]==nand[x1,or[x2,and[x3,x1]]]`

Proof

Note that the input for the rule:

`nand[x1_,or[x2_,and[x1_,x3_]]]→nand[x1,or[x2,x3]]`

contains a subpattern of the form:

`and[x1_,x3_]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Substitution Lemma 83 and Substitution Lemma 52 respectively.

Critical Pair Lemma 77

The following expressions are equivalent:

`and[x1,or[x2,and[x3,x1]]]==not[nand[x1,or[x2,x3]]]`

Proof

Note that the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,or[x2_,and[x3_,x1_]]]→nand[x1,or[x2,x3]]`

where these rules follow from Axiom 6 and Critical Pair Lemma 76 respectively.

Substitution Lemma 84

It can be shown that:

`and[x1,or[x2,and[x3,x1]]]==and[x1,or[x2,x3]]`

Proof

We start by taking Critical Pair Lemma 77, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Substitution Lemma 85

It can be shown that:

`and[not[or[x1,x2]],x3]==and[x3,not[or[x2,x1]]]`

Proof

We start by taking Substitution Lemma 78, and apply the substitution:

`and[not[x1_],and[x2_,not[x3_]]]→and[not[or[x1,x3]],x2]`

which follows from Critical Pair Lemma 55.

Critical Pair Lemma 78

The following expressions are equivalent:

`and[x1,not[or[and[x2,x3],and[x3,x2]]]]==and[not[and[x3,x2]],x1]`

Proof

Note that the input for the rule:

`and[not[or[x1_,x2_]],x3_]↔and[x3_,not[or[x2_,x1_]]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[and[x1_,x2_],and[x2_,x1_]]→and[x1,x2]`

where these rules follow from Substitution Lemma 85 and Substitution Lemma 74 respectively.

Substitution Lemma 86

It can be shown that:

`and[x1,and[nand[x2,x3],not[and[x3,x2]]]]==and[not[and[x3,x2]],x1]`

Proof

We start by taking Critical Pair Lemma 78, and apply the substitution:

`not[or[and[x1_,x2_],x3_]]→and[nand[x1,x2],not[x3]]`

which follows from Critical Pair Lemma 20.

Substitution Lemma 87

It can be shown that:

`and[x1,and[nand[x2,x3],nand[x3,x2]]]==and[not[and[x3,x2]],x1]`

Proof

We start by taking Substitution Lemma 86, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 7.

Substitution Lemma 88

It can be shown that:

`and[x1,nand[x2,x3]]==and[not[and[x3,x2]],x1]`

Proof

We start by taking Substitution Lemma 87, and apply the substitution:

`and[nand[x1_,x2_],nand[x2_,x1_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 69.

Substitution Lemma 89

It can be shown that:

`and[x1,nand[x2,x3]]==and[nand[x3,x2],x1]`

Proof

We start by taking Substitution Lemma 88, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 7.

Critical Pair Lemma 79

The following expressions are equivalent:

`nand[x1,nand[x2,x3]]==nand[x1,and[nand[x3,x2],x1]]`

Proof

Note that the input for the rule:

`nand[x1_,and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,nand[x2_,x3_]]↔and[nand[x3_,x2_],x1_]`

where these rules follow from Substitution Lemma 31 and Substitution Lemma 89 respectively.

Substitution Lemma 90

It can be shown that:

`nand[x1,nand[x2,x3]]==or[and[x3,x2],nand[x1,x1]]`

Proof

We start by taking Critical Pair Lemma 79, and apply the substitution:

`nand[x1_,and[nand[x2_,x3_],x4_]]→or[and[x2,x3],nand[x1,x4]]`

which follows from Critical Pair Lemma 33.

Substitution Lemma 91

It can be shown that:

`nand[x1,nand[x2,x3]]==or[and[x3,x2],not[x1]]`

Proof

We start by taking Substitution Lemma 90, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Substitution Lemma 92

It can be shown that:

`nand[x1,nand[x2,x3]]==nand[nand[x3,x2],x1]` nand[x1,nand[x2,x3]]==nand[nand[x3,x2],x1]

Proof

We start by taking Substitution Lemma 91, and apply the substitution:

`or[and[x1_,x2_],not[x3_]]→nand[nand[x1,x2],x3]`

which follows from Critical Pair Lemma 14.

Critical Pair Lemma 80

The following expressions are equivalent:

`nand[nand[x1,x2],not[x3]]==or[x3,not[nand[x2,x1]]]`

Proof

Note that the input for the rule:

`nand[x1_,nand[x2_,x3_]]↔nand[nand[x3_,x2_],x1_]`

contains a subpattern of the form:

`nand[x1_,nand[x2_,x3_]]`

which can be unified with the input for the rule:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

where these rules follow from Substitution Lemma 92 and Critical Pair Lemma 15 respectively.

Substitution Lemma 93

It can be shown that:

`or[not[nand[x1,x2]],x3]==or[x3,not[nand[x2,x1]]]`

Proof

We start by taking Critical Pair Lemma 80, and apply the substitution:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 8.

Substitution Lemma 94

It can be shown that:

`or[and[x1,x2],x3]==or[x3,not[nand[x2,x1]]]`

Proof

We start by taking Substitution Lemma 93, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Substitution Lemma 95

It can be shown that:

`or[and[x1,x2],x3]==or[x3,and[x2,x1]]`

Proof

We start by taking Substitution Lemma 94, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Substitution Lemma 96

It can be shown that:

`and[x1,or[x2,x3]]==or[and[x3,x1],and[or[x2,x3],x1]]`

Proof

We start by taking Critical Pair Lemma 63, and apply the substitution:

`or[and[x1_,x2_],x3_]→or[x3,and[x2,x1]]`

which follows from Substitution Lemma 95.

Critical Pair Lemma 81

The following expressions are equivalent:

`and[or[x1,x2],or[x3,x2]]==or[x2,and[or[x3,x2],or[x1,x2]]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],and[or[x3_,x1_],x2_]]→and[x2,or[x3,x1]]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,or[x2_,x1_]]→x1`

where these rules follow from Substitution Lemma 96 and Critical Pair Lemma 41 respectively.

Substitution Lemma 97

It can be shown that:

`and[or[x1,x2],or[x3,x2]]==or[x2,and[or[x1,x2],x3]]`

Proof

We start by taking Critical Pair Lemma 81, and apply the substitution:

`or[x1_,and[or[x2_,x1_],x3_]]→or[x1,and[x3,x2]]`

which follows from Substitution Lemma 81.

Substitution Lemma 98

It can be shown that:

`and[or[x1,x2],or[x3,x2]]==or[x2,and[x3,x1]]`

Proof

We start by taking Substitution Lemma 97, and apply the substitution:

`or[x1_,and[or[x2_,x1_],x3_]]→or[x1,and[x3,x2]]`

which follows from Substitution Lemma 81.

Critical Pair Lemma 82

The following expressions are equivalent:

`or[and[x1,x2],and[x3,x2]]==and[x2,or[x3,and[x1,x2]]]`

Proof

Note that the input for the rule:

`and[or[x1_,x2_],or[x3_,x2_]]→or[x2,and[x3,x1]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,and[x2_,x1_]]→x1`

where these rules follow from Substitution Lemma 98 and Substitution Lemma 45 respectively.

Substitution Lemma 99

It can be shown that:

`or[and[x1,x2],and[x3,x2]]==and[x2,or[x3,x1]]`

Proof

We start by taking Critical Pair Lemma 82, and apply the substitution:

`and[x1_,or[x2_,and[x3_,x1_]]]→and[x1,or[x2,x3]]`

which follows from Substitution Lemma 84.

Substitution Lemma 100

It can be shown that:

`or[and[a,b],and[c,a]]==and[a,or[b,c]]`

Proof

We start by taking Hypothesis 1, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Substitution Lemma 52.

Substitution Lemma 101

It can be shown that:

`or[and[b,a],and[c,a]]==and[a,or[b,c]]`

Proof

We start by taking Substitution Lemma 100, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Substitution Lemma 52.

Substitution Lemma 102

It can be shown that:

`or[and[b,a],and[c,a]]==and[a,or[c,b]]`

Proof

We start by taking Substitution Lemma 101, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Substitution Lemma 61.

Conclusion 1

We obtain the conclusion:

`True`

Proof

Take Substitution Lemma 102, and apply the substitution:

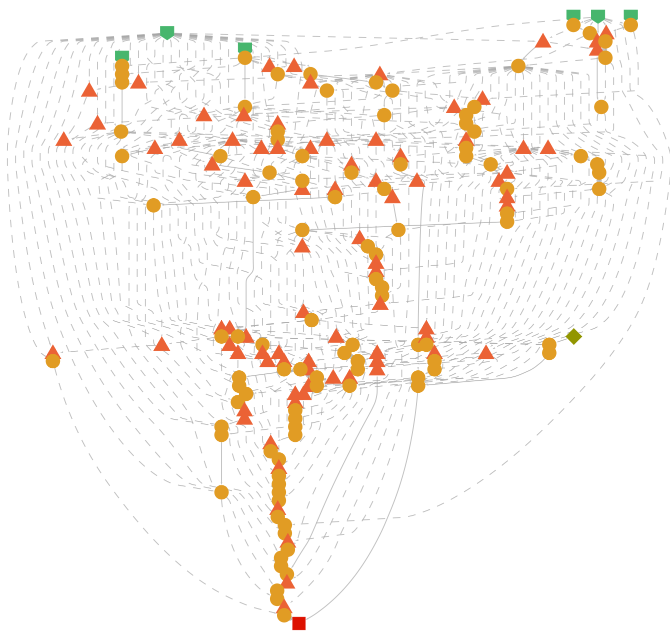
`or[and[x1_,x2_],and[x3_,x2_]]→and[x2,or[x3,x1]]`

which follows from Substitution Lemma 99.

large output | show less | show more | show all | set size limit...

In[∘]:= `proofAxB5fromShort["ProofGraph"]`

Out[∘]=



In[∘]:= `Clear[proofAxB5fromShort]`

In[∘]:= `proofAxB6fromShort["ProofNotebook"]`

Axiom 1

We are given that:

`x1==nand[nand[x1,x1],nand[x1,x2]]`

Axiom 2

We are given that:

`nand[x1,x1]==not[x1]`

Axiom 3

We are given that:

```
nand[x1,nand[x1,x2]]==nand[x1,nand[x2,x2]]
```

Axiom 4

We are given that:

```
nand[x1,nand[x1,nand[x2,x3]]]==nand[x2,nand[x2,nand[x1,x3]]]
```

Axiom 5

We are given that:

```
nand[nand[x1,x1],nand[x2,x2]]==or[x1,x2]
```

Axiom 6

We are given that:

```
not[nand[x1,x2]]==and[x1,x2]
```

Hypothesis 1

We would like to show that:

```
and[or[a,b],or[a,c]]==or[a,and[b,c]]
```

Substitution Lemma 1

It can be shown that:

```
nand[not[x1],nand[x1,x2]]==x1
```

Proof

We start by taking Axiom 1, and apply the substitution:

```
nand[x1_,x1_]→not[x1]
```

which follows from Axiom 2.

Substitution Lemma 2

It can be shown that:

```
nand[x1,nand[x1,x2]]==nand[x1,not[x2]]
```

Proof

We start by taking Axiom 3, and apply the substitution:

```
nand[x1_,x1_]→not[x1]
```

which follows from Axiom 2.

Substitution Lemma 3

It can be shown that:

```
nand[x1,not[nand[x2,x3]]]==nand[x2,nand[x2,nand[x1,x3]]]
```

Proof

We start by taking Axiom 4, and apply the substitution:

```
nand[x1_,nand[x1_,x2_]]→nand[x1,not[x2]]
```

which follows from Substitution Lemma 2.

Substitution Lemma 4

It can be shown that:

```
nand[x1,not[nand[x2,x3]]]==nand[x2,not[nand[x1,x3]]]
```

Proof

We start by taking Substitution Lemma 3, and apply the substitution:

`nand[x1_,nand[x1_,x2_]]→nand[x1,not[x2]]`

which follows from Substitution Lemma 2.

Substitution Lemma 5

It can be shown that:

`nand[not[x1],nand[x2,x2]]==or[x1,x2]`

Proof

We start by taking Axiom 5, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Substitution Lemma 6

It can be shown that:

`nand[not[x1],not[x2]]==or[x1,x2]`

Proof

We start by taking Substitution Lemma 5, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Critical Pair Lemma 1

The following expressions are equivalent:

`or[x1,x1]==not[not[x1]]`

Proof

Note that the input for the rule:

`nand[not[x1_],not[x2_]]→or[x1,x2]`

contains a subpattern of the form:

`nand[not[x1_],not[x2_]]`

which can be unified with the input for the rule:

`nand[x1_,x1_]→not[x1]`

where these rules follow from Substitution Lemma 6 and Axiom 2 respectively.

Critical Pair Lemma 2

The following expressions are equivalent:

`and[x1,x1]==not[not[x1]]`

Proof

Note that the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,x1_]→not[x1]`

where these rules follow from Axiom 6 and Axiom 2 respectively.

Critical Pair Lemma 3

The following expressions are equivalent:

`and[x1,nand[x1,x2]]==not[nand[x1,not[x2]]]`

Proof

Note that the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,nand[x1_,x2_]]→nand[x1,not[x2]]`

where these rules follow from Axiom 6 and Substitution Lemma 2 respectively.

Substitution Lemma 7

It can be shown that:

`and[x1,nand[x1,x2]]==and[x1,not[x2]]`

Proof

We start by taking Critical Pair Lemma 3, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Substitution Lemma 8

It can be shown that:

`nand[x1,and[x2,x3]]==nand[x2,not[nand[x1,x3]]]`

Proof

We start by taking Substitution Lemma 4, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Substitution Lemma 9

It can be shown that:

`nand[x1,and[x2,x3]]==nand[x2,and[x1,x3]]`

Proof

We start by taking Substitution Lemma 8, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Critical Pair Lemma 4

The following expressions are equivalent:

`and[x1,x1]==or[x1,x1]`

PROOF

Note that the input for the rule:

`and[x1_,x1_]↔not[not[x1_]]`

contains a subpattern of the form:

`not[not[x1_]]`

which can be unified with the input for the rule:

`or[x1_,x1_]↔not[not[x1_]]`

where these rules follow from Critical Pair Lemma 2 and Critical Pair Lemma 1 respectively.

Critical Pair Lemma 5

The following expressions are equivalent:

`x1==nand[not[x1],not[x1]]`

PROOF

Note that the input for the rule:

`nand[not[x1_],nand[x1_,x2_]]→x1`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,x1_]→not[x1]`

where these rules follow from Substitution Lemma 1 and Axiom 2 respectively.

Substitution Lemma 10

It can be shown that:

`x1==or[x1,x1]`

PROOF

We start by taking Critical Pair Lemma 5, and apply the substitution:

`nand[not[x1_],not[x2_]]→or[x1,x2]`

which follows from Substitution Lemma 6.

Substitution Lemma 11

It can be shown that:

`and[x1_,x1_]→x1`

PROOF

We start by taking Critical Pair Lemma 4, and apply the substitution:

`or[x1_,x1_]→x1`

which follows from Substitution Lemma 10.

Substitution Lemma 12

It can be shown that:

`x1==not[not[x1]]`

PROOF

PROOF

We start by taking Critical Pair Lemma 1, and apply the substitution:

```
or[x1_,x1_]→x1
```

which follows from Substitution Lemma 10.

Critical Pair Lemma 6

The following expressions are equivalent:

```
nand[x1,x2]==not[and[x1,x2]]
```

PROOF

Note that the input for the rule:

```
not[not[x1_]]→x1
```

contains a subpattern of the form:

```
not[x1_]
```

which can be unified with the input for the rule:

```
not[nand[x1_,x2_]]→and[x1,x2]
```

where these rules follow from Substitution Lemma 12 and Axiom 6 respectively.

Critical Pair Lemma 7

The following expressions are equivalent:

```
or[not[x1],x2]==nand[x1,not[x2]]
```

PROOF

Note that the input for the rule:

```
nand[not[x1_],not[x2_]]→or[x1,x2]
```

contains a subpattern of the form:

```
not[x1_]
```

which can be unified with the input for the rule:

```
not[not[x1_]]→x1
```

where these rules follow from Substitution Lemma 6 and Substitution Lemma 12 respectively.

Substitution Lemma 13

It can be shown that:

```
nand[x1_,nand[x1_,x2_]]→or[not[x1],x2]
```

PROOF

We start by taking Substitution Lemma 2, and apply the substitution:

```
nand[x1_,not[x2_]]→or[not[x1],x2]
```

which follows from Critical Pair Lemma 7.

Critical Pair Lemma 8

The following expressions are equivalent:

```
or[not[x1],and[x2,x3]]==nand[x1,nand[x2,x3]]
```

PROOF

Note that the input for the rule:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

where these rules follow from Critical Pair Lemma 7 and Critical Pair Lemma 6 respectively.

Critical Pair Lemma 9

The following expressions are equivalent:

`or[not[x1],nand[x2,x3]]==nand[x1,and[x2,x3]]`

Proof

Note that the input for the rule:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

where these rules follow from Critical Pair Lemma 7 and Axiom 6 respectively.

Critical Pair Lemma 10

The following expressions are equivalent:

`or[not[x1],not[x2]]==nand[x1,x2]`

Proof

Note that the input for the rule:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Critical Pair Lemma 7 and Substitution Lemma 12 respectively.

Critical Pair Lemma 11

The following expressions are equivalent:

`and[x1,not[x2]]==not[or[not[x1],x2]]`

Proof

Note that the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

where these rules follow from Axiom 6 and Critical Pair Lemma 7 respectively.

Critical Pair Lemma 12

The following expressions are equivalent:

```
nand[nand[x1,x2],x3]==or[and[x1,x2],not[x3]]
```

Proof

Note that the input for the rule:

```
or[not[x1_],not[x2_]]→nand[x1,x2]
```

contains a subpattern of the form:

```
not[x1_]
```

which can be unified with the input for the rule:

```
not[nand[x1_,x2_]]→and[x1,x2]
```

where these rules follow from Critical Pair Lemma 10 and Axiom 6 respectively.

Critical Pair Lemma 13

The following expressions are equivalent:

```
nand[not[x1],x2]==or[x1,not[x2]]
```

Proof

Note that the input for the rule:

```
or[not[x1_],not[x2_]]→nand[x1,x2]
```

contains a subpattern of the form:

```
not[x1_]
```

which can be unified with the input for the rule:

```
not[not[x1_]]→x1
```

where these rules follow from Critical Pair Lemma 10 and Substitution Lemma 12 respectively.

Critical Pair Lemma 14

The following expressions are equivalent:

```
or[x1,not[nand[not[x1],x2]]]==or[not[not[x1]],x2]
```

Proof

Note that the input for the rule:

```
nand[not[x1_],x2_]→or[x1,not[x2]]
```

contains a subpattern of the form:

```
nand[not[x1_],x2_]
```

which can be unified with the input for the rule:

```
Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[74,nand[x1_,no
```

where these rules follow from Critical Pair Lemma 13 and Substitution Lemma 13 respectively.

Substitution Lemma 14

It can be shown that:

```
or[x1,and[not[x1],x2]]==or[not[not[x1]],x2]
```

`or[x1,and[not[x1],x2]]==or[not[not[x1]],x2]`

Proof

We start by taking Critical Pair Lemma 14, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Substitution Lemma 15

It can be shown that:

`or[x1,and[not[x1],x2]]==or[x1,x2]`

Proof

We start by taking Substitution Lemma 14, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 12.

Substitution Lemma 16

It can be shown that:

`or[x1,not[nand[x1,x2]]]==x1`

Proof

We start by taking Substitution Lemma 1, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 13.

Substitution Lemma 17

It can be shown that:

`or[x1,and[x1,x2]]==x1`

Proof

We start by taking Substitution Lemma 16, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Critical Pair Lemma 15

The following expressions are equivalent:

`and[x1,not[not[x2]]]==and[x1,or[not[x1],x2]]`

Proof

Note that the input for the rule:

`and[x1_,nand[x1_,x2_]]→and[x1,not[x2]]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

where these rules follow from Substitution Lemma 7 and Critical Pair Lemma 7 respectively.

Substitution Lemma 18

Substitution Lemma 18

It can be shown that:

`and[x1,x2]==and[x1,or[not[x1],x2]]`

PROOF

We start by taking Critical Pair Lemma 15, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 12.

Critical Pair Lemma 16

The following expressions are equivalent:

`and[x1,not[and[not[x1],x2]]]==not[not[x1]]`

PROOF

Note that the input for the rule:

`not[or[not[x1_],x2_]]→and[x1,not[x2]]`

contains a subpattern of the form:

`or[not[x1_],x2_]`

which can be unified with the input for the rule:

`or[x1_,and[x1_,x2_]]→x1`

where these rules follow from Critical Pair Lemma 11 and Substitution Lemma 17 respectively.

Substitution Lemma 19

It can be shown that:

`and[x1,nand[not[x1],x2]]==not[not[x1]]`

PROOF

We start by taking Critical Pair Lemma 16, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 6.

Substitution Lemma 20

It can be shown that:

`and[x1,or[x1,not[x2]]]==not[not[x1]]`

PROOF

We start by taking Substitution Lemma 19, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 13.

Substitution Lemma 21

It can be shown that:

`and[x1,or[x1,not[x2]]]==x1`

PROOF

We start by taking Substitution Lemma 20, and apply the substitution:

We start by taking Substitution Lemma 20, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 12.

Critical Pair Lemma 17

The following expressions are equivalent:

`and[nand[x1,x2],not[x3]]==not[or[and[x1,x2],x3]]`

Proof

Note that the input for the rule:

`not[or[not[x1_],x2_]]→and[x1,not[x2]]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

where these rules follow from Critical Pair Lemma 11 and Axiom 6 respectively.

Critical Pair Lemma 18

The following expressions are equivalent:

`and[not[x1],not[x2]]==not[or[x1,x2]]`

Proof

Note that the input for the rule:

`not[or[not[x1_],x2_]]→and[x1,not[x2]]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Critical Pair Lemma 11 and Substitution Lemma 12 respectively.

Critical Pair Lemma 19

The following expressions are equivalent:

`x1==and[x1,or[x1,x2]]`

Proof

Note that the input for the rule:

`and[x1_,or[x1_,not[x2_]]]→x1`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Substitution Lemma 21 and Substitution Lemma 12 respectively.

Critical Pair Lemma 20

The following expressions are equivalent:

`not[x1]==and[not[x1],nand[x1,x2]]`

Proof

Note that the input for the rule:

`and[x1_,or[x1_,x2_]]→x1`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[not[x1_],not[x2_]]→nand[x1,x2]`

where these rules follow from Critical Pair Lemma 19 and Critical Pair Lemma 10 respectively.

Critical Pair Lemma 21

The following expressions are equivalent:

`nand[x1,or[x1,x2]]==not[x1]`

Proof

Note that the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,or[x1_,x2_]]→x1`

where these rules follow from Critical Pair Lemma 6 and Critical Pair Lemma 19 respectively.

Critical Pair Lemma 22

The following expressions are equivalent:

`not[or[x1,or[not[x2],x3]]]==and[not[x1],and[x2,not[x3]]]`

Proof

Note that the input for the rule:

`and[not[x1_],not[x2_]]→not[or[x1,x2]]`

contains a subpattern of the form:

`not[x2_]`

which can be unified with the input for the rule:

`not[or[not[x1_],x2_]]→and[x1,not[x2]]`

where these rules follow from Critical Pair Lemma 18 and Critical Pair Lemma 11 respectively.

Critical Pair Lemma 23

The following expressions are equivalent:

`not[and[x1,x2]]==and[nand[x1,x2],nand[and[x1,x2],x3]]`

Proof

Note that the input for the rule:

`and[not[x1_],nand[x1_,x2_]]→not[x1]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

where these rules follow from Critical Pair Lemma 20 and Critical Pair Lemma 6 respectively.

Substitution Lemma 22

It can be shown that:

`nand[x1,x2]==and[nand[x1,x2],nand[and[x1,x2],x3]]`

Proof

We start by taking Critical Pair Lemma 23, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 6.

Critical Pair Lemma 24

The following expressions are equivalent:

`or[x1,not[x2]]==or[x1,not[or[x1,x2]]]`

Proof

Note that the input for the rule:

`or[x1_,and[not[x1_],x2_]]→or[x1,x2]`

contains a subpattern of the form:

`and[not[x1_],x2_]`

which can be unified with the input for the rule:

`and[not[x1_],not[x2_]]→not[or[x1,x2]]`

where these rules follow from Substitution Lemma 15 and Critical Pair Lemma 18 respectively.

Critical Pair Lemma 25

The following expressions are equivalent:

`and[x1,and[not[not[x1]],x2]]==and[x1,or[not[x1],x2]]`

Proof

Note that the input for the rule:

`and[x1_,or[not[x1_],x2_]]→and[x1,x2]`

contains a subpattern of the form:

`or[not[x1_],x2_]`

which can be unified with the input for the rule:

`or[x1_,and[not[x1_],x2_]]→or[x1,x2]`

where these rules follow from Substitution Lemma 18 and Substitution Lemma 15 respectively.

Substitution Lemma 23

It can be shown that:

It can be shown that:

`and[x1,and[x1,x2]]==and[x1,or[not[x1],x2]]`

PROOF

We start by taking Critical Pair Lemma 25, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 12.

Substitution Lemma 24

It can be shown that:

`and[x1,and[x1,x2]]==and[x1,x2]`

PROOF

We start by taking Substitution Lemma 23, and apply the substitution:

`and[x1_,or[not[x1_],x2_]]→and[x1,x2]`

which follows from Substitution Lemma 18.

Critical Pair Lemma 26

The following expressions are equivalent:

`nand[x1,or[not[x1],x2]]==not[and[x1,x2]]`

PROOF

Note that the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,or[not[x1_],x2_]]→and[x1,x2]`

where these rules follow from Critical Pair Lemma 6 and Substitution Lemma 18 respectively.

Substitution Lemma 25

It can be shown that:

`nand[x1,or[not[x1],x2]]==nand[x1,x2]`

PROOF

We start by taking Critical Pair Lemma 26, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 6.

Critical Pair Lemma 27

The following expressions are equivalent:

`nand[x1,and[x1,x2]]==not[and[x1,x2]]`

PROOF

Note that the input for the rule:

`not[and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,and[x1_,x2_]]→and[x1,x2]`

where these rules follow from Critical Pair Lemma 6 and Substitution Lemma 24 respectively.

Substitution Lemma 26

It can be shown that:

`nand[x1,and[x1,x2]]==nand[x1,x2]`

Proof

We start by taking Critical Pair Lemma 27, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 6.

Critical Pair Lemma 28

The following expressions are equivalent:

`nand[x1,and[not[x1],x2]]==nand[x1,not[x1]]`

Proof

Note that the input for the rule:

`nand[x1_,or[not[x1_],x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`or[not[x1_],x2_]`

which can be unified with the input for the rule:

`or[x1_,and[x1_,x2_]]→x1`

where these rules follow from Substitution Lemma 25 and Substitution Lemma 17 respectively.

Substitution Lemma 27

It can be shown that:

`nand[x1,and[not[x1],x2]]==or[not[x1],x1]`

Proof

We start by taking Critical Pair Lemma 28, and apply the substitution:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 7.

Critical Pair Lemma 29

The following expressions are equivalent:

`nand[nand[x1,x2],nand[x3,x4]]==or[and[x1,x2],and[x3,x4]]`

Proof

Note that the input for the rule:

`or[not[x1_],and[x2_,x3_]]→nand[x1,nand[x2,x3]]`

contains a subpattern of the form:

`not[x1_]`

not[x1_]

which can be unified with the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

where these rules follow from Critical Pair Lemma 8 and Axiom 6 respectively.

Critical Pair Lemma 30

The following expressions are equivalent:

`nand[nand[x1,x2],and[x3,x4]]⩵or[and[x1,x2],nand[x3,x4]]`

Proof

Note that the input for the rule:

`or[not[x1_],nand[x2_,x3_]]→nand[x1,and[x2,x3]]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

where these rules follow from Critical Pair Lemma 9 and Axiom 6 respectively.

Critical Pair Lemma 31

The following expressions are equivalent:

`nand[x1,and[nand[x2,x3],x4]]⩵or[and[x2,x3],nand[x1,x4]]`

Proof

Note that the input for the rule:

`nand[x1_,and[x2_,x3_]]↔nand[x2_,and[x1_,x3_]]`

contains a subpattern of the form:

`nand[x1_,and[x2_,x3_]]`

which can be unified with the input for the rule:

`nand[nand[x1_,x2_],and[x3_,x4_]]→or[and[x1,x2],nand[x3,x4]]`

where these rules follow from Substitution Lemma 9 and Critical Pair Lemma 30 respectively.

Critical Pair Lemma 32

The following expressions are equivalent:

`nand[x1,and[not[x2],x3]]⩵or[x2,not[and[x1,x3]]]`

Proof

Note that the input for the rule:

`nand[x1_,and[x2_,x3_]]↔nand[x2_,and[x1_,x3_]]`

contains a subpattern of the form:

`nand[x1_,and[x2_,x3_]]`

which can be unified with the input for the rule:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

where these rules follow from Substitution Lemma 9 and Critical Pair Lemma 13 respectively.

Substitution Lemma 28

It can be shown that:

It can be shown that:

`nand[x1,and[not[x2],x3]]==or[x2,nand[x1,x3]]`

PROOF

We start by taking Critical Pair Lemma 32, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 6.

Critical Pair Lemma 33

The following expressions are equivalent:

`nand[x1,and[x2,x1]]==nand[x2,x1]`

PROOF

Note that the input for the rule:

`nand[x1_,and[x2_,x3_]]↔nand[x2_,and[x1_,x3_]]`

contains a subpattern of the form:

`and[x2_,x3_]`

which can be unified with the input for the rule:

`Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[62,or[x1_,x1_]`

where these rules follow from Substitution Lemma 9 and Substitution Lemma 11 respectively.

Critical Pair Lemma 34

The following expressions are equivalent:

`nand[x1,and[x2,or[x1,x3]]]==nand[x2,x1]`

PROOF

Note that the input for the rule:

`nand[x1_,and[x2_,x3_]]↔nand[x2_,and[x1_,x3_]]`

contains a subpattern of the form:

`and[x2_,x3_]`

which can be unified with the input for the rule:

`and[x1_,or[x1_,x2_]]→x1`

where these rules follow from Substitution Lemma 9 and Critical Pair Lemma 19 respectively.

Critical Pair Lemma 35

The following expressions are equivalent:

`or[not[x1],and[x2,x3]]==nand[x1,nand[x2,and[x1,x3]]]`

PROOF

Note that the input for the rule:

`Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[74,nand[x1_,no`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,and[x2_,x3_]]↔nand[x2_,and[x1_,x3_]]`

where these rules follow from Substitution Lemma 13 and Substitution Lemma 9 respectively.

where these rules follow from Substitution Lemma 13 and Substitution Lemma 9 respectively.

Substitution Lemma 29

It can be shown that:

`nand[x1,nand[x2,x3]]==nand[x1,nand[x2,and[x1,x3]]]`

Proof

We start by taking Critical Pair Lemma 35, and apply the substitution:

`or[not[x1_],and[x2_,x3_]]→nand[x1,nand[x2,x3]]`

which follows from Critical Pair Lemma 8.

Critical Pair Lemma 36

The following expressions are equivalent:

`nand[not[x1],x1]==or[not[x1],x1]`

Proof

Note that the input for the rule:

`nand[x1_,and[x2_,x1_]]→nand[x2,x1]`

contains a subpattern of the form:

`nand[x1_,and[x2_,x1_]]`

which can be unified with the input for the rule:

`nand[x1_,and[not[x1_],x2_]]→or[not[x1],x1]`

where these rules follow from Critical Pair Lemma 33 and Substitution Lemma 27 respectively.

Substitution Lemma 30

It can be shown that:

`or[x1,not[x1]]==or[not[x1],x1]`

Proof

We start by taking Critical Pair Lemma 36, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 13.

Critical Pair Lemma 37

The following expressions are equivalent:

`nand[x1,or[x1,x2]]==nand[or[x1,x2],x1]`

Proof

Note that the input for the rule:

`nand[x1_,and[x2_,x1_]]→nand[x2,x1]`

contains a subpattern of the form:

`and[x2_,x1_]`

which can be unified with the input for the rule:

`and[x1_,or[x1_,x2_]]→x1`

where these rules follow from Critical Pair Lemma 33 and Critical Pair Lemma 19 respectively.

Substitution Lemma 31

It can be shown that:

`not[x1]==nand[or[x1,x2],x1]`

Proof

We start by taking Critical Pair Lemma 37, and apply the substitution:

`nand[x1_,or[x1_,x2_]]→not[x1]`

which follows from Critical Pair Lemma 21.

Critical Pair Lemma 38

The following expressions are equivalent:

`nand[not[x1],not[x2]]==nand[not[x2],not[or[x1,x2]]]`

Proof

Note that the input for the rule:

`nand[x1_,and[x2_,x1_]]→nand[x2,x1]`

contains a subpattern of the form:

`and[x2_,x1_]`

which can be unified with the input for the rule:

`and[not[x1_],not[x2_]]→not[or[x1,x2]]`

where these rules follow from Critical Pair Lemma 33 and Critical Pair Lemma 18 respectively.

Substitution Lemma 32

It can be shown that:

`or[x1,not[not[x2]]]==nand[not[x2],not[or[x1,x2]]]`

Proof

We start by taking Critical Pair Lemma 38, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 13.

Substitution Lemma 33

It can be shown that:

`or[x1,x2]==nand[not[x2],not[or[x1,x2]]]`

Proof

We start by taking Substitution Lemma 32, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 12.

Substitution Lemma 34

It can be shown that:

`or[x1,x2]==or[x2,not[not[or[x1,x2]]]]`

Proof

We start by taking Substitution Lemma 33, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 13.

Substitution Lemma 35

It can be shown that:

`or[x1,x2]==or[x2,or[x1,x2]]`

Proof

We start by taking Substitution Lemma 34, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 12.

Critical Pair Lemma 39

The following expressions are equivalent:

`and[x1,and[x2,x1]]==not[nand[x2,x1]]`

Proof

Note that the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,and[x2_,x1_]]→nand[x2,x1]`

where these rules follow from Axiom 6 and Critical Pair Lemma 33 respectively.

Substitution Lemma 36

It can be shown that:

`and[x1,and[x2,x1]]==and[x2,x1]`

Proof

We start by taking Critical Pair Lemma 39, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Critical Pair Lemma 40

The following expressions are equivalent:

`not[x1]==nand[or[x2,x1],x1]`

Proof

Note that the input for the rule:

`nand[or[x1_,x2_],x1_]→not[x1]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,x1_]]→or[x2,x1]`

where these rules follow from Substitution Lemma 31 and Substitution Lemma 35

where these rules follow from Substitution Lemma 31 and Substitution Lemma 35 respectively.

Critical Pair Lemma 41

The following expressions are equivalent:

`not[x1]==nand[x1,or[x2,x1]]`

Proof

Note that the input for the rule:

`nand[x1_,or[x1_,x2_]]→not[x1]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,x1_]]→or[x2,x1]`

where these rules follow from Critical Pair Lemma 21 and Substitution Lemma 35 respectively.

Critical Pair Lemma 42

The following expressions are equivalent:

`x1==and[x1,or[x2,x1]]`

Proof

Note that the input for the rule:

`and[x1_,or[x1_,x2_]]→x1`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,x1_]]→or[x2,x1]`

where these rules follow from Critical Pair Lemma 19 and Substitution Lemma 35 respectively.

Critical Pair Lemma 43

The following expressions are equivalent:

`nand[x1,and[x2,or[x3,x1]]]==nand[x2,x1]`

Proof

Note that the input for the rule:

`nand[x1_,and[x2_,x3_]]↔nand[x2_,and[x1_,x3_]]`

contains a subpattern of the form:

`and[x2_,x3_]`

which can be unified with the input for the rule:

`and[x1_,or[x2_,x1_]]→x1`

where these rules follow from Substitution Lemma 9 and Critical Pair Lemma 42 respectively.

Critical Pair Lemma 44

The following expressions are equivalent:

The following expressions are equivalent:

`not[not[x1]]==nand[nand[x2,x1],not[x1]]`

PROOF

Note that the input for the rule:

`nand[or[x1_,x2_],x2_]→not[x2]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[not[x1_],not[x2_]]→nand[x1,x2]`

where these rules follow from Critical Pair Lemma 40 and Critical Pair Lemma 10 respectively.

Substitution Lemma 37

It can be shown that:

`x1==nand[nand[x2,x1],not[x1]]`

PROOF

We start by taking Critical Pair Lemma 44, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 12.

Substitution Lemma 38

It can be shown that:

`x1==or[not[nand[x2,x1]],x1]`

PROOF

We start by taking Substitution Lemma 37, and apply the substitution:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 7.

Substitution Lemma 39

It can be shown that:

`x1==or[and[x2,x1],x1]`

PROOF

We start by taking Substitution Lemma 38, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Critical Pair Lemma 45

The following expressions are equivalent:

`not[not[x1]]==nand[not[x1],nand[x2,x1]]`

PROOF

Note that the input for the rule:

`nand[x1_,or[x2_,x1_]]→not[x1]`

contains a subpattern of the form:

`or[x2_,x1_]`

which can be unified with the input for the rule:

`or[not[x1_],not[x2_]]→nand[x1,x2]`

where these rules follow from Critical Pair Lemma 41 and Critical Pair Lemma 10 respectively.

Substitution Lemma 40

It can be shown that:

`x1==nand[not[x1],nand[x2,x1]]`

Proof

We start by taking Critical Pair Lemma 45, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 12.

Substitution Lemma 41

It can be shown that:

`x1==or[x1,not[nand[x2,x1]]]`

Proof

We start by taking Substitution Lemma 40, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 13.

Substitution Lemma 42

It can be shown that:

`x1==or[x1,and[x2,x1]]`

Proof

We start by taking Substitution Lemma 41, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Critical Pair Lemma 46

The following expressions are equivalent:

`nand[x1,and[x2,not[x1]]]==nand[x1,not[x1]]`

Proof

Note that the input for the rule:

`nand[x1_,or[not[x1_],x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`or[not[x1_],x2_]`

which can be unified with the input for the rule:

`or[x1_,and[x2_,x1_]]→x1`

where these rules follow from Substitution Lemma 25 and Substitution Lemma 42 respectively.

Substitution Lemma 43

It can be shown that:

`nand[x1,and[x2,not[x1]]]==or[not[x1],x1]`

Proof

We start by taking Critical Pair Lemma 46, and apply the substitution:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 7.

Substitution Lemma 44

It can be shown that:

`nand[x1,and[x2,not[x1]]]==or[x1,not[x1]]`

Proof

We start by taking Substitution Lemma 43, and apply the substitution:

`or[not[x1_],x1_]→or[x1,not[x1]]`

which follows from Substitution Lemma 30.

Critical Pair Lemma 47

The following expressions are equivalent:

`or[x1,not[x1]]==nand[x2,and[x1,not[x1]]]`

Proof

Note that the input for the rule:

`nand[x1_,and[x2_,not[x1_]]]→or[x1,not[x1]]`

contains a subpattern of the form:

`nand[x1_,and[x2_,not[x1_]]]`

which can be unified with the input for the rule:

`nand[x1_,and[x2_,x3_]]↔nand[x2_,and[x1_,x3_]]`

where these rules follow from Substitution Lemma 44 and Substitution Lemma 9 respectively.

Critical Pair Lemma 48

The following expressions are equivalent:

`or[x1,not[x1]]==or[x2,not[and[x1,not[x1]]]]`

Proof

Note that the input for the rule:

`nand[x1_,and[x2_,not[x2_]]]→or[x2,not[x2]]`

contains a subpattern of the form:

`nand[x1_,and[x2_,not[x2_]]]`

which can be unified with the input for the rule:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

where these rules follow from Critical Pair Lemma 47 and Critical Pair Lemma 13 respectively.

Substitution Lemma 45

It can be shown that:

It can be shown that:

`or[x1,not[x1]]==or[x2,nand[x1,not[x1]]]`

Proof

We start by taking Critical Pair Lemma 48, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 6.

Substitution Lemma 46

It can be shown that:

`or[x1,not[x1]]==or[x2,or[not[x1],x1]]`

Proof

We start by taking Substitution Lemma 45, and apply the substitution:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 7.

Substitution Lemma 47

It can be shown that:

`or[x1,not[x1]]==or[x2,or[x1,not[x1]]]`

Proof

We start by taking Substitution Lemma 46, and apply the substitution:

`or[not[x1_],x1_]→or[x1,not[x1]]`

which follows from Substitution Lemma 30.

Critical Pair Lemma 49

The following expressions are equivalent:

`x1==and[x1,or[x2,not[x2]]]`

Proof

Note that the input for the rule:

`and[x1_,or[x1_,x2_]]→x1`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,not[x2_]]]→or[x2,not[x2]]`

where these rules follow from Critical Pair Lemma 19 and Substitution Lemma 47 respectively.

Critical Pair Lemma 50

The following expressions are equivalent:

`nand[x1,and[x2,or[x3,not[x3]]]]==nand[x2,x1]`

Proof

Note that the input for the rule:

`nand[x1_,and[x2_,x3_]]↔nand[x2_,and[x1_,x3_]]`

contains a subpattern of the form:

`and[x2_,x3_]`

which can be unified with the input for the rule:

`and[x1_,or[x2_,not[x2_]]]→x1`

where these rules follow from Substitution Lemma 9 and Critical Pair Lemma 49 respectively.

Substitution Lemma 48

It can be shown that:

`nand[x1,x2]==nand[x2,x1]`

Proof

We start by taking Critical Pair Lemma 50, and apply the substitution:

`and[x1_,or[x2_,not[x2_]]]→x1`

which follows from Critical Pair Lemma 49.

Critical Pair Lemma 51

The following expressions are equivalent:

`nand[and[x1,x2],x3]==nand[x1,and[x3,x2]]`

Proof

Note that the input for the rule:

`nand[x1_,x2_]↔nand[x2_,x1_]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,and[x2_,x3_]]↔nand[x2_,and[x1_,x3_]]`

where these rules follow from Substitution Lemma 48 and Substitution Lemma 9 respectively.

Critical Pair Lemma 52

The following expressions are equivalent:

`or[not[x1],x2]==nand[x1,nand[x2,x1]]`

Proof

Note that the input for the rule:

`` Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[74,nand[x1_,no ``

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,x2_]↔nand[x2_,x1_]`

where these rules follow from Substitution Lemma 13 and Substitution Lemma 48 respectively.

Critical Pair Lemma 53

The following expressions are equivalent:

`and[x1,not[x2]]==and[x1,nand[x2,x1]]`

Proof

Note that the input for the rule:

`and[x1_,nand[x1_,x2_]]→and[x1,not[x2]]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,x2_]↔nand[x2_,x1_]`

where these rules follow from Substitution Lemma 7 and Substitution Lemma 48 respectively.

Critical Pair Lemma 54

The following expressions are equivalent:

`and[x1,x2]==not[nand[x2,x1]]`

Proof

Note that the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,x2_]↔nand[x2_,x1_]`

where these rules follow from Axiom 6 and Substitution Lemma 48 respectively.

Substitution Lemma 49

It can be shown that:

`and[x1,x2]==and[x2,x1]`

Proof

We start by taking Critical Pair Lemma 54, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Critical Pair Lemma 55

The following expressions are equivalent:

`or[x1,x2]==or[x1,and[x2,not[x1]]]`

Proof

Note that the input for the rule:

`or[x1_,and[not[x1_],x2_]]→or[x1,x2]`

contains a subpattern of the form:

`and[not[x1_],x2_]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Substitution Lemma 15 and Substitution Lemma 49 respectively.

Critical Pair Lemma 56

The following expressions are equivalent:

`or[not[nand[x1,x2]],x2]==nand[nand[x1,x2],or[not[x2],x1]]`

Proof

Note that the input for the rule:

`nand[x1_,nand[x2_,x1_]]→or[not[x1],x2]`

contains a subpattern of the form:

`nand[x2_,x1_]`

which can be unified with the input for the rule:

`nand[x1_,nand[x2_,x1_]]→or[not[x1],x2]`

where these rules follow from Critical Pair Lemma 52 and Critical Pair Lemma 52 respectively.

Substitution Lemma 50

It can be shown that:

`or[and[x1,x2],x2]==nand[nand[x1,x2],or[not[x2],x1]]`

Proof

We start by taking Critical Pair Lemma 56, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Substitution Lemma 51

It can be shown that:

`x1==nand[nand[x2,x1],or[not[x1],x2]]`

Proof

We start by taking Substitution Lemma 50, and apply the substitution:

`or[and[x1_,x2_],x2_]→x2`

which follows from Substitution Lemma 39.

Critical Pair Lemma 57

The following expressions are equivalent:

`and[x1,not[not[x2]]]==and[x1,or[x2,not[x1]]]`

Proof

Note that the input for the rule:

`and[x1_,nand[x2_,x1_]]→and[x1,not[x2]]`

contains a subpattern of the form:

`nand[x2_,x1_]`

which can be unified with the input for the rule:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

where these rules follow from Critical Pair Lemma 53 and Critical Pair Lemma 13 respectively.

Substitution Lemma 52

It can be shown that:

`and[x1,x2]==and[x1,or[x2,not[x1]]]`

PROOF

We start by taking Critical Pair Lemma 57, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 12.

Critical Pair Lemma 58

The following expressions are equivalent:

`nand[not[x1],and[x2,not[x3]]]==nand[not[or[x1,x3]],x2]`

PROOF

Note that the input for the rule:

`nand[and[x1_,x2_],x3_]→nand[x1,and[x3,x2]]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[not[x1_],not[x2_]]→not[or[x1,x2]]`

where these rules follow from Critical Pair Lemma 51 and Critical Pair Lemma 18 respectively.

Substitution Lemma 53

It can be shown that:

`or[x1,not[and[x2,not[x3]]]]==nand[not[or[x1,x3]],x2]`

PROOF

We start by taking Critical Pair Lemma 58, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 13.

Substitution Lemma 54

It can be shown that:

`or[x1,nand[x2,not[x3]]]==nand[not[or[x1,x3]],x2]`

PROOF

We start by taking Substitution Lemma 53, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 6.

Substitution Lemma 55

It can be shown that:

`or[x1,or[not[x2],x3]]==nand[not[or[x1,x3]],x2]`

PROOF

We start by taking Substitution Lemma 54, and apply the substitution:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 7.

which follows from Critical Pair Lemma 7.

Substitution Lemma 56

It can be shown that:

`or[x1,or[not[x2],x3]]==or[or[x1,x3],not[x2]]`

Proof

We start by taking Substitution Lemma 55, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 13.

Critical Pair Lemma 59

The following expressions are equivalent:

`and[and[x1,x2],x3]==not[nand[x1,and[x3,x2]]]`

Proof

Note that the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[and[x1_,x2_],x3_]→nand[x1,and[x3,x2]]`

where these rules follow from Axiom 6 and Critical Pair Lemma 51 respectively.

Substitution Lemma 57

It can be shown that:

`and[and[x1,x2],x3]==and[x1,and[x3,x2]]`

Proof

We start by taking Critical Pair Lemma 59, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Critical Pair Lemma 60

The following expressions are equivalent:

`and[x1,and[x2,or[x3,x1]]]==and[x1,x2]`

Proof

Note that the input for the rule:

`and[and[x1_,x2_],x3_]→and[x1,and[x3,x2]]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,or[x2_,x1_]]→x1`

where these rules follow from Substitution Lemma 57 and Critical Pair Lemma 42 respectively.

Critical Pair Lemma 61

The following expressions are equivalent:

`and[x1,and[x2,or[x1,x3]]]==and[x1,x2]`

Proof

Note that the input for the rule:

`and[and[x1_,x2_],x3_]→and[x1,and[x3,x2]]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,or[x1_,x2_]]→x1`

where these rules follow from Substitution Lemma 57 and Critical Pair Lemma 19 respectively.

Critical Pair Lemma 62

The following expressions are equivalent:

`and[not[x1],and[x2,not[x3]]]==and[not[or[x1,x3]],x2]`

Proof

Note that the input for the rule:

`and[and[x1_,x2_],x3_]→and[x1,and[x3,x2]]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

Out[◦]=

`and[not[x1_],not[x2_]]→not[or[x1,x2]]`

where these rules follow from Substitution Lemma 57 and Critical Pair Lemma 18 respectively.

Critical Pair Lemma 63

The following expressions are equivalent:

`x1==or[x1,and[x2,and[x1,x3]]]`

Proof

Note that the input for the rule:

`or[x1_,and[x2_,x1_]]→x1`

contains a subpattern of the form:

`and[x2_,x1_]`

which can be unified with the input for the rule:

`and[and[x1_,x2_],x3_]→and[x1,and[x3,x2]]`

where these rules follow from Substitution Lemma 42 and Substitution Lemma 57 respectively.

Critical Pair Lemma 64

The following expressions are equivalent:

`or[x1,not[x2]]==or[x1,not[or[x2,x1]]]`

Proof

Note that the input for the rule:

`or[x1_,and[x2_,not[x1_]]]→or[x1,x2]`

contains a subpattern of the form:

`and[x2_,not[x1_]]`

which can be unified with the input for the rule:

`and[not[x1_],not[x2_]]→not[or[x1,x2]]`

where these rules follow from Critical Pair Lemma 55 and Critical Pair Lemma 18 respectively.

Critical Pair Lemma 65

The following expressions are equivalent:

`and[not[x1],x2]==and[not[x1],or[x2,x1]]`

Proof

Note that the input for the rule:

`and[x1_,or[x2_,not[x1_]]]→and[x1,x2]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Substitution Lemma 52 and Substitution Lemma 12 respectively.

Critical Pair Lemma 66

The following expressions are equivalent:

`or[x1,or[x2,not[not[x1]]]]==or[x1,and[not[x1],x2]]`

Proof

Note that the input for the rule:

`or[x1_,and[not[x1_],x2_]]→or[x1,x2]`

contains a subpattern of the form:

`and[not[x1_],x2_]`

which can be unified with the input for the rule:

`and[x1_,or[x2_,not[x1_]]]→and[x1,x2]`

where these rules follow from Substitution Lemma 15 and Substitution Lemma 52 respectively.

Substitution Lemma 58

It can be shown that:

`or[x1,or[x2,x1]]==or[x1,and[not[x1],x2]]`

Proof

We start by taking Critical Pair Lemma 66, and apply the substitution:

`not[not[x1_]]→x1`

`not[not[x1_]]→x1`

which follows from Substitution Lemma 12.

Substitution Lemma 59

It can be shown that:

`or[x1,x2]==or[x2,and[not[x2],x1]]`

Proof

We start by taking Substitution Lemma 58, and apply the substitution:

`or[x1_,or[x2_,x1_]]→or[x2,x1]`

which follows from Substitution Lemma 35.

Substitution Lemma 60

It can be shown that:

`or[x1,x2]==or[x2,x1]`

Proof

We start by taking Substitution Lemma 59, and apply the substitution:

`or[x1_,and[not[x1_],x2_]]→or[x1,x2]`

which follows from Substitution Lemma 15.

Critical Pair Lemma 67

The following expressions are equivalent:

`x1==or[x1,and[x2,and[x3,x1]]]`

Proof

Note that the input for the rule:

`or[x1_,and[x2_,and[x1_,x3_]]]→x1`

contains a subpattern of the form:

`and[x1_,x3_]`

which can be unified with the input for the rule:

`and[x1_,and[x2_,x1_]]→and[x2,x1]`

where these rules follow from Critical Pair Lemma 63 and Substitution Lemma 36 respectively.

Critical Pair Lemma 68

The following expressions are equivalent:

`or[x1,x2]==or[or[x1,x2],and[x3,x2]]`

Proof

Note that the input for the rule:

`or[x1_,and[x2_,and[x3_,x1_]]]→x1`

contains a subpattern of the form:

`and[x3_,x1_]`

which can be unified with the input for the rule:

`and[x1_,or[x2_,x1_]]→x1`

where these rules follow from Critical Pair Lemma 67 and Critical Pair Lemma 42 respectively.

Critical Pair Lemma 69

The following expressions are equivalent:

`nand[not[x1],and[x2,or[x3,x1]]]==nand[and[not[x1],x3],x2]`

Proof

Note that the input for the rule:

`nand[and[x1_,x2_],x3_]→nand[x1,and[x3,x2]]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[not[x1_],or[x2_,x1_]]→and[not[x1],x2]`

where these rules follow from Critical Pair Lemma 51 and Critical Pair Lemma 65 respectively.

Substitution Lemma 61

It can be shown that:

`or[x1,not[and[x2,or[x3,x1]]]]==nand[and[not[x1],x3],x2]`

Proof

We start by taking Critical Pair Lemma 69, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 13.

Substitution Lemma 62

It can be shown that:

`or[x1,nand[x2,or[x3,x1]]]==nand[and[not[x1],x3],x2]`

Proof

We start by taking Substitution Lemma 61, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 6.

Substitution Lemma 63

It can be shown that:

`or[x1,nand[x2,or[x3,x1]]]==nand[not[x1],and[x2,x3]]`

Proof

We start by taking Substitution Lemma 62, and apply the substitution:

`nand[and[x1_,x2_],x3_]→nand[x1,and[x3,x2]]`

which follows from Critical Pair Lemma 51.

Substitution Lemma 64

It can be shown that:

`or[x1,nand[x2,or[x3,x1]]]==or[x1,not[and[x2,x3]]]`

Proof

We start by taking Substitution Lemma 63, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 13.

Substitution Lemma 65

It can be shown that:

`or[x1,nand[x2,or[x3,x1]]]==or[x1,nand[x2,x3]]`

Proof

We start by taking Substitution Lemma 64, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 6.

Critical Pair Lemma 70

The following expressions are equivalent:

`and[x1,not[and[not[x2],x3]]]==and[x1,or[x2,nand[x1,x3]]]`

Proof

Note that the input for the rule:

`and[x1_,nand[x1_,x2_]]→and[x1,not[x2]]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,and[not[x2_],x3_]]→or[x2,nand[x1,x3]]`

where these rules follow from Substitution Lemma 7 and Substitution Lemma 28 respectively.

Substitution Lemma 66

It can be shown that:

`and[x1,nand[not[x2],x3]]==and[x1,or[x2,nand[x1,x3]]]`

Proof

We start by taking Critical Pair Lemma 70, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 6.

Substitution Lemma 67

It can be shown that:

`and[x1,or[x2,not[x3]]]==and[x1,or[x2,nand[x1,x3]]]`

Proof

We start by taking Substitution Lemma 66, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 13.

Critical Pair Lemma 71

The following expressions are equivalent:

```
nand[x1,x2]==nand[x2,and[or[x2,x3],x1]]
```

PROOF

Note that the input for the rule:

```
nand[x1_,and[x2_,or[x1_,x3_]]]→nand[x2,x1]
```

contains a subpattern of the form:

```
and[x2_,or[x1_,x3_]]
```

which can be unified with the input for the rule:

```
and[x1_,x2_]↔and[x2_,x1_]
```

where these rules follow from Critical Pair Lemma 34 and Substitution Lemma 49 respectively.

Critical Pair Lemma 72

The following expressions are equivalent:

```
nand[x1,x2]==nand[x2,and[or[x3,x2],x1]]
```

PROOF

Note that the input for the rule:

```
nand[x1_,and[x2_,or[x3_,x1_]]]→nand[x2,x1]
```

contains a subpattern of the form:

```
and[x2_,or[x3_,x1_]]
```

which can be unified with the input for the rule:

```
and[x1_,x2_]↔and[x2_,x1_]
```

where these rules follow from Critical Pair Lemma 43 and Substitution Lemma 49 respectively.

Critical Pair Lemma 73

The following expressions are equivalent:

```
and[x1,or[x2,x3]]==or[and[x1,or[x2,x3]],and[x3,x1]]
```

PROOF

Note that the input for the rule:

```
or[x1_,and[x2_,x1_]]→x1
```

contains a subpattern of the form:

```
and[x2_,x1_]
```

which can be unified with the input for the rule:

```
and[x1_,and[x2_,or[x3_,x1_]]]→and[x1,x2]
```

where these rules follow from Substitution Lemma 42 and Critical Pair Lemma 60 respectively.

Critical Pair Lemma 74

The following expressions are equivalent:

```
or[x1,x2]==or[or[x1,x2],and[x1,x3]]
```

PROOF

Note that the input for the rule:

`or[x1_,and[x2_,and[x3_,x1_]]]→x1`

contains a subpattern of the form:

`and[x2_,and[x3_,x1_]]`

which can be unified with the input for the rule:

`and[x1_,and[x2_,or[x1_,x3_]]]→and[x1,x2]`

where these rules follow from Critical Pair Lemma 67 and Critical Pair Lemma 61 respectively.

Critical Pair Lemma 75

The following expressions are equivalent:

`or[x1,x2]==or[and[x3,x2],or[x1,x2]]`

Proof

Note that the input for the rule:

`or[or[x1_,x2_],and[x3_,x2_]]→or[x1,x2]`

contains a subpattern of the form:

`or[or[x1_,x2_],and[x3_,x2_]]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Critical Pair Lemma 68 and Substitution Lemma 60 respectively.

Critical Pair Lemma 76

The following expressions are equivalent:

`nand[or[x1,or[x2,x3]],x2]==nand[x2,or[x2,x3]]`

Proof

Note that the input for the rule:

`nand[x1_,and[or[x1_,x2_],x3_]]→nand[x3,x1]`

contains a subpattern of the form:

`and[or[x1_,x2_],x3_]`

which can be unified with the input for the rule:

`and[x1_,or[x2_,x1_]]→x1`

where these rules follow from Critical Pair Lemma 71 and Critical Pair Lemma 42 respectively.

Substitution Lemma 68

It can be shown that:

`nand[or[x1,or[x2,x3]],x2]==not[x2]`

Proof

We start by taking Critical Pair Lemma 76, and apply the substitution:

`nand[x1_,or[x1_,x2_]]→not[x1]`

which follows from Critical Pair Lemma 21.

Substitution Lemma 69

It can be shown that:

`nand[x1,or[x2,or[x1,x3]]]==not[x1]`

PROOF

We start by taking Substitution Lemma 68, and apply the substitution:

`nand[x1_,x2_]→nand[x2,x1]`

which follows from Substitution Lemma 48.

Critical Pair Lemma 77

The following expressions are equivalent:

`not[x1]==nand[x1,or[x2,or[x3,x1]]]`

PROOF

Note that the input for the rule:

`nand[x1_,or[x2_,or[x1_,x3_]]]→not[x1]`

contains a subpattern of the form:

`or[x1_,x3_]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,x1_]]→or[x2,x1]`

where these rules follow from Substitution Lemma 69 and Substitution Lemma 35 respectively.

Critical Pair Lemma 78

The following expressions are equivalent:

`nand[or[x1,not[or[x2,x3]]],x3]==nand[x3,and[or[x2,x3],x1]]`

PROOF

Note that the input for the rule:

`nand[x1_,and[or[x2_,x1_],x3_]]→nand[x3,x1]`

contains a subpattern of the form:

`and[or[x2_,x1_],x3_]`

which can be unified with the input for the rule:

`and[x1_,or[x2_,not[x1_]]]→and[x1,x2]`

where these rules follow from Critical Pair Lemma 72 and Substitution Lemma 52 respectively.

Substitution Lemma 70

It can be shown that:

`nand[or[x1,not[or[x2,x3]]],x3]==nand[x1,x3]`

PROOF

We start by taking Critical Pair Lemma 78, and apply the substitution:

`nand[x1_,and[or[x2_,x1_],x3_]]→nand[x3,x1]`

which follows from Critical Pair Lemma 72.

Critical Pair Lemma 79

The following expressions are equivalent:

`or[x1,or[x2,x3]]==or[x3,or[x1,or[x2,x3]]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],or[x3_,x2_]]→or[x3,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,or[x2_,x1_]]→x1`

where these rules follow from Critical Pair Lemma 75 and Critical Pair Lemma 42 respectively.

Critical Pair Lemma 80

The following expressions are equivalent:

`x1==nand[nand[x2,x1],or[x2,not[x1]]]`

Proof

Note that the input for the rule:

`nand[nand[x1_,x2_],or[not[x2_],x1_]]→x2`

contains a subpattern of the form:

`or[not[x2_],x1_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Substitution Lemma 51 and Substitution Lemma 60 respectively.

Critical Pair Lemma 81

The following expressions are equivalent:

`not[x1]==nand[or[not[x2],x1],or[x2,not[not[x1]]]]`

Proof

Note that the input for the rule:

`nand[nand[x1_,x2_],or[x1_,not[x2_]]]→x2`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

where these rules follow from Critical Pair Lemma 80 and Critical Pair Lemma 7 respectively.

Substitution Lemma 71

It can be shown that:

`not[x1]==nand[or[not[x2],x1],or[x2,x1]]`

Proof

We start by taking Critical Pair Lemma 81, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 12.

Critical Pair Lemma 82

The following expressions are equivalent:

`or[x1,or[not[not[x2]],x3]]==or[or[x1,x3],x2]`

Proof

Note that the input for the rule:

`or[or[x1_,x2_],not[x3_]]→or[x1,or[not[x3],x2]]`

contains a subpattern of the form:

`not[x3_]`

which can be unified with the input for the rule:

`not[not[x1_]]→x1`

where these rules follow from Substitution Lemma 56 and Substitution Lemma 12 respectively.

Substitution Lemma 72

It can be shown that:

`or[x1,or[x2,x3]]==or[or[x1,x3],x2]`

Proof

We start by taking Critical Pair Lemma 82, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 12.

Critical Pair Lemma 83

The following expressions are equivalent:

`or[x1,or[x2,and[x3,x1]]]==or[x1,x2]`

Proof

Note that the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x3,x2]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,and[x2_,x1_]]→x1`

where these rules follow from Substitution Lemma 72 and Substitution Lemma 42 respectively.

Critical Pair Lemma 84

The following expressions are equivalent:

`or[x1,not[or[x2,x3]]]==or[x1,not[or[x2,or[x1,x3]]]]`

Proof

Note that the input for the rule:

Note that the input for the rule:

`or[x1_,not[or[x2_,x1_]]]→or[x1,not[x2]]`

contains a subpattern of the form:

`or[x2_,x1_]`

which can be unified with the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x3,x2]]`

where these rules follow from Critical Pair Lemma 64 and Substitution Lemma 72 respectively.

Substitution Lemma 73

It can be shown that:

`or[x1,or[and[x1,x2],x3]]==or[x1,x3]`

Proof

We start by taking Critical Pair Lemma 74, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x3,x2]]`

which follows from Substitution Lemma 72.

Critical Pair Lemma 85

The following expressions are equivalent:

`or[or[x1,x2],x3]==or[or[x1,x2],or[x3,x1]]`

Proof

Note that the input for the rule:

`or[x1_,or[x2_,and[x3_,x1_]]]→or[x1,x2]`

contains a subpattern of the form:

`and[x3_,x1_]`

which can be unified with the input for the rule:

`and[x1_,or[x1_,x2_]]→x1`

where these rules follow from Critical Pair Lemma 83 and Critical Pair Lemma 19 respectively.

Substitution Lemma 74

It can be shown that:

`or[x1,or[x2,x3]]==or[or[x1,x3],or[x2,x1]]`

Proof

We start by taking Critical Pair Lemma 85, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x3,x2]]`

which follows from Substitution Lemma 72.

Substitution Lemma 75

It can be shown that:

`or[x1,or[x2,x3]]==or[x1,or[or[x2,x1],x3]]`

Proof

We start by taking Substitution Lemma 74, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x3,x2]]`

which follows from Substitution Lemma 72.

Substitution Lemma 76

It can be shown that:

`or[x1,or[x2,x3]]==or[x1,or[x2,or[x3,x1]]]`

Proof

We start by taking Substitution Lemma 75, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x3,x2]]`

which follows from Substitution Lemma 72.

Critical Pair Lemma 86

The following expressions are equivalent:

`or[x1,and[not[and[x1,x2]],x3]]==or[x1,or[and[x1,x2],x3]]`

Proof

Note that the input for the rule:

`or[x1_,or[and[x1_,x2_],x3_]]→or[x1,x3]`

contains a subpattern of the form:

`or[and[x1_,x2_],x3_]`

which can be unified with the input for the rule:

`or[x1_,and[not[x1_],x2_]]→or[x1,x2]`

where these rules follow from Substitution Lemma 73 and Substitution Lemma 15 respectively.

Substitution Lemma 77

It can be shown that:

`or[x1,and[nand[x1,x2],x3]]==or[x1,or[and[x1,x2],x3]]`

Proof

We start by taking Critical Pair Lemma 86, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 6.

Substitution Lemma 78

It can be shown that:

`or[x1,and[nand[x1,x2],x3]]==or[x1,x3]`

Proof

We start by taking Substitution Lemma 77, and apply the substitution:

`or[x1_,or[and[x1_,x2_],x3_]]→or[x1,x3]`

which follows from Substitution Lemma 73.

Critical Pair Lemma 87

The following expressions are equivalent:

`or[nand[x1,x2],x3]==or[nand[x1,x2],and[x2,x3]]` or[nand[x1,x2],x3]==or[nand[x1,x2],and[x2,x3]]

Proof

Note that the input for the rule:

`or[x1_,and[nand[x1_,x2_],x3_]]→or[x1,x3]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[nand[x1_,x2_],or[x1_,not[x2_]]]→x2`

where these rules follow from Substitution Lemma 78 and Critical Pair Lemma 80 respectively.

Substitution Lemma 79

It can be shown that:

`nand[x1,x2]==and[nand[x1,x2],nand[x1,and[x3,x2]]]`

Proof

We start by taking Substitution Lemma 22, and apply the substitution:

`nand[and[x1_,x2_],x3_]→nand[x1,and[x3,x2]]`

which follows from Critical Pair Lemma 51.

Critical Pair Lemma 88

The following expressions are equivalent:

`nand[x1,x2]==and[nand[x1,x2],nand[x2,x1]]`

Proof

Note that the input for the rule:

`and[nand[x1_,x2_],nand[x1_,and[x3_,x2_]]]→nand[x1,x2]`

contains a subpattern of the form:

`nand[x1_,and[x3_,x2_]]`

which can be unified with the input for the rule:

`nand[x1_,and[or[x1_,x2_],x3_]]→nand[x3,x1]`

where these rules follow from Substitution Lemma 79 and Critical Pair Lemma 71 respectively.

Critical Pair Lemma 89

The following expressions are equivalent:

`nand[nand[x1,x2],nand[x2,x1]]==nand[nand[x1,x2],nand[x1,x2]]`

Proof

Note that the input for the rule:

`nand[x1_,and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[nand[x1_,x2_],nand[x2_,x1_]]→nand[x1,x2]`

where these rules follow from Substitution Lemma 26 and Critical Pair Lemma 88 respectively.

Substitution Lemma 80

It can be shown that:

```
or[and[x1,x2],and[x2,x1]]==nand[nand[x1,x2],nand[x1,x2]]
```

Proof

We start by taking Critical Pair Lemma 89, and apply the substitution:

```
nand[nand[x1_,x2_],nand[x3_,x4_]]→or[and[x1,x2],and[x3,x4]]
```

which follows from Critical Pair Lemma 29.

Substitution Lemma 81

It can be shown that:

```
or[and[x1,x2],and[x2,x1]]==or[and[x1,x2],and[x1,x2]]
```

Proof

We start by taking Substitution Lemma 80, and apply the substitution:

```
nand[nand[x1_,x2_],nand[x3_,x4_]]→or[and[x1,x2],and[x3,x4]]
```

which follows from Critical Pair Lemma 29.

Substitution Lemma 82

It can be shown that:

```
or[and[x1,x2],and[x2,x1]]==and[x1,x2]
```

Proof

We start by taking Substitution Lemma 81, and apply the substitution:

```
or[x1_,x1_]→x1
```

which follows from Substitution Lemma 10.

Substitution Lemma 83

It can be shown that:

```
not[x1]==nand[or[x2,x1],or[not[x2],x1]]
```

Proof

We start by taking Substitution Lemma 71, and apply the substitution:

```
nand[x1_,x2_]→nand[x2,x1]
```

which follows from Substitution Lemma 48.

Critical Pair Lemma 90

The following expressions are equivalent:

```
nand[x1,nand[x2,nand[x1,x3]]]==nand[x1,nand[x2,and[x1,not[x3]]]]
```

Proof

Note that the input for the rule:

```
nand[x1_,nand[x2_,and[x1_,x3_]]]→nand[x1,nand[x2,x3]]
```

contains a subpattern of the form:

contains a subpattern of the form:

`and[x1_,x3_]`

which can be unified with the input for the rule:

`and[x1_,nand[x1_,x2_]]→and[x1,not[x2]]`

where these rules follow from Substitution Lemma 29 and Substitution Lemma 7 respectively.

Substitution Lemma 84

It can be shown that:

`nand[x1,nand[x2,nand[x1,x3]]]==nand[x1,nand[x2,not[x3]]]`

Proof

We start by taking Critical Pair Lemma 90, and apply the substitution:

`nand[x1_,nand[x2_,and[x1_,x3_]]]→nand[x1,nand[x2,x3]]`

which follows from Substitution Lemma 29.

Substitution Lemma 85

It can be shown that:

`nand[x1,nand[x2,nand[x1,x3]]]==nand[x1,or[not[x2],x3]]`

Proof

We start by taking Substitution Lemma 84, and apply the substitution:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 7.

Critical Pair Lemma 91

The following expressions are equivalent:

`or[x1,nand[x2,x3]]==or[x1,nand[or[x3,x1],x2]]`

Proof

Note that the input for the rule:

`or[x1_,nand[x2_,or[x3_,x1_]]]→or[x1,nand[x2,x3]]`

contains a subpattern of the form:

`nand[x2_,or[x3_,x1_]]`

which can be unified with the input for the rule:

`nand[x1_,x2_]↔nand[x2_,x1_]`

where these rules follow from Substitution Lemma 65 and Substitution Lemma 48 respectively.

Substitution Lemma 86

It can be shown that:

`or[x1,or[x2,x3]]==or[x2,or[x3,x1]]`

Proof

We start by taking Substitution Lemma 76, and apply the substitution:

`or[x1_,or[x2_,or[x3_,x1_]]]→or[x2,or[x3,x1]]`

which follows from Critical Pair Lemma 79.

Critical Pair Lemma 92

The following expressions are equivalent:

`and[not[x1],and[x2,not[x3]]]==not[or[not[x2],or[x3,x1]]]`

Proof

Note that the input for the rule:

`not[or[x1_,or[not[x2_],x3_]]]→and[not[x1],and[x2,not[x3]]]`

contains a subpattern of the form:

`or[x1_,or[not[x2_],x3_]]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,x3_]]↔or[x2_,or[x3_,x1_]]`

where these rules follow from Critical Pair Lemma 22 and Substitution Lemma 86 respectively.

Substitution Lemma 87

It can be shown that:

`and[not[x1],and[x2,not[x3]]]==and[x2,not[or[x3,x1]]]`

Proof

We start by taking Critical Pair Lemma 92, and apply the substitution:

`not[or[not[x1_],x2_]]→and[x1,not[x2]]`

which follows from Critical Pair Lemma 11.

Critical Pair Lemma 93

The following expressions are equivalent:

`not[or[x1,x2]]==nand[or[x1,x2],or[x2,or[x3,x1]]]`

Proof

Note that the input for the rule:

`nand[x1_,or[x2_,x1_]]→not[x1]`

contains a subpattern of the form:

`or[x2_,x1_]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,x3_]]↔or[x2_,or[x3_,x1_]]`

where these rules follow from Critical Pair Lemma 41 and Substitution Lemma 86 respectively.

Critical Pair Lemma 94

The following expressions are equivalent:

`or[x1,not[nand[or[x2,x1],x3]]]==or[x1,not[or[x1,nand[x3,x2]]]]`

Proof

Note that the input for the rule:

`or[x1_,not[or[x1_,x2_]]]→or[x1,not[x2]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,nand[or[x2_,x1_],x3_]]→or[x1,nand[x3,x2]]`

where these rules follow from Critical Pair Lemma 24 and Critical Pair Lemma 91 respectively.

Substitution Lemma 88

It can be shown that:

`or[x1,and[or[x2,x1],x3]]==or[x1,not[or[x1,nand[x3,x2]]]]`

Proof

We start by taking Critical Pair Lemma 94, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Substitution Lemma 89

It can be shown that:

`or[x1,and[or[x2,x1],x3]]==or[x1,not[nand[x3,x2]]]`

Proof

We start by taking Substitution Lemma 88, and apply the substitution:

`or[x1_,not[or[x1_,x2_]]]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 24.

Substitution Lemma 90

It can be shown that:

`or[x1,and[or[x2,x1],x3]]==or[x1,and[x3,x2]]`

Proof

We start by taking Substitution Lemma 89, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Substitution Lemma 91

It can be shown that:

`nand[x1,or[x2,not[or[x3,x1]]]]==nand[x2,x1]`

Proof

We start by taking Substitution Lemma 70, and apply the substitution:

`nand[x1_,x2_]→nand[x2,x1]`

which follows from Substitution Lemma 48.

Critical Pair Lemma 95

The following expressions are equivalent:

`and[x1,or[x2,not[x3]]]==and[x1,or[nand[x1,x3],x2]]`

Proof

Note that the input for the rule:

Note that the input for the rule:

`and[x1_,or[x2_,nand[x1_,x3_]]]→and[x1,or[x2,not[x3]]]`

contains a subpattern of the form:

`or[x2_,nand[x1_,x3_]]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Substitution Lemma 67 and Substitution Lemma 60 respectively.

Critical Pair Lemma 96

The following expressions are equivalent:

`nand[x1,or[not[not[x2]],x3]]==nand[x1,or[x2,not[nand[x1,x3]]]]`

Proof

Note that the input for the rule:

`nand[x1_,nand[x2_,nand[x1_,x3_]]]→nand[x1,or[not[x2],x3]]`

contains a subpattern of the form:

`nand[x2_,nand[x1_,x3_]]`

which can be unified with the input for the rule:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

where these rules follow from Substitution Lemma 85 and Critical Pair Lemma 13 respectively.

Substitution Lemma 92

It can be shown that:

`nand[x1,or[x2,x3]]==nand[x1,or[x2,not[nand[x1,x3]]]]`

Proof

We start by taking Critical Pair Lemma 96, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 12.

Substitution Lemma 93

It can be shown that:

`nand[x1,or[x2,x3]]==nand[x1,or[x2,and[x1,x3]]]`

Proof

We start by taking Substitution Lemma 92, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Critical Pair Lemma 97

The following expressions are equivalent:

`nand[x1,or[x2,x3]]==nand[x1,or[and[x1,x3],x2]]`

Proof

Note that the input for the rule:

`nand[x1_,or[x2_,and[x1_,x3_]]]→nand[x1,or[x2,x3]]`

contains a subpattern of the form:

`or[x2_,and[x1_,x3_]]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Substitution Lemma 93 and Substitution Lemma 60 respectively.

Critical Pair Lemma 98

The following expressions are equivalent:

`and[x1,or[x2,and[x1,x3]]]==not[nand[x1,or[x2,x3]]]`

Proof

Note that the input for the rule:

`not[nand[x1_,x2_]]→and[x1,x2]`

contains a subpattern of the form:

`nand[x1_,x2_]`

which can be unified with the input for the rule:

`nand[x1_,or[x2_,and[x1_,x3_]]]→nand[x1,or[x2,x3]]`

where these rules follow from Axiom 6 and Substitution Lemma 93 respectively.

Substitution Lemma 94

It can be shown that:

`and[x1,or[x2,and[x1,x3]]]==and[x1,or[x2,x3]]`

Proof

We start by taking Critical Pair Lemma 98, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Critical Pair Lemma 99

The following expressions are equivalent:

`nand[x1,or[x2,x3]]==nand[x1,or[and[x3,x1],x2]]`

Proof

Note that the input for the rule:

`nand[x1_,or[and[x1_,x2_],x3_]]→nand[x1,or[x3,x2]]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Critical Pair Lemma 97 and Substitution Lemma 49 respectively.

Substitution Lemma 95

Substitution Lemma 95

It can be shown that:

`and[not[or[x1,x2]],x3]==and[x3,not[or[x2,x1]]]`

Proof

We start by taking Substitution Lemma 87, and apply the substitution:

`and[not[x1_],and[x2_,not[x3_]]]→and[not[or[x1,x3]],x2]`

which follows from Critical Pair Lemma 62.

Critical Pair Lemma 100

The following expressions are equivalent:

`and[x1,not[or[and[x2,x3],and[x3,x2]]]]==and[not[and[x3,x2]],x1]`

Proof

Note that the input for the rule:

`and[not[or[x1_,x2_]],x3_]↔and[x3_,not[or[x2_,x1_]]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[and[x1_,x2_],and[x2_,x1_]]→and[x1,x2]`

where these rules follow from Substitution Lemma 95 and Substitution Lemma 82 respectively.

Substitution Lemma 96

It can be shown that:

`and[x1,and[nand[x2,x3],not[and[x3,x2]]]]==and[not[and[x3,x2]],x1]`

Proof

We start by taking Critical Pair Lemma 100, and apply the substitution:

`not[or[and[x1_,x2_],x3_]]→and[nand[x1,x2],not[x3]]`

which follows from Critical Pair Lemma 17.

Substitution Lemma 97

It can be shown that:

`and[x1,and[nand[x2,x3],nand[x3,x2]]]==and[not[and[x3,x2]],x1]`

Proof

We start by taking Substitution Lemma 96, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 6.

Substitution Lemma 98

It can be shown that:

`and[x1,nand[x2,x3]]==and[not[and[x3,x2]],x1]`

Proof

We start by taking Substitution Lemma 97, and apply the substitution:

We start by taking Substitution Lemma 97, and apply the substitution:

`and[nand[x1_,x2_],nand[x2_,x1_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 88.

Substitution Lemma 99

It can be shown that:

`and[x1,nand[x2,x3]]==and[nand[x3,x2],x1]`

Proof

We start by taking Substitution Lemma 98, and apply the substitution:

`not[and[x1_,x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 6.

Critical Pair Lemma 101

The following expressions are equivalent:

`nand[x1,nand[x2,x3]]==nand[x1,and[nand[x3,x2],x1]]`

Proof

Note that the input for the rule:

`nand[x1_,and[x1_,x2_]]→nand[x1,x2]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,nand[x2_,x3_]]↔and[nand[x3_,x2_],x1_]`

where these rules follow from Substitution Lemma 26 and Substitution Lemma 99 respectively.

Substitution Lemma 100

It can be shown that:

`nand[x1,nand[x2,x3]]==or[and[x3,x2],nand[x1,x1]]`

Proof

We start by taking Critical Pair Lemma 101, and apply the substitution:

`nand[x1_,and[nand[x2_,x3_],x4_]]→or[and[x2,x3],nand[x1,x4]]`

which follows from Critical Pair Lemma 31.

Substitution Lemma 101

It can be shown that:

`nand[x1,nand[x2,x3]]==or[and[x3,x2],not[x1]]`

Proof

We start by taking Substitution Lemma 100, and apply the substitution:

`nand[x1_,x1_]→not[x1]`

which follows from Axiom 2.

Substitution Lemma 102

It can be shown that:

It can be shown that:

`nand[x1,nand[x2,x3]]==nand[nand[x3,x2],x1]`

PROOF

We start by taking Substitution Lemma 101, and apply the substitution:

`or[and[x1_,x2_],not[x3_]]→nand[nand[x1,x2],x3]`

which follows from Critical Pair Lemma 12.

Critical Pair Lemma 102

The following expressions are equivalent:

`nand[nand[x1,x2],not[x3]]==or[x3,not[nand[x2,x1]]]`

PROOF

Note that the input for the rule:

`nand[x1_,nand[x2_,x3_]]↔nand[nand[x3_,x2_],x1_]`

contains a subpattern of the form:

`nand[x1_,nand[x2_,x3_]]`

which can be unified with the input for the rule:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

where these rules follow from Substitution Lemma 102 and Critical Pair Lemma 13 respectively.

Substitution Lemma 103

It can be shown that:

`or[not[nand[x1,x2]],x3]==or[x3,not[nand[x2,x1]]]`

PROOF

We start by taking Critical Pair Lemma 102, and apply the substitution:

`nand[x1_,not[x2_]]→or[not[x1],x2]`

which follows from Critical Pair Lemma 7.

Substitution Lemma 104

It can be shown that:

`or[and[x1,x2],x3]==or[x3,not[nand[x2,x1]]]`

PROOF

We start by taking Substitution Lemma 103, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Substitution Lemma 105

It can be shown that:

`or[and[x1,x2],x3]==or[x3,and[x2,x1]]`

PROOF

We start by taking Substitution Lemma 104, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Critical Pair Lemma 103

The following expressions are equivalent:

`nand[x1,or[x1,x2]]==nand[or[x1,x2],or[x1,not[or[x3,x2]]]]`

Proof

Note that the input for the rule:

`nand[x1_,or[x2_,not[or[x3_,x1_]]]]→nand[x2,x1]`

contains a subpattern of the form:

`or[x2_,not[or[x3_,x1_]]]`

which can be unified with the input for the rule:

`or[x1_,not[or[x2_,or[x1_,x3_]]]]→or[x1,not[or[x2,x3]]]`

where these rules follow from Substitution Lemma 91 and Critical Pair Lemma 84 respectively.

Substitution Lemma 106

It can be shown that:

`not[x1]==nand[or[x1,x2],or[x1,not[or[x3,x2]]]]`

Proof

We start by taking Critical Pair Lemma 103, and apply the substitution:

`nand[x1_,or[x1_,x2_]]→not[x1]`

which follows from Critical Pair Lemma 21.

Critical Pair Lemma 104

The following expressions are equivalent:

`not[not[x1]]==nand[nand[x1,x2],or[not[x1],not[or[x3,not[x2]]]]]`

Proof

Note that the input for the rule:

`nand[or[x1_,x2_],or[x1_,not[or[x3_,x2_]]]]→not[x1]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[not[x1_],not[x2_]]→nand[x1,x2]`

where these rules follow from Substitution Lemma 106 and Critical Pair Lemma 10 respectively.

Substitution Lemma 107

It can be shown that:

`x1==nand[nand[x1,x2],or[not[x1],not[or[x3,not[x2]]]]]`

Proof

We start by taking Critical Pair Lemma 104, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 12.

Substitution Lemma 108

It can be shown that:

`x1==nand[nand[x1,x2],nand[x1,or[x3,not[x2]]]]`

Proof

We start by taking Substitution Lemma 107, and apply the substitution:

`or[not[x1_],not[x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 10.

Substitution Lemma 109

It can be shown that:

`x1==or[and[x1,x2],and[x1,or[x3,not[x2]]]]`

Proof

We start by taking Substitution Lemma 108, and apply the substitution:

`nand[nand[x1_,x2_],nand[x3_,x4_]]→or[and[x1,x2],and[x3,x4]]`

which follows from Critical Pair Lemma 29.

Critical Pair Lemma 105

The following expressions are equivalent:

`x1==or[and[x1,x2],and[x1,or[not[x2],not[x3]]]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],and[x1_,or[x3_,not[x2_]]]]→x1`

contains a subpattern of the form:

`and[x1_,or[x3_,not[x2_]]]`

which can be unified with the input for the rule:

`and[x1_,or[nand[x1_,x2_],x3_]]→and[x1,or[x3,not[x2]]]`

where these rules follow from Substitution Lemma 109 and Critical Pair Lemma 95 respectively.

Substitution Lemma 110

It can be shown that:

`x1==or[and[x1,x2],and[x1,nand[x2,x3]]]`

Proof

We start by taking Critical Pair Lemma 105, and apply the substitution:

`or[not[x1_],not[x2_]]→nand[x1,x2]`

which follows from Critical Pair Lemma 10.

Critical Pair Lemma 106

The following expressions are equivalent:

`x1==or[and[x2,x1],and[x1,nand[x2,x3]]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],and[x1_,nand[x2_,x3_]]]→x1`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,x2_]↔and[x2_,x1_]`

where these rules follow from Substitution Lemma 110 and Substitution Lemma 49 respectively.

Critical Pair Lemma 107

The following expressions are equivalent:

`x1==or[and[x1,or[x2,x3]],and[x1,not[x3]]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],and[x1_,nand[x2_,x3_]]]→x1`

contains a subpattern of the form:

`nand[x2_,x3_]`

which can be unified with the input for the rule:

`nand[or[x1_,x2_],or[not[x1_],x2_]]→not[x2]`

where these rules follow from Substitution Lemma 110 and Substitution Lemma 83 respectively.

Critical Pair Lemma 108

The following expressions are equivalent:

`x1==or[and[x2,x1],and[nand[x3,x2],x1]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],and[x2_,nand[x1_,x3_]]]→x2`

contains a subpattern of the form:

`and[x2_,nand[x1_,x3_]]`

which can be unified with the input for the rule:

`and[x1_,nand[x2_,x3_]]↔and[nand[x3_,x2_],x1_]`

where these rules follow from Critical Pair Lemma 106 and Substitution Lemma 99 respectively.

Critical Pair Lemma 109

The following expressions are equivalent:

`not[or[and[x1,x2],and[nand[x3,x1],x2]]]==nand[x2,or[and[nand[x3,x1],x2],or[x4,and[x1,x`

Proof

Note that the input for the rule:

```
nand[or[x1_,x2_],or[x2_,or[x3_,x1_]]]→not[or[x1,x2]]
```

contains a subpattern of the form:

```
or[x1_,x2_]
```

which can be unified with the input for the rule:

```
or[and[x1_,x2_],and[nand[x3_,x1_],x2_]]→x2
```

where these rules follow from Critical Pair Lemma 93 and Critical Pair Lemma 108 respectively.

Substitution Lemma 111

It can be shown that:

```
and[nand[x1,x2],not[and[nand[x3,x1],x2]]]==nand[x2,or[and[nand[x3,x1],x2],or[x4,and[x1
```

Proof

We start by taking Critical Pair Lemma 109, and apply the substitution:

```
not[or[and[x1_,x2_],x3_]]→and[nand[x1,x2],not[x3]]
```

which follows from Critical Pair Lemma 17.

Substitution Lemma 112

It can be shown that:

```
and[nand[x1,x2],nand[nand[x3,x1],x2]]==nand[x2,or[and[nand[x3,x1],x2],or[x4,and[x1,x2]
```

Proof

We start by taking Substitution Lemma 111, and apply the substitution:

```
not[and[x1_,x2_]]→nand[x1,x2]
```

which follows from Critical Pair Lemma 6.

Substitution Lemma 113

It can be shown that:

```
and[nand[x1,x2],nand[nand[x3,x1],x2]]==nand[x2,or[or[x4,and[x1,x2]],nand[x3,x1]]]
```

Proof

We start by taking Substitution Lemma 112, and apply the substitution:

```
nand[x1_,or[and[x2_,x1_],x3_]]→nand[x1,or[x3,x2]]
```

which follows from Critical Pair Lemma 99.

Substitution Lemma 114

It can be shown that:

```
and[nand[x1,x2],nand[nand[x3,x1],x2]]==nand[x2,or[x4,or[nand[x3,x1],and[x1,x2]]]]
```

Proof

We start by taking Substitution Lemma 113, and apply the substitution:

```
or[or[x1_,x2_],x3_]→or[x1,or[x3,x2]]
```

which follows from Substitution Lemma 72.

Substitution Lemma 115

It can be shown that:

```
and[nand[x1,x2],nand[nand[x3,x1],x2]]==nand[x2,or[x4,or[nand[x3,x1],x2]]]
```

Proof

We start by taking Substitution Lemma 114, and apply the substitution:

```
or[nand[x1_,x2_],and[x2_,x3_]]→or[nand[x1,x2],x3]
```

which follows from Critical Pair Lemma 87.

Substitution Lemma 116

It can be shown that:

```
and[nand[x1,x2],nand[nand[x3,x1],x2]]==not[x2]
```

Proof

We start by taking Substitution Lemma 115, and apply the substitution:

```
nand[x1_,or[x2_,or[x3_,x1_]]]→not[x1]
```

which follows from Critical Pair Lemma 77.

Critical Pair Lemma 110

The following expressions are equivalent:

```
not[not[x1]]==and[or[not[x2],x1],nand[nand[x3,x2],not[x1]]]
```

Proof

Note that the input for the rule:

```
and[nand[x1_,x2_],nand[nand[x3_,x1_],x2_]]→not[x2]
```

contains a subpattern of the form:

```
nand[x1_,x2_]
```

which can be unified with the input for the rule:

```
nand[x1_,not[x2_]]→or[not[x1],x2]
```

where these rules follow from Substitution Lemma 116 and Critical Pair Lemma 7 respectively.

Substitution Lemma 117

It can be shown that:

```
x1==and[or[not[x2],x1],nand[nand[x3,x2],not[x1]]]
```

Proof

We start by taking Critical Pair Lemma 110, and apply the substitution:

```
not[not[x1_]]→x1
```

which follows from Substitution Lemma 12.

Substitution Lemma 118

It can be shown that:

```
x1==and[or[not[x2],x1],or[not[nand[x3,x2]],x1]]
```

Proof

We start by taking Substitution Lemma 117, and apply the substitution:

```
nand[x1_,not[x2_]]→or[not[x1],x2]
```

which follows from Critical Pair Lemma 7.

which follows from Critical Pair Lemma 7.

Substitution Lemma 119

It can be shown that:

`x1==and[or[not[x2],x1],or[and[x3,x2],x1]]`

Proof

We start by taking Substitution Lemma 118, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Substitution Lemma 120

It can be shown that:

`x1==or[and[x1,not[x2]],and[or[x3,x2],x1]]`

Proof

We start by taking Critical Pair Lemma 107, and apply the substitution:

`or[and[x1_,x2_],x3_]→or[x3,and[x2,x1]]`

which follows from Substitution Lemma 105.

Critical Pair Lemma 111

The following expressions are equivalent:

`and[or[x1,x2],x3]==and[or[not[not[x2]],and[or[x1,x2],x3]],x3]`

Proof

Note that the input for the rule:

`and[or[not[x1_],x2_],or[and[x3_,x1_],x2_]]→x2`

contains a subpattern of the form:

`or[and[x3_,x1_],x2_]`

which can be unified with the input for the rule:

`or[and[x1_,not[x2_]],and[or[x3_,x2_],x1_]]→x1`

where these rules follow from Substitution Lemma 119 and Substitution Lemma 120 respectively.

Substitution Lemma 121

It can be shown that:

`and[or[x1,x2],x3]==and[nand[not[x2],nand[or[x1,x2],x3]],x3]`

Proof

We start by taking Critical Pair Lemma 111, and apply the substitution:

`or[not[x1_],and[x2_,x3_]]→nand[x1,nand[x2,x3]]`

which follows from Critical Pair Lemma 8.

Substitution Lemma 122

It can be shown that:

`and[or[x1,x2],x3]==and[or[x2,not[nand[or[x1,x2],x3]]],x3]`

Proof

PROOF

We start by taking Substitution Lemma 121, and apply the substitution:

`nand[not[x1_],x2_]→or[x1,not[x2]]`

which follows from Critical Pair Lemma 13.

Substitution Lemma 123

It can be shown that:

`and[or[x1,x2],x3]==and[or[x2,and[or[x1,x2],x3]],x3]`

PROOF

We start by taking Substitution Lemma 122, and apply the substitution:

`not[nand[x1_,x2_]]→and[x1,x2]`

which follows from Axiom 6.

Substitution Lemma 124

It can be shown that:

`and[or[x1,x2],x3]==and[or[x2,and[x3,x1]],x3]`

PROOF

We start by taking Substitution Lemma 123, and apply the substitution:

`or[x1_,and[or[x2_,x1_],x3_]]→or[x1,and[x3,x2]]`

which follows from Substitution Lemma 90.

Substitution Lemma 125

It can be shown that:

`and[or[x1,x2],x3]==and[x3,or[x2,and[x3,x1]]]`

PROOF

We start by taking Substitution Lemma 124, and apply the substitution:

`and[x1_,x2_]→and[x2,x1]`

which follows from Substitution Lemma 49.

Substitution Lemma 126

It can be shown that:

`and[or[x1,x2],x3]==and[x3,or[x2,x1]]`

PROOF

We start by taking Substitution Lemma 125, and apply the substitution:

`and[x1_,or[x2_,and[x1_,x3_]]]→and[x1,or[x2,x3]]`

which follows from Substitution Lemma 94.

Substitution Lemma 127

It can be shown that:

`and[x1,or[x2,x3]]==or[and[x3,x1],and[or[x2,x3],x1]]`

PROOF

We start by taking Critical Pair Lemma 73, and apply the substitution:

`or[and[x1_,x2_],x3_]→or[x3,and[x2,x1]]`

which follows from Substitution Lemma 105.

Critical Pair Lemma 112

The following expressions are equivalent:

`and[or[x1,x2],or[x3,x2]]==or[x2,and[or[x3,x2],or[x1,x2]]]`

Proof

Note that the input for the rule:

`or[and[x1_,x2_],and[or[x3_,x1_],x2_]]→and[x2,or[x3,x1]]`

contains a subpattern of the form:

`and[x1_,x2_]`

which can be unified with the input for the rule:

`and[x1_,or[x2_,x1_]]→x1`

where these rules follow from Substitution Lemma 127 and Critical Pair Lemma 42 respectively.

Substitution Lemma 128

It can be shown that:

`and[or[x1,x2],or[x3,x2]]==or[x2,and[or[x1,x2],x3]]`

Proof

We start by taking Critical Pair Lemma 112, and apply the substitution:

`or[x1_,and[or[x2_,x1_],x3_]]→or[x1,and[x3,x2]]`

which follows from Substitution Lemma 90.

Substitution Lemma 129

It can be shown that:

`and[or[x1,x2],or[x3,x2]]==or[x2,and[x3,x1]]`

Proof

We start by taking Substitution Lemma 128, and apply the substitution:

`or[x1_,and[or[x2_,x1_],x3_]]→or[x1,and[x3,x2]]`

which follows from Substitution Lemma 90.

Substitution Lemma 130

It can be shown that:

`and[or[a,b],or[c,a]]==or[a,and[b,c]]`

Proof

We start by taking Hypothesis 1, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Substitution Lemma 60.

Substitution Lemma 131

It can be shown that:

```
and[or[c,a],or[b,a]]==or[a,and[b,c]]
```

PROOF

We start by taking Substitution Lemma 130, and apply the substitution:

```
and[or[x1_,x2_],x3_]→and[x3,or[x2,x1]]
```

which follows from Substitution Lemma 126.

Conclusion 1

We obtain the conclusion:

```
True
```

PROOF

Take Substitution Lemma 131, and apply the substitution:

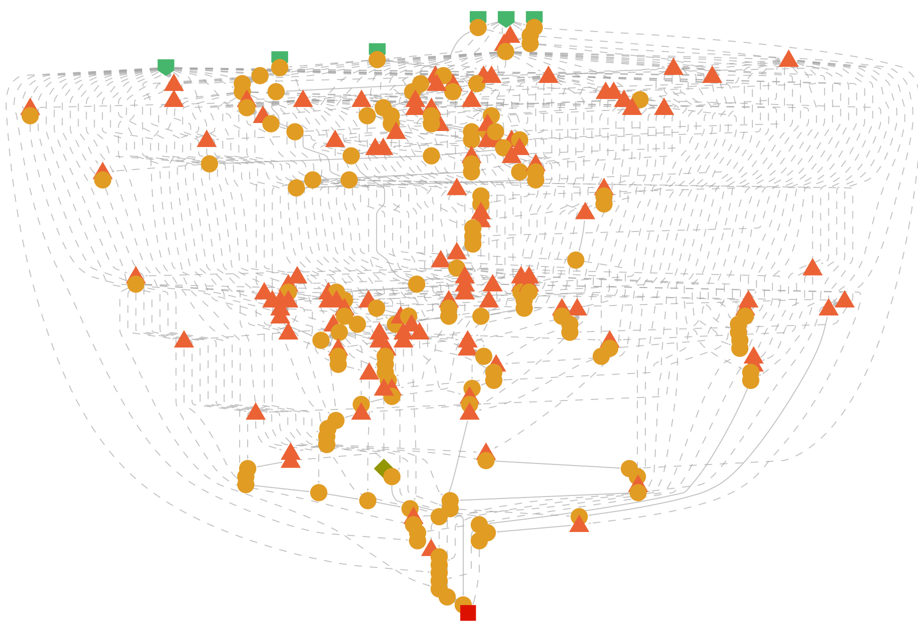
```
and[or[x1_,x2_],or[x3_,x2_]]→or[x2,and[x3,x1]]
```

which follows from Substitution Lemma 129.

large output | show less | show more | show all | set size limit...

In[◦]:=
```
proofAxB6fromShort["ProofGraph"]
```

Out[◦]=



In[◦]:=
```
Clear[proofAxB6fromShort]
```

Appendix 13. Derivation of the Law of Double Negation from Robbins logic

In[∘]:= `proofDNfromRobbins["ProofNotebook"]`

Axiom 1

We are given that:

`x1==not[or[not[or[x1,x2]],not[or[x1,not[x2]]]]]`

Axiom 2

We are given that:

`or[x1,x2]==or[x2,x1]`

Axiom 3

We are given that:

`or[x1,or[x2,x3]]==or[or[x1,x2],x3]`

Axiom 4

We are given that:

`or[x1,not[x1]]==1`

Hypothesis 1

We would like to show that:

`not[not[a]]==a`

Critical Pair Lemma 1

The following expressions are equivalent:

`not[or[x1,x2]]==not[or[x1,not[or[not[or[x1,x2]],not[not[or[x1,not[x2]]]]]]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[x1_,not[x2_]]]]]→x1`

contains a subpattern of the form:

`not[or[x1_,x2_]]`

which can be unified with the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[x1_,not[x2_]]]]]→x1`

where these rules follow from Axiom 1 and Axiom 1 respectively.

Critical Pair Lemma 2

The following expressions are equivalent:

`1==or[x1,or[x2,not[or[x1,x2]]]]`

Proof

Note that the input for the rule:

`or[x1_,not[x1_]]→1`

contains a subpattern of the form:

`or[x1_,not[x1_]]`

or[x1_,not[x1_]]

which can be unified with the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

where these rules follow from Axiom 4 and Axiom 3 respectively.

Critical Pair Lemma 3

The following expressions are equivalent:

`or[x1,or[not[x1],x2]]==or[1,x2]`

Proof

Note that the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,not[x1_]]→1`

where these rules follow from Axiom 3 and Axiom 4 respectively.

Critical Pair Lemma 4

The following expressions are equivalent:

`x1==not[or[not[or[x1,x1]],not[1]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[x1_,not[x2_]]]]]→x1`

contains a subpattern of the form:

`or[x1_,not[x2_]]`

which can be unified with the input for the rule:

`or[x1_,not[x1_]]→1`

where these rules follow from Axiom 1 and Axiom 4 respectively.

Critical Pair Lemma 5

The following expressions are equivalent:

`or[1,not[not[x1]]]==or[x1,1]`

Proof

Note that the input for the rule:

`or[x1_,or[not[x1_],x2_]]→or[1,x2]`

contains a subpattern of the form:

`or[not[x1_],x2_]`

which can be unified with the input for the rule:

`or[x1_,not[x1_]]→1`

where these rules follow from Critical Pair Lemma 3 and Axiom 4 respectively.

Critical Pair Lemma 6

The following expressions are equivalent:

The following expressions are equivalent:

`or[1,x1]⩵or[x2,or[x1,not[x2]]]`

Proof

Note that the input for the rule:

`or[x1_,or[not[x1_],x2_]]→or[1,x2]`

contains a subpattern of the form:

`or[not[x1_],x2_]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Critical Pair Lemma 3 and Axiom 2 respectively.

Critical Pair Lemma 7

The following expressions are equivalent:

`or[1,1]⩵or[not[x1],or[x1,1]]`

Proof

Note that the input for the rule:

`or[x1_,or[x2_,not[x1_]]]→or[1,x2]`

contains a subpattern of the form:

`or[x2_,not[x1_]]`

which can be unified with the input for the rule:

`or[1,not[not[x1_]]]→or[x1,1]`

where these rules follow from Critical Pair Lemma 6 and Critical Pair Lemma 5 respectively.

Critical Pair Lemma 8

The following expressions are equivalent:

`or[1,or[x1,x2]]⩵or[x3,or[x1,or[x2,not[x3]]]]`

Proof

Note that the input for the rule:

`or[x1_,or[x2_,not[x1_]]]→or[1,x2]`

contains a subpattern of the form:

`or[x2_,not[x1_]]`

which can be unified with the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

where these rules follow from Critical Pair Lemma 6 and Axiom 3 respectively.

Critical Pair Lemma 9

The following expressions are equivalent:

`or[1,1]⩵or[not[x1],or[1,x1]]`

Proof

Note that the input for the rule:

`or[not[x1_],or[x1_,1]]→or[1,1]`

contains a subpattern of the form:

contains a subpattern of the form:

`or[x1_,1]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Critical Pair Lemma 7 and Axiom 2 respectively.

Critical Pair Lemma 10

The following expressions are equivalent:

`or[not[x1],or[or[1,x1],x2]]==or[or[1,1],x2]`

Proof

Note that the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[not[x1_],or[1,x1_]]→or[1,1]`

where these rules follow from Axiom 3 and Critical Pair Lemma 9 respectively.

Substitution Lemma 1

It can be shown that:

`or[not[x1],or[1,or[x1,x2]]]==or[or[1,1],x2]`

Proof

We start by taking Critical Pair Lemma 10, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Substitution Lemma 2

It can be shown that:

`or[not[x1],or[1,or[x1,x2]]]==or[1,or[1,x2]]`

Proof

We start by taking Substitution Lemma 1, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Substitution Lemma 3

It can be shown that:

`x1==not[or[not[1],not[or[x1,x1]]]]`

Proof

We start by taking Critical Pair Lemma 4, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Critical Pair Lemma 11

The following expressions are equivalent:

`or[1,1]==or[x1,or[1,or[not[1],not[or[x1,x1]]]]]`

PROOF

Note that the input for the rule:

`or[not[x1_],or[1,x1_]]→or[1,1]`

contains a subpattern of the form:

`not[x1_]`

which can be unified with the input for the rule:

`not[or[not[1],not[or[x1_,x1_]]]]→x1`

where these rules follow from Critical Pair Lemma 9 and Substitution Lemma 3 respectively.

Substitution Lemma 4

It can be shown that:

`or[1,1]==or[x1,or[1,not[or[x1,x1]]]]`

PROOF

We start by taking Critical Pair Lemma 11, and apply the substitution:

`or[x1_,or[not[x1_],x2_]]→or[1,x2]`

which follows from Critical Pair Lemma 3.

Critical Pair Lemma 12

The following expressions are equivalent:

`or[1,1]==1`

PROOF

Note that the input for the rule:

`or[x1_,or[1,not[or[x1_,x1_]]]]→or[1,1]`

contains a subpattern of the form:

`or[x1_,or[1,not[or[x1_,x1_]]]]`

which can be unified with the input for the rule:

`or[x1_,or[x2_,not[or[x1_,x2_]]]]→1`

where these rules follow from Substitution Lemma 4 and Critical Pair Lemma 2 respectively.

Critical Pair Lemma 13

The following expressions are equivalent:

`or[1,or[x1,1]]==or[or[x1,x1],or[1,1]]`

PROOF

Note that the input for the rule:

`or[x1_,or[x2_,or[x3_,not[x1_]]]]→or[1,or[x2,x3]]`

contains a subpattern of the form:

`or[x2_,or[x3_,not[x1_]]]`

which can be unified with the input for the rule:

`or[x1_,or[1,not[or[x1_,x1_]]]]→or[1,1]`

where these rules follow from Critical Pair Lemma 8 and Substitution Lemma 4 respectively.

Substitution Lemma 5

It can be shown that:

`or[1,or[x1,1]]==or[x1,or[x1,or[1,1]]]`

Proof

We start by taking Critical Pair Lemma 13, and apply the substitution:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

which follows from Axiom 3.

Critical Pair Lemma 14

The following expressions are equivalent:

`or[1,or[1,x1]]==or[1,x1]`

Proof

Note that the input for the rule:

`or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[1,1]→1`

where these rules follow from Axiom 3 and Critical Pair Lemma 12 respectively.

Critical Pair Lemma 15

The following expressions are equivalent:

`1==not[or[not[1],not[1]]]`

Proof

Note that the input for the rule:

`not[or[not[1],not[or[x1_,x1_]]]]→x1`

contains a subpattern of the form:

`or[x1_,x1_]`

which can be unified with the input for the rule:

`or[1,1]→1`

where these rules follow from Substitution Lemma 3 and Critical Pair Lemma 12 respectively.

Substitution Lemma 6

It can be shown that:

`or[not[x1_],or[1,or[x1_,x2_]]]→or[1,x2]`

Proof

We start by taking Substitution Lemma 2, and apply the substitution:

`or[1,or[1,x1_]]→or[1,x1]`

which follows from Critical Pair Lemma 14.

Out[ ]=

Critical Pair Lemma 16

The following expressions are equivalent:

`or[1,not[not[x1]]]==or[1,or[x1,1]]`

Proof

Note that the input for the rule:

`or[1,or[1,x1_]]→or[1,x1]`

contains a subpattern of the form:

`or[1,x1_]`

which can be unified with the input for the rule:

`or[1,not[not[x1_]]]→or[x1,1]`

where these rules follow from Critical Pair Lemma 14 and Critical Pair Lemma 5 respectively.

Substitution Lemma 7

It can be shown that:

`or[x1,1]==or[1,or[x1,1]]`

Proof

We start by taking Critical Pair Lemma 16, and apply the substitution:

`or[1,not[not[x1_]]]→or[x1,1]`

which follows from Critical Pair Lemma 5.

Substitution Lemma 8

It can be shown that:

`or[x1,1]==or[x1,or[x1,or[1,1]]]`

Proof

We start by taking Substitution Lemma 5, and apply the substitution:

`or[1,or[x1_,1]]→or[x1,1]`

which follows from Substitution Lemma 7.

Substitution Lemma 9

It can be shown that:

`or[x1,1]==or[x1,or[x1,1]]`

Proof

We start by taking Substitution Lemma 8, and apply the substitution:

`or[1,1]→1`

which follows from Critical Pair Lemma 12.

Critical Pair Lemma 17

The following expressions are equivalent:

`or[1,or[x1,1]]==or[not[x1],or[1,or[x1,1]]]`

Proof

Note that the input for the rule:

`Language`EquationalProofDump`getConstructRule[EquationalProof`ApplyLemma[641,or[1,or[1,x1`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,or[x1_,1]]→or[x1,1]`

where these rules follow from Substitution Lemma 6 and Substitution Lemma 9 respectively.

Substitution Lemma 10

It can be shown that:

`or[x1,1]==or[not[x1],or[1,or[x1,1]]]`

Proof

We start by taking Critical Pair Lemma 17, and apply the substitution:

`or[1,or[x1_,1]]→or[x1,1]`

which follows from Substitution Lemma 7.

Substitution Lemma 11

It can be shown that:

`or[x1,1]==or[1,1]`

Proof

We start by taking Substitution Lemma 10, and apply the substitution:

`or[not[x1_],or[1,or[x1_,x2_]]]→or[1,x2]`

which follows from Substitution Lemma 6.

Substitution Lemma 12

It can be shown that:

`or[x1,1]==1`

Proof

We start by taking Substitution Lemma 11, and apply the substitution:

`or[1,1]→1`

which follows from Critical Pair Lemma 12.

Critical Pair Lemma 18

The following expressions are equivalent:

`x1==not[or[not[1],not[or[x1,not[1]]]]]`

Proof

Note that the input for the rule:

`not[or[not[or[x1_,x2_]],not[or[x1_,not[x2_]]]]]→x1`

contains a subpattern of the form:

`or[x1_,x2_]`

which can be unified with the input for the rule:

`or[x1_,1]→1`

where these rules follow from Axiom 1 and Substitution Lemma 12 respectively.

Critical Pair Lemma 19

The following expressions are equivalent:

`x1==not[or[not[1],not[or[not[1],x1]]]]`

Proof

Note that the input for the rule:

`not[or[not[1],not[or[x1_,not[1]]]]]→x1`

contains a subpattern of the form:

`or[x1_,not[1]]`

which can be unified with the input for the rule:

`or[x1_,x2_]↔or[x2_,x1_]`

where these rules follow from Critical Pair Lemma 18 and Axiom 2 respectively.

Critical Pair Lemma 20

The following expressions are equivalent:

`not[not[1]]==not[or[not[1],not[1]]]`

Proof

Note that the input for the rule:

`not[or[not[1],not[or[not[1],x1_]]]]→x1`

contains a subpattern of the form:

`or[not[1],x1_]`

which can be unified with the input for the rule:

`or[x1_,not[x1_]]→1`

where these rules follow from Critical Pair Lemma 19 and Axiom 4 respectively.

Substitution Lemma 13

It can be shown that:

`not[not[1]]==1`

Proof

We start by taking Critical Pair Lemma 20, and apply the substitution:

`not[or[not[1],not[1]]]→1`

which follows from Critical Pair Lemma 15.

Critical Pair Lemma 21

The following expressions are equivalent:

`or[x1,not[or[not[1],x1]]]==not[or[not[1],not[1]]]`

Proof

Note that the input for the rule:

`not[or[not[1],not[or[not[1],x1_]]]]→x1`

contains a subpattern of the form:

`or[not[1],x1_]`

which can be unified with the input for the rule:

which can be unified with the input for the rule:

`or[x1_,or[x2_,not[or[x1_,x2_]]]]→1`

where these rules follow from Critical Pair Lemma 19 and Critical Pair Lemma 2 respectively.

Substitution Lemma 14

It can be shown that:

`or[x1,not[or[not[1],x1]]]==1`

Proof

We start by taking Critical Pair Lemma 21, and apply the substitution:

`not[or[not[1],not[1]]]→1`

which follows from Critical Pair Lemma 15.

Critical Pair Lemma 22

The following expressions are equivalent:

`1==or[not[or[x1,x1]],x1]`

Proof

Note that the input for the rule:

`or[x1_,not[or[not[1],x1_]]]→1`

contains a subpattern of the form:

`not[or[not[1],x1_]]`

which can be unified with the input for the rule:

`not[or[not[1],not[or[x1_,x1_]]]]→x1`

where these rules follow from Substitution Lemma 14 and Substitution Lemma 3 respectively.

Substitution Lemma 15

It can be shown that:

`1==or[x1,not[or[x1,x1]]]`

Proof

We start by taking Critical Pair Lemma 22, and apply the substitution:

`or[x1_,x2_]→or[x2,x1]`

which follows from Axiom 2.

Critical Pair Lemma 23

The following expressions are equivalent:

`not[or[x1,or[x1,x1]]]==not[or[x1,not[or[not[or[x1,or[x1,x1]]],not[not[1]]]]]]`

Proof

Note that the input for the rule:

`not[or[x1_,not[or[not[or[x1_,x2_]],not[not[or[x1_,not[x2_]]]]]]]]→not[or[x1,x2]]`

contains a subpattern of the form:

`or[x1_,not[x2_]]`

which can be unified with the input for the rule:

`or[x1_,not[or[x1_,x1_]]]→1`

where these rules follow from Critical Pair Lemma 1 and Substitution Lemma 15 respectively.

Substitution Lemma 16

It can be shown that:

`not[or[x1,or[x1,x1]]]==not[or[x1,not[or[not[or[x1,or[x1,x1]]],1]]]]`

Proof

We start by taking Critical Pair Lemma 23, and apply the substitution:

`not[not[1]]→1`

which follows from Substitution Lemma 13.

Substitution Lemma 17

It can be shown that:

`not[or[x1,or[x1,x1]]]==not[or[x1,not[1]]]`

Proof

We start by taking Substitution Lemma 16, and apply the substitution:

`or[x1_,1]→1`

which follows from Substitution Lemma 12.

Critical Pair Lemma 24

The following expressions are equivalent:

`or[not[1],not[1]]==not[or[not[1],not[or[not[1],not[1]]]]]`

Proof

Note that the input for the rule:

`not[or[not[1],not[or[not[1],x1_]]]]→x1`

contains a subpattern of the form:

`not[or[not[1],x1_]]`

which can be unified with the input for the rule:

`not[or[x1_,or[x1_,x1_]]]→not[or[x1,not[1]]]`

where these rules follow from Critical Pair Lemma 19 and Substitution Lemma 17 respectively.

Substitution Lemma 18

It can be shown that:

`or[not[1],not[1]]==not[1]`

Proof

We start by taking Critical Pair Lemma 24, and apply the substitution:

`not[or[not[1],not[or[not[1],x1_]]]]→x1`

which follows from Critical Pair Lemma 19.

Critical Pair Lemma 25

The following expressions are equivalent:

`or[not[1],or[not[1],x1]]==or[not[1],x1]`

Proof

Note that the input for the rule:

```
or[or[x1_,x2_],x3_]→or[x1,or[x2,x3]]
```

contains a subpattern of the form:

```
or[x1_,x2_]
```

which can be unified with the input for the rule:

```
or[not[1],not[1]]→not[1]
```

where these rules follow from Axiom 3 and Substitution Lemma 18 respectively.

Critical Pair Lemma 26

The following expressions are equivalent:

```
or[not[1],x1]==not[or[not[1],not[or[not[1],x1]]]]
```

Proof

Note that the input for the rule:

```
not[or[not[1],not[or[not[1],x1_]]]]→x1
```

contains a subpattern of the form:

```
or[not[1],x1_]
```

which can be unified with the input for the rule:

```
or[not[1],or[not[1],x1_]]→or[not[1],x1]
```

where these rules follow from Critical Pair Lemma 19 and Critical Pair Lemma 25 respectively.

Substitution Lemma 19

It can be shown that:

```
or[not[1],x1]==x1
```

Proof

We start by taking Critical Pair Lemma 26, and apply the substitution:

```
not[or[not[1],not[or[not[1],x1_]]]]→x1
```

which follows from Critical Pair Lemma 19.

Substitution Lemma 20

It can be shown that:

```
not[not[or[not[1],x1]]]==x1
```

Proof

We start by taking Critical Pair Lemma 19, and apply the substitution:

```
or[not[1],x1_]→x1
```

which follows from Substitution Lemma 19.

Substitution Lemma 21

It can be shown that:

```
not[not[x1]]==x1
```

Proof

We start by taking Substitution Lemma 20, and apply the substitution:

`or[not[1],x1_]→x1`

which follows from Substitution Lemma 19.

Conclusion 1

We obtain the conclusion:

`True`

Proof

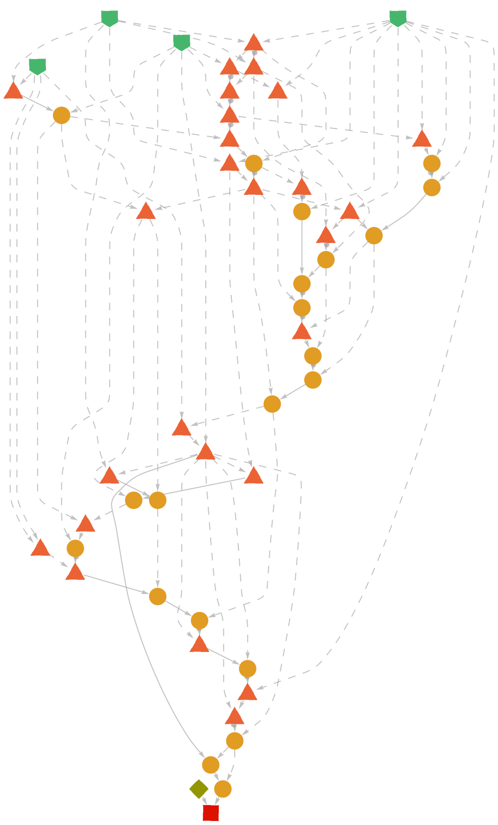
Take Hypothesis 1, and apply the substitution:

`not[not[x1_]]→x1`

which follows from Substitution Lemma 21.

In[∘]:= `proofDNfromRobbins["ProofGraph"]`

Out[∘]=



In[∘]:= `Clear[proofDNfromRobbins]`

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In[ ]:= ClearAll
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