Persistence of Plummer-Distributed Small Globular Clusters as a Function of Primordial-Binary Population Size

Jack K. Horner
Last modified: 3/4/2019 7:15 AM

Abstract

Globular stellar clusters are relatively common and are among the oldest known structures in galaxies. Most globular clusters observed to date are relatively large (~10^4 - 10^6 stars). In the absence of other influences, many if not all globular clusters continually lose mass as stars escape their gravitational hold. It has been hypothesized that the presence of primordial binaries helps to increase the persistence of small (~1000-star) globular clusters. Here I use N-body gravitational simulation of isotropic, Plummer-distributed, small globular clusters, with stellar evolution, to assess the persistence of such clusters as a function of initial populations of 100-500 primordial binaries (representing 0.1 - 0.5 of the initial cluster mass). The simulation predicts that in such clusters (a) star-loss is roughly linear in time up to ~300 Myr after t_0, (b) cluster persistence is, more or less, an increasing function of the fraction of primordial binaries at t_0, when such binaries account for 0.1-0.5 of the initial cluster mass.

Keywords: globular cluster, primordial binaries, N-body simulation

1.0 Introduction

1.1 Overview of globular clusters

Globular stellar clusters are roughly spherical, gravitationally bound collections of stars. A large fraction of the total mass of a globular cluster tends to be concentrated in a cluster "core".

Most globular clusters observed to date are relatively large (10,000 to 1,000,000 stars). They are relatively common in galactic halos: ~150 have been detected in the Milky Way ([25]); the galaxy M87 may contain ~13,000 ([24]).

Large globular clusters are among the oldest structures in many galaxies. If a globular cluster arises in a galaxy, therefore, its presence provides a rough lower bound on the age of that galaxy.[1]

In the absence of external influences, all globular clusters continually lose mass as stars escape their gravitational hold. Small clusters (~1000 stars) may not persist for more than ~500 Myr because their core does not exert a large enough gravitational force to retain stars of average mass and velocity.

---

[1] It is at least theoretically possible that some globular clusters have been captured by existing galaxies, but this mode of incorporation in a galaxy is considered to be rare.

Generally speaking, we would not expect small globular clusters to persist as long as larger clusters, especially near the center of the galaxy, where disruptive gravitational forces would be expected to be much larger than in parts of the galaxy distant from the galaxy core. Contrary to this expectation, however, the Hubble Space Telescope has provided evidence of small, relatively old, small globular clusters near the center of the Milky Way. The presence of these small, old clusters would seem to indicate the existence some distinctive gravitational regimes, or possibly, that these small clusters are relatively recent captures from structures that were once outside the Milky Way.

It has been conjectured that the persistence of small globular clusters may be enhanced by the presence of primordial binaries. This paper reports the results of an N-body-gravitational-simulator assessment of that hypothesis.

2.0 Method

2.1 Overview of the *nbody6* simulator

nbody6 ([1],[2]) is a highly parameterized gravitational N-body ([1]) simulator that has been widely used to simulate globular cluster evolution ([26]).

The *nbody6* equations of motion are derived and discussed extensively in [1].

In *nbody6*, single particles and center-of-mass systems are integrated by the neighbor scheme described in [6], using the fourth-order Hermite method ([7]).

Binaries and close two-body encounters are simulated in *nbody6* by the Stumpff version of Kustaanheimo–Stiefel ([8]) regularization ([9]), while interactions of compact subsystems are described by the chain regularization method ([10], [11], [12]). Strong interactions in unperturbed triples and quadruples are treated by three-body ([13]) and Heggie ([14]) global regularization ([15]). Hard triples and higher-order systems satisfying a stability criterion ([16],[17]) are reduced to two-body configurations (so-called mergers as opposed to collisions).

Several aspects of synthetic stellar evolution, mass loss, tidal circularization, and collisions are included in the simulator. Binary evolution and collisions ([18]), metallicities ([19]), Roche lobe overflow, and spin–orbit coupling are also included in the simulator.

Further details of the theory and methods implemented in *nbody6* can be found in [1] and [4].

The *nbody6* software is ~30,000 source lines, distributed over ~300 program units, of a non-standards-conforming variant of Fortran77 ([5]). By modern engineering standards ([23]), the *nbody6* detailed design, testing, and maintenance documentation is meager.

The GRAPE special-purpose supercomputer ([22]) was for several years the principal target platform for *nbody* development. Vestiges of GRAPE-oriented code remain in the *nbody6* baseline. The PC version of *nbody6* appears to lag the development of ports to other platforms.

Here, I use *nbody6* to investigate the effect of primordial binaries on the persistence of small globular clusters. N-body simulation of a globular cluster requires a specification of the initial mass distribution of the cluster. I assume an isotropic Plummer density distribution ([20]) at t_0. The isotropic Plummer density distribution is given by

$$\rho_P(r) = \left(\frac{3M}{4\pi a^3}\right)\left(1 + \frac{r^2}{a^2}\right)^{-\frac{5}{2}},$$

where *M* is the total mass of the cluster, and *a* is the *Plummer radius*, a scale parameter which sets the size of the cluster core.

Although the Plummer distribution diverges from the distribution of observed globular clusters, it is a mathematically convenient and plausible approximation of the distribution of ~80% of the mass of many observed globular clusters.

nbody6 source code ([2]) was obtained in April 2012 from the URL identified in [2] and ported to the *Vista*/64-bit-*Cygwin* environment ([21]). As distributed [2], the *nbody6* source code contains over 1000 instances of mixed-precision arithmetic. It also contains ~30 Fortran INTRINSIC function calls that violate [5]. Initial testing of executables built from [2] revealed tens of fatal numeric pathologies in the *Cygwin*/gfortran-compiler environment, all resulting from these problems. The source was accordingly re-engineered to eliminate mixed-precision arithmetic and to make the INTRINSIC function calls consistent with [5]. Inaccessible ("dead") code (~300 Fortran statements) was removed.

The makefile distributed at [2] has been maintained primarily support the platform described in [22]. Accordingly, the makefile was modified to be consistent with the *Cygwin*/gfortran compiler environment.

The re-engineered source was rebuilt from the re-engineered makefile.

Selected setup (input file) parameter values (in *nbody6* simulator units unless otherwise noted) for these experiments were (names in caps are input parameter names, as identified in file *define.f* in [2]):

- total wall clock time limit (TCOMP): 10 minutes
- initial number of single stars (N): 800, 700, 600, 500
- minimum number of particles (NCRIT): 5
- maximum number of neighbors (NNBMAX): 95
- time step for irregular force polynomial (ETAI): 0.02
- time step for regular force polynomial (ETAR): 0.03
- initial radius of neighborhood sphere (RS0): 0.3
- time interval for parameter adjustment (DTADJ): 2.0
- termination time (TCRIT): 500.0 *nbody* time units
- energy tolerance (QE): 2.0D-04
- virial cluster radius (RBAR): 1.0 pc
- mean mass in solar units (ZMBAR): 0.5
- assume isotropic Plummer distribution at t_0 (KZ5 = 1)

- use standard tidal field (KZ14 = 1)
- enable standard treatment of stable triples and quadruples (KZ15 = 1)
- enable updating of regularization parameters RMIN, DTMIN, and ECLOSE (KZ16 = 1)
- use Eggleton-Tout-Hurley mass-loss algorithm (KZ19 = 3)
- use Kroupa initial mass function (KZ20 = 4)
- enable logging of escaped stars to file ESC (KZ23 = 2)
- enable slow-down of two-body motion at chain level (KZ26 = 2)
- enable multiple regularization for all particles (KZ30 = 1)
- assume no unique density center (KZ39 = 1)
- time-step criterion for regularization search (DTMIN): 1.0E-05
- distance criterion for regularization search (RMIN): 1.0D-03
- regularized time-step parameter (ETAU): 0.2
- binding energy per unit mass for hard binary (ECLOSE): 1.0
- relative two-body perturbation for unperturbed motion (GMIN): 1.0E-06
- secondary termination parameter for soft KS binaries (GMAX): 0.001
- pre-scaling maximum particle mass (BODY1): 10.0
- pre-scaling minimum particle mass (BODYN): 0.2
- number of primordial binary systems (NBIN0): 200, 300, 400, 500
- number of primordial hierarchies (NHI0): 0
- metal abundance (ZMET): 0.02
- evolutionary epoch (EPOCH0): 0
- virial ratio (Q): 0.5
- maximum time step (SMAX): 1.0
- maximum semi-major axis (SEMI): 0.0005
- initial eccentricity (ECC): -1.0 (for thermal distribution)
- mass ratio (RATIO): 0.0
- range in SEMI for uniform logarithmic distribution (RANGE): 100.0

Given the above, all other parameters are ignored by *nbody6*, or were set to zero. These setups assume that the cluster does not interact with anything outside the cluster.

Preliminary experiments showed that the simulator typically exceeded the QE maximum-energy-error, then terminated, for simulated times greater than ~500.0 simulator time units (~300 Myr) in the above configurations.[2]

An example of a full setup file is shown in Figure 1.

```
1 10.0 0
800 1 5 45 95 1
0.02 0.03 0.3  2.0 10.0 500.0 2.0D-04 1.0 0.5
0 0 1 0 1 0 6 3 0 0
0 1 0 1 1 1 0 1 3 4
1 0 2 0 0 2 0 0 0 1
0 0 0 0 0 0 0 0 0 1
0 0 0 0 0 0 0 0 0 0
```

[2] Whether exceeding this tolerance is an artifact of the numerical methods used in *nbody6*, or has some physical significance, or some combination of the two, has not, to my knowledge, been systematically investigated.

```
1.0E-05 1.0D-03 0.2 1.0 1.0E-06 0.001
2.3 10.0 0.2 200 0 0.02 0 10.0
0.5 0 0 0 1.0
0.0005 -1.0 0.0 100. 0 0 0
```

Figure 1. An example an *nbody6* setup file used in this study (the setup for 200 primordial binaries is shown).

See [1], [3], [4], and file *define.f* in [2] for a more detailed description of the input file.

The reengineered *nbody6* was executed on a Dell Inspiron 545 with an Intel Core2 Quad CPU Q8200 clocked at 2.33 GHz, with 8.00 GB RAM, under *Windows Vista HomePremium(SP2)/64-bit-Cygwin* ([21]).

nbody6 produces an "escaped star" log, ESC, one per setup, if parameter KZ23 ≥ 1. The data in the ESC files generated by the method described above were imported as space-delimited text to an Excel spreadsheet and graphed using Excel graphing functions (see Figure 2 in Section 3.0).

3.0 Results

Figure 2 shows stellar escapes vs. time as a function of number of primordial binaries, generated under the conditions described in Section 2.0.

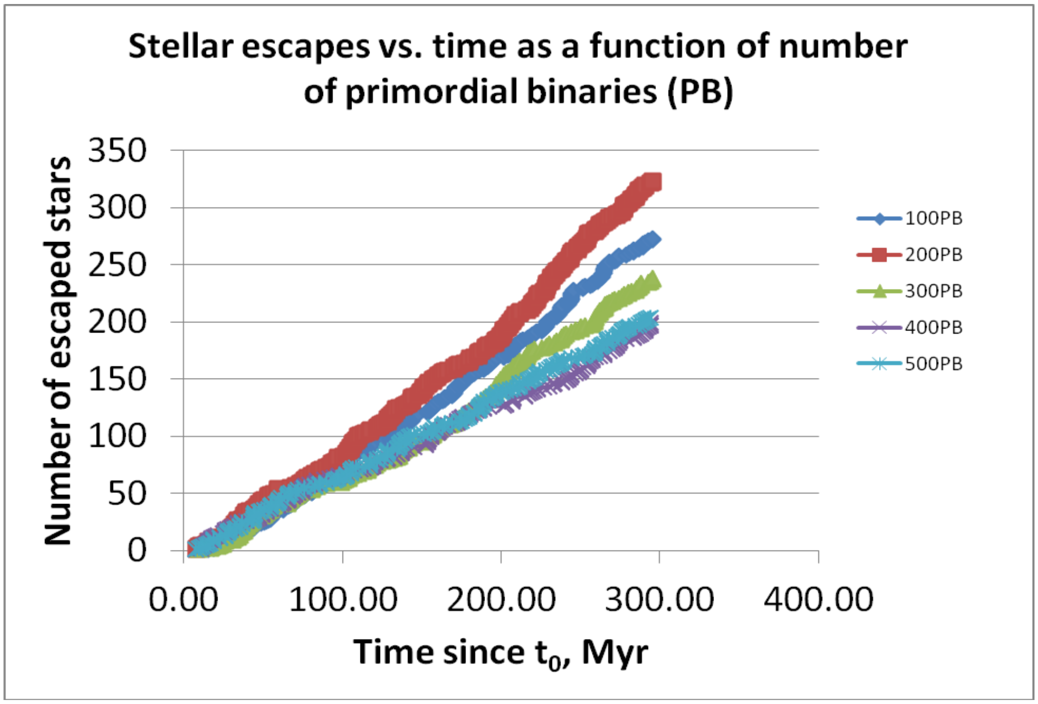

Figure 2. Stellar escapes vs. time as a function of number of primordial binaries in an isotropic, 1000-star, Plummer-distributed globular clusters, generated by the method described in Section 2.0.

Typical CPU utilization per setup on the platform described in Section 2.0 was 25% and typical memory utilization was 1.4 GB. Each setup required 5-9 minutes to complete on that platform.

4.0 Conclusions and discussion

The simulation described in Sections 2.0 and 3.0 predicts that a small globular cluster satisfying the conditions of Section 2.0 (not subject to interaction with anything outside the cluster)

(a) experiences star-loss at a rate that is roughly constant up to ~300 Myr after t_0,

(b) persists approximately as an increasing function of the fraction of primordial binaries at t_0, when such binaries account for 0.1-0.5 of the initial cluster mass.

The mechanism by which primordial binaries enhance the persistence of small globular clusters is not well understood. However, all else being the same, it requires twice as much energy to boost a binary system with mass 2M to escape velocity as it does to boost a single star of mass M to that velocity ([27], esp. Chap. 6).

5.0 Acknowledgements

This work benefited from discussions with Tony Pawlicki and Bill Spangenberg. I am also indebted to discussions with Dick Stutzke and Dave Davin, whose passion for astrophysics was an inspiration for all those privileged to have known them. For any errors that remain, I am solely responsible.

6.0 References

[1] Aarseth SJ. *Gravitational N-Body Simulations: Tools and Algorithms*. Cambridge. 2003. Note: in this document, the description of the *nbody* software distributed at the site identified in [2] have been significantly inconsistent since at least December 2011. [1] remains the best single description of the general physics implemented in *nbody6.*

[2] Aarseth SJ et al. *nbody6.* Circa April 2012. http://www.ast.cam.ac.uk/~sverre/web/pages/nbody.htm. Accessed 9 January 2019. Note: at least as early as December 2011, none of the sample input files distributed at this site would run with the version of *nbody6* distributed at this site. Sample input files that will run with the versions distributed between December 2011 and March 2013 are available from me on request.

[3] Aarseth SJ. *Introduction to Running Simulations with NBODY4 and NBODY6.* July 2006. http://www.ast.cam.ac.uk/~sverre/web/pages/nbody.htm. Note: this document is significantly inconsistent with [2]. Accessed 9 January 2019.

[4] Aarseth SJ. *NBODY6 User Manual.* December 2009. http://www.ast.cam.ac.uk/~sverre/web/pages/nbody.htm. Note: this document is significantly inconsistent with [2]. Accessed 9 January 2009.

[5] American National Standards Institute. *American National Standard Programming Language Fortran.* ANSI X3.9-1978. 1978.

[6] Ahmad A and Cohen L. A numerical integration scheme for the N-body gravitational problem. *Journal of Computational Physics* 12 (1973), 389–402.

[7] Makino J and Aarseth SJ. On a Hermite integrator with Ahmad–Cohen scheme for gravitational many-body problems. *Publications of the Astronomical Society of Japan* 44 (1992), 141–151.

[8] Kustaanheimo P and Stiefel E. Perturbation theory of Kepler motion based on spinor regularization. *Journal für die reine und angewandte Mathematik* 218 (1965), 204–219.

[9] Mikkola S and Aarseth SJ. An efficient integration method for binaries in N-body simulations. New Astronomy 3 (1998), 309–320.

[10] Mikkola S and Aarseth SJ. A chain regularization method for the few-body problem. *Celestial Mechanics and Dynamical Astronomy* 47 (1990), 375–390.

[11] Mikkola S and Aarseth SJ. An implementation of N-body chain regularization. *Celestial Mechanics and Dynamical Astronomy* 57 (1993), 439–459.

[12] Mikkola S and Aarseth SJ. A slow-down treatment for close binaries. *Celestial Mechanics and Dynamical Astronomy* 64 (1996), 197–208.

[13] Aarseth SJ and Zare K. A regularization of the three-body problem. *Celestial Mechanics* 10 (1974), 185–205.

[14] Heggie DC and Ramamani N. Approximate self-consistent models for tidally truncated star clusters. *Monthly Notices of the Royal Astronomical Society* 272 (1995), 317–322.

[15] Mikkola S. A practical and regular formulation of the N-body equations. *Monthly Notices of the Royal Astronomical Society* 215 (1985), 171–177.

[16] Mardling RA and Aarseth SJ. Dynamics and stability of three-body systems. In *The Dynamics of Small Bodies in the Solar System*, ed. BA Steves and A Roy. Kluwer. 1999. pp. 385–392.

[17] Mardling RA. The three-body problem in astrophysics. In SJ Aarseth, CA Tout, and RA Mardling, eds. *The Cambridge N-Body Lectures.* Springer. 2008. pp. 59-96.

[18] Tout CA, Aarseth SJ, Pols O, and Eggleton P. Rapid binary star evolution for N-body simulations and population synthesis. *Monthly Notices of the Royal Astronomical Society* 291 (1997), 732–748.

[19] Hurley JR, Pols OR, and Tout CA. Comprehensive analytical formulae for stellar evolution as a function of mass and metallicity. *Monthly Notices of the Royal Astronomical Society* 315 (2000), 543–569.

[20] Plummer HC. On the problem of distribution in globular star clusters. *Monthly Notices of the Royal Astronomical Society* 71 (1911), 460-470.

[21] *Cygwin* development team. *Cygwin* CYGWIN_nT-6.0-WOW64 v1.7.15 (0.260/5/3) i686. http://www.cygwin.com/. 2012.

[22] Makino J. Current status of the GRAPE Project. *Proceedings of the International Astronomical Union* 3 (1997), 457-466.

[23] International Standards Organization. *ISO/IEC 12207:2008. Systems and Software Engineering -- Software Life Cycle Processes.* http://www.iso.org/iso/iso_catalogue/catalogue_tc/catalogue_detail.htm?csnumber=43447. 2008. Accessed 9 January 2019.

[24] Kundu A and Whitmore BC. New insights from HST studies of globular cluster systems. I. Colors, distances, and specific frequencies of 28 elliptical galaxies. *The Astronomical Journal* 121 (June 2001), 2950–2973.

[25] Harris WE. *Catalog of Parameters for Milky Way Globular Clusters: The Database*. http://messier.seds.org/xtra/data/mwgc.dat.txt. December 2010. Accessed 9 January 2019.

[26] Aarseth SJ. From NBODY1 to NBODY6: The growth of an industry. *Publications of the Astronomical Society of the Pacific* 111 (November 1999), 1333-1346.

[27] Battin RH. *An Introduction to the Mathematics and Methods of Astrodynamics*. American Institute of Aeronautics and Astronautics. 1987